RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 You are given that matrix $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ and matrix $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$.

(i) Find **BA**. [2]

(ii) A plane shape of area 3 square units is transformed using matrix **A**. The image is transformed using matrix **B**. What is the area of the resulting shape? [3]

2 You are given that $\alpha = -3 + 4\text{j}$.

(i) Calculate α^2. [2]

(ii) Express α in modulus-argument form. [3]

3 **(i)** Show that $z = 3$ is a root of the cubic equation $z^3 + z^2 - 7z - 15 = 0$ and find the other roots. [5]

(ii) Show the roots on an Argand diagram. [2]

4 Using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{n} [(r+1)(r-2)] = \frac{1}{3}n(n^2 - 7)$. [6]

5 The equation $x^3 + px^2 + qx + r = 0$ has roots α, β and γ, where

$$\alpha + \beta + \gamma = 3,$$
$$\alpha\beta\gamma = -7,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 13.$$

(i) Write down the values of p and r. [2]

(ii) Find the value of q. [3]

6 A sequence is defined by $a_1 = 7$ and $a_{k+1} = 7a_k - 3$.

(i) Calculate the value of the third term, a_3. [2]

(ii) Prove by induction that $a_n = \dfrac{(13 \times 7^{n-1}) + 1}{2}$. [6]

Section B (36 marks)

7 The sketch below shows part of the graph of $y = \dfrac{x-1}{(x-2)(x+3)(2x+3)}$. One section of the graph has been omitted.

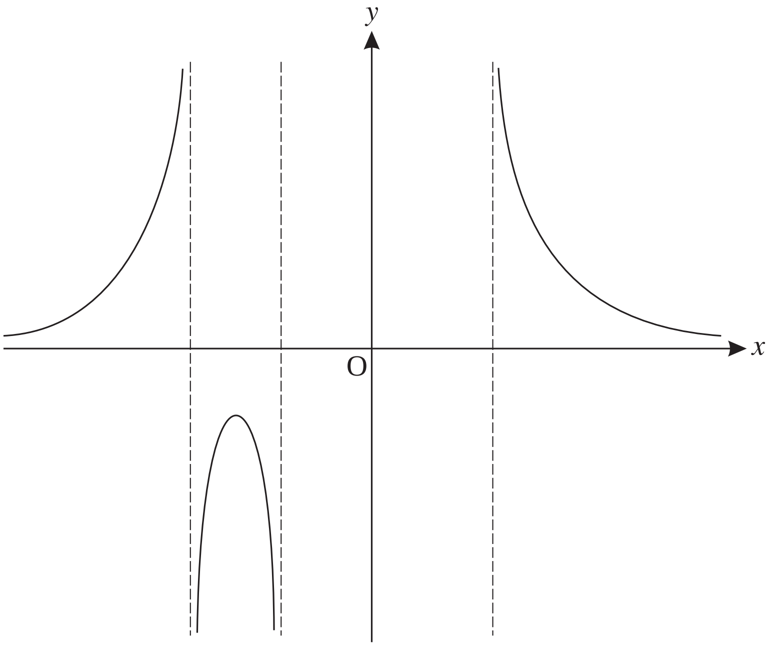

Fig. 7

(i) Find the coordinates of the points where the curve crosses the axes. [2]

(ii) Write down the equations of the three vertical asymptotes and the one horizontal asymptote. [4]

(iii) Copy the sketch and draw in the missing section. [2]

(iv) Solve the inequality $\dfrac{x-1}{(x-2)(x+3)(2x+3)} \geqslant 0$. [3]

8 **(i)** On a single Argand diagram, sketch the locus of points for which

(A) $|z - 3\text{j}| = 2$, [3]

(B) $\arg(z+1) = \frac{1}{4}\pi$. [3]

(ii) Indicate clearly on your Argand diagram the set of points for which

$$|z - 3\text{j}| \leqslant 2 \quad \text{and} \quad \arg(z+1) \leqslant \tfrac{1}{4}\pi. \qquad [2]$$

(iii) (A) By drawing an appropriate line through the origin, indicate on your Argand diagram the point for which $|z - 3\text{j}| = 2$ and $\arg z$ has its minimum possible value. [2]

(B) Calculate the value of $\arg z$ at this point. [2]

9 A transformation T acts on all points in the plane. The image of a general point P is denoted by P′. P′ always lies on the line $y = x$ and has the same x-coordinate as P. This is illustrated in Fig. 9.

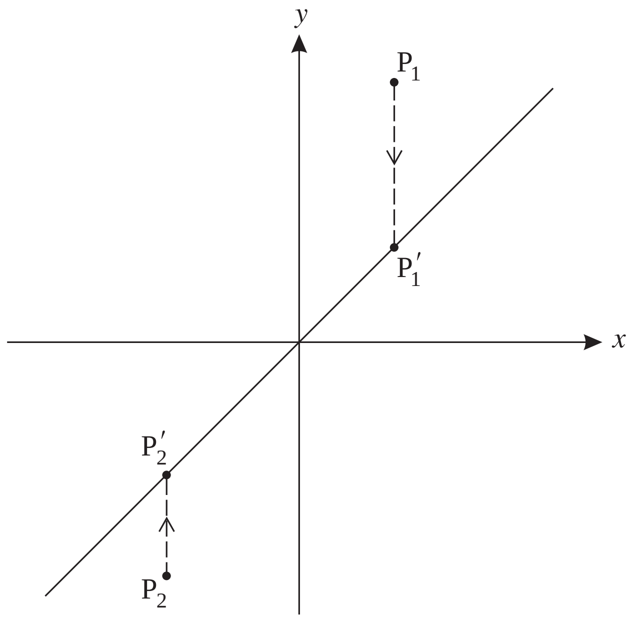

Fig. 9

(i) Write down the image of the point (−3, 7) under transformation T. [1]

(ii) Write down the image of the point (x, y) under transformation T. [2]

(iii) Find the 2×2 matrix which represents the transformation. [3]

(iv) Describe the transformation M represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [2]

(v) Find the matrix representing the composite transformation of T followed by M. [2]

(vi) Find the image of the point (x, y) under this composite transformation. State the equation of the line on which all of these images lie. [3]