OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4754/01A**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

WEDNESDAY 21 MAY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **4** printed pages.

Section A (36 marks)

1 Express $\dfrac{x}{x^2 - 4} + \dfrac{2}{x + 2}$ as a single fraction, simplifying your answer. [3]

2 Fig. 2 shows the curve $y = \sqrt{1 + e^{2x}}$.

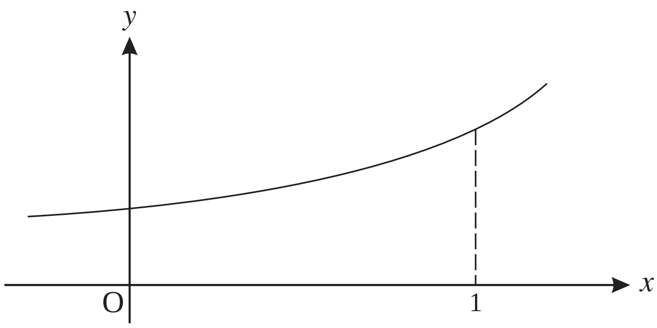

Fig. 2

The region bounded by the curve, the x-axis, the y-axis and the line $x = 1$ is rotated through 360° about the x-axis.

Show that the volume of the solid of revolution produced is $\frac{1}{2}\pi(1 + e^2)$. [4]

3 Solve the equation $\cos 2\theta = \sin\theta$ for $0 \leqslant \theta \leqslant 2\pi$, giving your answers in terms of π. [7]

4 Given that $x = 2\sec\theta$ and $y = 3\tan\theta$, show that $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$. [3]

5 A curve has parametric equations $x = 1 + u^2$, $y = 2u^3$.

(i) Find $\dfrac{dy}{dx}$ in terms of u. [3]

(ii) Hence find the gradient of the curve at the point with coordinates (5, 16). [2]

6 **(i)** Find the first three non-zero terms of the binomial series expansion of $\dfrac{1}{\sqrt{1 + 4x^2}}$, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the first three non-zero terms of the series expansion of $\dfrac{1 - x^2}{\sqrt{1 + 4x^2}}$. [3]

7 Express $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Express α in the form $k\pi$.

Find the exact coordinates of the maximum point of the curve $y = \sqrt{3}\sin x - \cos x$ for which $0 < x < 2\pi$. [6]

Section B (36 marks)

8 The upper and lower surfaces of a coal seam are modelled as planes ABC and DEF, as shown in Fig. 8. All dimensions are metres.

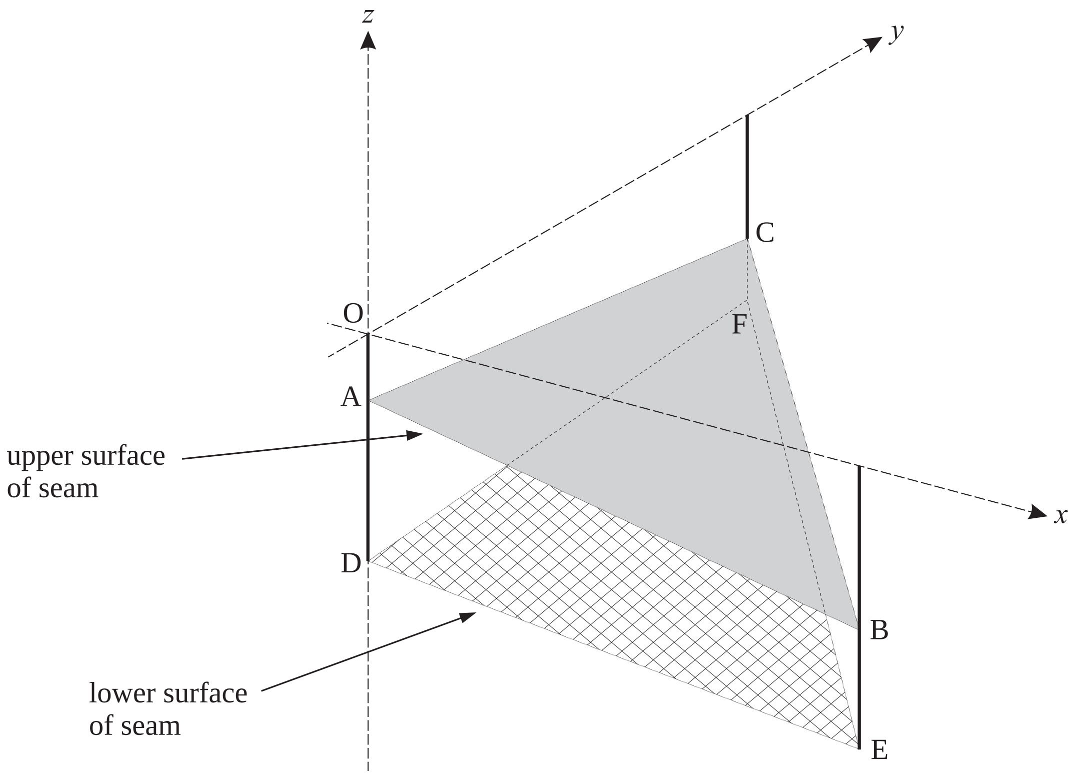

Fig. 8

Relative to axes Ox (due east), Oy (due north) and Oz (vertically upwards), the coordinates of the points are as follows.

A: (0, 0, −15) B: (100, 0, −30) C: (0, 100, −25)
D: (0, 0, −40) E: (100, 0, −50) F: (0, 100, −35)

(i) Verify that the cartesian equation of the plane ABC is $3x + 2y + 20z + 300 = 0$. [3]

(ii) Find the vectors $\overrightarrow{DE}$ and $\overrightarrow{DF}$. Show that the vector $2\mathbf{i} - \mathbf{j} + 20\mathbf{k}$ is perpendicular to each of these vectors. Hence find the cartesian equation of the plane DEF. [6]

(iii) By calculating the angle between their normal vectors, find the angle between the planes ABC and DEF. [4]

It is decided to drill down to the seam from a point R (15, 34, 0) in a line perpendicular to the upper surface of the seam. This line meets the plane ABC at the point S.

(iv) Write down a vector equation of the line RS.

Calculate the coordinates of S. [5]

[Turn over

9 A skydiver drops from a helicopter. Before she opens her parachute, her speed $v\,\mathrm{m\,s^{-1}}$ after time t seconds is modelled by the differential equation

$$\frac{dv}{dt} = 10e^{-\frac{1}{2}t}.$$

When $t = 0$, $v = 0$.

(i) Find v in terms of t. [4]

(ii) According to this model, what is the speed of the skydiver in the long term? [2]

She opens her parachute when her speed is $10\,\mathrm{m\,s^{-1}}$. Her speed t seconds after this is $w\,\mathrm{m\,s^{-1}}$, and is modelled by the differential equation

$$\frac{dw}{dt} = -\tfrac{1}{2}(w-4)(w+5).$$

(iii) Express $\dfrac{1}{(w-4)(w+5)}$ in partial fractions. [4]

(iv) Using this result, show that $\dfrac{w-4}{w+5} = 0.4e^{-4.5t}$. [6]

(v) According to this model, what is the speed of the skydiver in the long term? [2]