

Mathematics

Advanced GCE

Unit **4731**: Mechanics 4

Mark Scheme for June 2011

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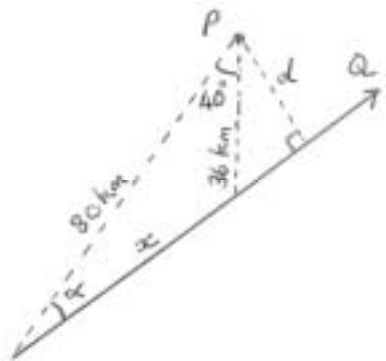
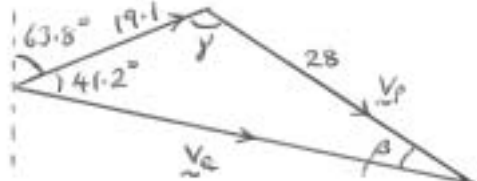
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1 (i)	Using $\omega_2 = \omega_1 + \alpha t$, $750 = 950 - 0.8t$ Time taken is 250 s	M1 A1 [2]	
(ii)	Using $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $200^2 = 220^2 - 1.6\theta$ Angle is 5250 rad	M1 A1 [2]	
(iii)	Angle is 20π rad Using $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$, $20\pi = 0 + 0.4t^2$ Time taken is 12.5 s (3 sf)	B1 M1 A1 [3]	or equivalent; e.g. finding $\omega_1 = 10.03$ and then $t = \omega_1 \div 0.8$ <i>Accept $\sqrt{50\pi}$ or $5\sqrt{2\pi}$</i>
2	$m = \int_0^a k e^{-\frac{x}{a}} dx$ $= k \left[-a e^{-\frac{x}{a}} \right]_0^a \quad (= ka(1 - e^{-1}))$ $m\bar{x} = \int_0^a x k e^{-\frac{x}{a}} dx$ $= k \left[-ax e^{-\frac{x}{a}} - a^2 e^{-\frac{x}{a}} \right]_0^a$ $= ka^2(1 - 2e^{-1})$ $\bar{x} = \frac{ka^2(1 - 2e^{-1})}{ka(1 - e^{-1})}$ $= \frac{a(1 - 2e^{-1})}{1 - e^{-1}} = \frac{a(e - 2)}{e - 1}$	M1 A1 M1 M1 A1 A1 A1 [7]	For $\int e^{-\frac{x}{a}} dx$ For $-a e^{-\frac{x}{a}}$ For $\int x e^{-\frac{x}{a}} dx$ Integration by parts For $-ax e^{-\frac{x}{a}} - a^2 e^{-\frac{x}{a}}$ For $a^2(1 - 2e^{-1})$ or exact equivalent
3 (i)	WD by couple is $C \times \frac{\pi}{2}$ Change in PE is $5 \times 9.8 \times 0.9$ By conservation of energy, $C \times \frac{\pi}{2} = 5 \times 9.8 \times 0.9$ Moment of couple is 28.1 Nm (3 sf)	B1 B1 M1 A1 [4]	Must clearly be PE (not moment) Equation involving WD and PE
(ii) (a)	$I = \frac{4}{3} \times 5 \times 0.9^2 \quad (= 5.4)$ $28.075 = 5.4\alpha$ Angular acceleration is 5.20 rad s^{-2} (3 sf)	B1 M1 A1 ft [3]	<i>Can be earned anywhere in the question</i> Applying $C = I\alpha$ ft is $C \div I$
(ii) (b)	$28.075 - 5 \times 9.8 \times 0.9 = 5.4\alpha$ Angular acceleration is $(-)$ 2.97 rad s^{-2} (3 sf)	M1 A1 [2]	Rotational equation of motion (3 terms) <i>(Allow 1.8 instead of 0.9 etc)</i>

<p>4 (i)</p>	<p>GPE is $-mg(\frac{1}{2}a \sin 2\theta)$ EPE is $\frac{3mg}{2a} AD^2 + \frac{4mg}{2a} BD^2$ $= \frac{3mg}{2a} (2a \cos \theta)^2 + \frac{4mg}{2a} (2a \sin \theta)^2$ $= mga(6 \cos^2 \theta + 8 \sin^2 \theta)$ $= mga(3 + 3 \cos 2\theta + 4 - 4 \cos 2\theta)$ $= mga(7 - \cos 2\theta)$ Total PE is $V = mga(7 - \cos 2\theta) - \frac{1}{2}mga \sin 2\theta$ $= \frac{1}{2}mga(14 - 2 \cos 2\theta - \sin 2\theta)$</p>	<p>B1 M1 A1 M1 A1 ag [5]</p>	<p><i>Negative sign is essential, but may be implied later</i> Any correct form Expressing EPE in terms of $\cos 2\theta$</p>
<p>(ii)</p>	<p>$\frac{dV}{d\theta} = \frac{1}{2}mga(4 \sin 2\theta - 2 \cos 2\theta)$ $\frac{dV}{d\theta} = 0$ when $4 \sin 2\theta = 2 \cos 2\theta$ $\tan 2\theta = 0.5$ $\theta = 0.232$ (3 sf)</p>	<p>B1 M1 A1 [3]</p>	<p>Equating to zero and solving Accept 13.3°</p>
<p>(iii)</p>	<p>$\frac{d^2V}{d\theta^2} = \frac{1}{2}mga(8 \cos 2\theta + 4 \sin 2\theta)$ When $\theta = 0.232$, $\frac{d^2V}{d\theta^2} > 0$ So the equilibrium is stable</p>	<p>M1 A1 [2]</p>	<p>Fully correct working only</p>

<p>5 (i)</p>	$\left(\frac{4}{3}\pi a^3\right)\rho = 10M, \text{ so } \rho = \frac{15M}{2\pi a^3}$ $I = \sum \frac{1}{2}(\rho\pi y^2 \delta x)y^2 = \frac{1}{2}\rho\pi \int y^4 dx$ $= \frac{1}{2}\rho\pi \int_{-a}^a (a^2 - x^2)^2 dx$ $= \frac{1}{2}\rho\pi \left[a^4x - \frac{2}{3}a^2x^3 + \frac{1}{5}x^5 \right]_{-a}^a$ $= \frac{1}{2}\rho\pi \left(a^5 - \frac{2}{3}a^5 + \frac{1}{5}a^5 \right) \times 2$ $= \frac{8}{15}\rho\pi a^5$ $= \frac{8}{15} \times \frac{15M}{2\pi a^3} \times \pi a^5 = 4Ma^2$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1 ag</p> <p>[6]</p>	<p>For $\int y^4 dx$</p> <p>Correct integral expression including limits</p> <p>For $a^4x - \frac{2}{3}a^2x^3 + \frac{1}{5}x^5$</p>
<p>(ii)</p>	$MI \text{ is } 4Ma^2 + Ma^2$ $= 5Ma^2$ <hr/> $-Mga \sin \theta = 5Ma^2 \ddot{\theta}$ $\ddot{\theta} \approx -\frac{g}{5a} \theta$ <p>Period is $2\pi \sqrt{\frac{5a}{g}}$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>Equation of motion</p> <p>Obtaining period</p>
	<p><i>Alternative for last 3 marks of (ii)</i></p> $11M \bar{x} = 10M(0) + Ma$ $\bar{x} = \frac{1}{11}a$ $\text{Period is } 2\pi \sqrt{\frac{I}{mgh}} = 2\pi \sqrt{\frac{5Ma^2}{11Mg \frac{1}{11}a}}$ $= 2\pi \sqrt{\frac{5a}{g}}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Finding centre of mass</p> <p>Using formula <i>Dependent on previous M1</i></p> <p>Note $2\pi \sqrt{\frac{I}{Mgh}} = 2\pi \sqrt{\frac{5Ma^2}{Mga}}$ is M0</p>

<p>6 (i)</p>	<p>As viewed from P</p>  $x^2 = 80^2 + 36^2 - 2 \times 80 \times 36 \cos 40^\circ$ $x = 57.30$ <p>Relative velocity has magnitude $\frac{x}{3} = 19.1 \text{ km h}^{-1}$</p> $\frac{\sin \alpha}{36} = \frac{\sin 40^\circ}{57.30}$ $\alpha = 23.82^\circ$ <p>Relative velocity has bearing $40 + \alpha = 063.8^\circ$</p>	<p>M1 M1 A1 ag M1 A1 ag [5]</p>	<p>Suitable diagram showing relative velocity <i>May be implied</i></p> <p>Or other valid method for finding a relevant angle</p>
	<p>OR, using components, Diagram M1 East $\frac{80 \sin 40^\circ}{3}$ (= 17.14) M1 North $\frac{80 \cos 40^\circ - 36}{3}$ (= 8.428) M1 Speed $\sqrt{17.14^2 + 8.428^2} = 19.1$ A1 ag Bearing $\tan^{-1} \frac{17.14}{8.428} = 063.8^\circ$ A1 ag</p>		<p>Implied by both components correct</p>
<p>(ii)</p>	<p>Shortest distance $d = 80 \sin 23.82^\circ$ $= 32.3 \text{ km}$ (3 sf)</p>	<p>M1 A1 [2]</p>	<p>or $36 \sin 63.8^\circ$</p>
<p>(iii)</p>	 $\frac{\sin \beta}{19.10} = \frac{\sin 41.18^\circ}{28}$ $\beta = 26.69^\circ$ <p>Bearing of P is $105 + \beta = 131.7^\circ$ (1 dp)</p>	<p>M1 M1 A1 [3]</p>	<p>Velocity diagram <i>May be implied</i> (28 opposite a known angle between sides with positive and negative slopes)</p> <p><i>Using components for (iii) and (iv)</i> <i>M2A1 for $\theta = 131.7^\circ$ or $v = 39.4$</i> <i>M1A1 for other quantity</i></p>
<p>(iv)</p>	$\frac{v_Q}{\sin 112.13^\circ} = \frac{28}{\sin 41.18^\circ}$ <p>Speed of Q is 39.4 km h^{-1} (3 sf)</p>	<p>M1 A1 [2]</p>	<p>Or other valid method for finding speed</p>

7 (i)	$XG = \sqrt{5}a$ $I = \frac{1}{3}m\{a^2 + (3a)^2\} + m(\sqrt{5}a)^2$ $= \frac{25}{3}ma^2$	B1 M1 A1 [3]	For $I_G = \frac{1}{3}m\{a^2 + (3a)^2\}$ Using parallel axes rule
	OR, other complete method, e.g. $\frac{4}{3}\left(\frac{1}{6}m\right)\left(\left(\frac{1}{2}a\right)^2 + a^2\right) + \frac{4}{3}\left(\frac{5}{6}m\right)\left(\left(\frac{5}{2}a\right)^2 + a^2\right)$ $I = \frac{25}{3}ma^2$	M1 A1 A1	Correct expression for I
(ii)	$mg(\sqrt{5}a) = I\alpha$ $\sqrt{5}mga = \frac{25}{3}ma^2\alpha$ $\alpha = \frac{3\sqrt{5}g}{25a}$	M1 A1 ag [2]	Allow, e.g. $mg(2a) = I\alpha$
(iii)	$\frac{1}{2}I\omega^2 = mga$ $\frac{25}{6}ma^2\omega^2 = mga$ $\omega = \sqrt{\frac{6g}{25a}}$	M1 A1 ft A1 [3]	Equation involving KE and PE
(iv)	$H = m(XG)\omega^2$ $= m(\sqrt{5}a)\left(\frac{6g}{25a}\right)$ $= \frac{6\sqrt{5}}{25}mg$ $mg - V = m(XG)\alpha$ $V = mg - m(\sqrt{5}a)\left(\frac{3\sqrt{5}g}{25a}\right)$ $= \frac{2}{5}mg$ Force has magnitude $\sqrt{H^2 + V^2}$ $= \frac{2}{25}mg\sqrt{(3\sqrt{5})^2 + 5^2}$ $= \frac{2\sqrt{70}}{25}mg$	M1 A1 A1 ft M1 A1 A1 M1 A1 ag [8]	For using acceleration $r\omega^2$ Or (F parallel to BA , θ is angle GXB) $F - mg \sin \theta = m((AG)\omega^2 \cos \theta - (AG)\alpha \sin \theta)$ ft from incorrect ω only Or $F = \frac{mg(2\sqrt{5} + 12)}{25}$ For using acceleration $r\alpha$ Or (R parallel to AD) $mg \cos \theta - R = m((AG)\omega^2 \sin \theta + (AG)\alpha \cos \theta)$ Or $R = \frac{mg(4\sqrt{5} - 6)}{25}$ Or $\sqrt{F^2 + R^2}$

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