OCR
Oxford Cambridge and RSA

Monday 05 October 2020 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Find $\sum_{r=2}^{50}\left(\frac{1}{r-1}-\frac{1}{r+1}\right)$, expressing the answer as an exact fraction. **[3]**

2 Fig. 2 shows two complex numbers z_1 and z_2 represented on an Argand diagram.

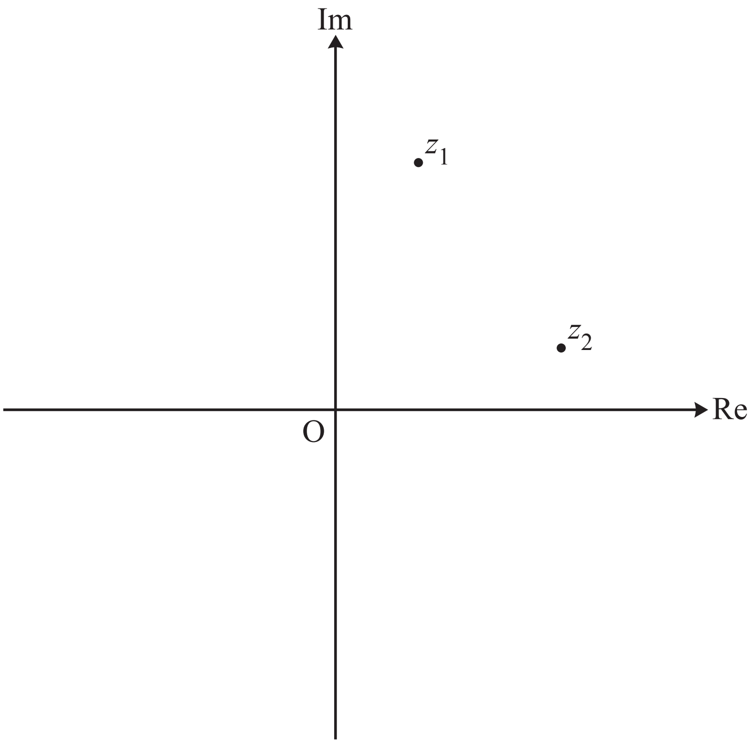

Fig. 2

(a) On the copy of Fig. 2 in the Printed Answer Booklet, mark points representing each of the following complex numbers.

- z_1^*
- $z_2 - z_1$ **[2]**

(b) **In this question you must show detailed reasoning.**

In the case where $z_1 = 1+2\text{i}$ and $z_2 = 3+\text{i}$, find $\frac{z_2 - z_1}{z_1^*}$ in the form $a+\text{i}b$, where a and b are real numbers. **[2]**

3 **In this question you must show detailed reasoning.**

The roots of the equation $x^2 - 2x + 4 = 0$ are α and β.

(a) Find α and β in modulus-argument form. **[4]**

(b) Hence or otherwise show that α and β are both roots of $x^3 + \lambda = 0$, where λ is a real constant to be determined. **[3]**

4 The matrix $\mathbf{M}$ is $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(a) **(i)** Calculate $\det \mathbf{M}$. **[1]**

(ii) State two geometrical consequences of this value for the transformation associated with $\mathbf{M}$. **[2]**

(b) Describe fully the transformation associated with $\mathbf{M}$. **[1]**

5 You are given that $u_1 = 5$ and $u_{n+1} = u_n + 2n + 4$.

Prove by induction that $u_n = n^2 + 3n + 1$ for all positive integers n. **[6]**

6 The matrices $\mathbf{M}$ and $\mathbf{N}$ are $\begin{pmatrix} \lambda & 2 \\ 2 & \lambda \end{pmatrix}$ and $\begin{pmatrix} \mu & 1 \\ 1 & \mu \end{pmatrix}$ respectively, where λ and μ are constants.

(a) Investigate whether $\mathbf{M}$ and $\mathbf{N}$ are commutative under multiplication. **[2]**

(b) You are now given that $\mathbf{MN} = \mathbf{I}$.

(i) Write down a relationship between $\det \mathbf{M}$ and $\det \mathbf{N}$. **[1]**

(ii) Given that $\lambda > 0$, find the exact values of λ and μ. **[3]**

(iii) Hence verify your answer to part **(i)**. **[2]**

7 In the quartic equation $2x^4 - 20x^3 + ax^2 + bx + 250 = 0$, the coefficients a and b are real. One root of the equation is $2 + \mathrm{i}$.

Find the other roots. **[7]**

8 **(a)** The matrix **M** is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

(i) Find $\mathbf{M}^2$. **[1]**

(ii) Write down the transformation represented by **M**. **[1]**

(iii) Hence state the geometrical significance of the result of part **(i)**. **[1]**

(b) The matrix **N** is $\begin{pmatrix} k+1 & 0 \\ k & k+2 \end{pmatrix}$, where k is a constant.

Using determinants, investigate whether **N** can represent a reflection. **[4]**

9 Three planes have equations

$$kx + y - 2z = 0$$
$$2x + 3y - 6z = -5$$
$$3x - 2y + 5z = 1$$

where k is a constant.

Investigate the arrangement of the planes for each of the following cases. If in either case the planes meet at a unique point, find the coordinates of that point.

(a) $k = -1$ **[3]**

(b) $k = \frac{2}{3}$ **[4]**

10 A vector $\mathbf{v}$ has magnitude 1 unit. The angle between $\mathbf{v}$ and the positive z-axis is 60°, and $\mathbf{v}$ is parallel to the plane $x - 2y = 0$.

Given that $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, where a, b and c are all positive, find $\mathbf{v}$. **[7]**

END OF QUESTION PAPER

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