OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Report on the Units

January 2008

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Oxford Cambridge and RSA Examinations

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GCE Mathematics and Further Mathematics Certification

From the January 2008 Examination session, there are important changes to the certification rules for GCE Mathematics and Further Mathematics.

1 In previous sessions, GCE Mathematics and Further Mathematics have been aggregated using 'least-best' i.e. the candidate was awarded the highest possible grade in their GCE Mathematics using the lowest possible number of uniform marks. The intention of this was to allow the greatest number of uniform marks to be available to grade Further Mathematics.

From January 2008 QCA have decided that this will no longer be the case. Candidates certificating for AS and/or GCE Mathematics will be awarded the highest grade with the highest uniform mark. For candidates entering for Further Mathematics, both Mathematics and Further Mathematics will be initially graded using 'least-best' to obtain the best pair of grades available. Allowable combinations of units will then be considered, in order to give the candidate the highest uniform mark possible for the GCE Mathematics that allows this pre-determined pair of grades. See page 2 for an example.

As before, the maximisation process will award a grade combination of AU above, say, BE. Where a candidate's grade combination includes a U grade a request from centres to change to an aggregation will be granted. No other requests to change grading combinations will be accepted. e.g. A candidate who has been awarded a grade combination of AD cannot request a grading change that would result in BC.

2 In common with other subjects, candidates are no longer permitted to decline AS and GCE grades. Once a grade has been issued for a certification title, the units used in that certification are locked into that qualification. Candidates wishing to improve their grades by retaking units, or who have aggregated GCE Mathematics or AS Further Mathematics in a previous session should re-enter the certification codes in order to ensure that all units are unlocked and so available for use. For example, a candidate who has certificated AS Mathematics and AS Further Mathematics at the end of Year 12, and who is certificating for GCE Mathematics at the end of Year 13, should put in certification entries for AS Mathematics and AS Further Mathematics in addition to the GCE Mathematics.

Grading Example

A candidate is entered for Mathematics and Further Mathematics with the following units and uniform marks.

Unit	Uniform marks	Unit	Uniform marks
C1	90	M1	80
C2	90	M2	100
C3	90	M3	90
C4	80	S1	70
FP1	100	S2	70
FP2	80	D1	60

Grading this candidate using least-best gives the following unit combinations:

Mathematics		Further Mathematics	
Unit	Uniform marks	Unit	Uniform marks
C1	90	FP1	100
C2	90	FP2	80
C3	90	M1	80
C4	80	M2	100
S1	70	M3	90
D1	60	S2	70
Total	480 (Grade A)	Total	520 (Grade A)

Under the new system, having fixed the best pair of grades as two As, the mark for the Mathematics would be increased by combining the units in a more advantageous manner. The table below shows the allowable combination of units.

Option	Applied units used for Maths	Total uniform marks for Mathematics	Applied units used for Mathematics	Total uniform marks for Further Mathematics
1	M1, S1	500	M2, M3, S2, D1	500
2	M1, D1	490	M2, M3, S1, S2	510
3	S1, D1	480	M1, M2, M3, S2	520
4	M1, M2	530	M3, S1, S2, D1	470
5	S1, S2	490	M1, M2, M3, D1	510

Option 4 gives the highest uniform mark for Mathematics. However, this would only give a grade B in the Further Mathematics, and so is discarded. Option 1 is the next highest uniform mark for Mathematics and gives an A in Further Mathematics, and so this is the combination of units that would be used.

Chief Examiner's Report – Pure Mathematics

Units 4721, 4722 and 4725 are AS units and the requests in these units are generally familiar and accessible to any candidates who have prepared thoroughly for the examinations. Care is needed in answering the questions but most candidates are able to identify the techniques needed very readily.

The remaining Pure Mathematics units are A2 units and, as such, are designed to make rather greater demands on candidates, in terms of both breadth and depth of mathematical knowledge. The evidence from this session of examinations was that many candidates took an AS-type approach to A2 units, proceeding too quickly with solutions and without giving due thought and consideration to what each question was asking and to what would therefore be the most effective method of solution. It is acknowledged that, under examination conditions, a candidate might not judge it a wise tactic to pause periodically to think about a solution but, when such thought means that the appropriate technique is recalled or an efficient method is adopted, the benefits can be considerable.

An example where the considered and thoughtful approach was seen to particularly good effect occurred in the response to Q9(i) of unit 4723. For many candidates, their attempt at a proof was lengthy, haphazard and unconvincing, sometimes extending over more than one page. A little planning and some thought on how to communicate most persuasively would have led to a solution such as:

$$\begin{aligned}
\text{Left-hand side} &= 4(\cos\theta\cos 60° - \sin\theta\sin 60°)(\cos\theta\cos 30° - \sin\theta\sin 30°) \\
&= 4(\tfrac{1}{2}\cos\theta - \tfrac{1}{2}\sqrt{3}\sin\theta)(\tfrac{1}{2}\sqrt{3}\cos\theta - \tfrac{1}{2}\sin\theta) \\
&= 4(\tfrac{1}{4}\sqrt{3}\cos^2\theta - \tfrac{3}{4}\sin\theta\cos\theta - \tfrac{1}{4}\cos\theta\sin\theta + \tfrac{1}{4}\sqrt{3}\sin^2\theta) \\
&= \sqrt{3}\cos^2\theta - 3\sin\theta\cos\theta - \sin\theta\cos\theta + \sqrt{3}\sin^2\theta \\
&= \sqrt{3}(\cos^2\theta + \sin^2\theta) - 4\sin\theta\cos\theta \\
&= \sqrt{3}\times 1 \quad - \quad 2\times 2\sin\theta\cos\theta \\
&= \sqrt{3} \quad - \quad 2\sin 2\theta\,.
\end{aligned}$$

This is clearly set out and the proof is logically and accurately developed. There is sufficient detail and no examiner would have been in any doubt that such a solution merited all the marks available.

4721 Core Mathematics 1

General Comments

This paper proved accessible to the majority of candidates, most of whom made good attempts at all questions. In general, answers were clearly presented with an appropriate amount of working shown although there were plenty of exceptions to this.

Most candidates worked through the questions in numerical order and appeared to have enough time to complete the paper. Many candidates' scripts showed evidence of checking and correcting answers. There was often only a minimal attempt at the second part of Q10 but it was difficult to ascertain if this was because of its difficulty or because of a lack of time.

It was disappointing that a few centres are still issuing graph paper to every candidate. This is unnecessary and may result in candidates spending too much time drawing accurate graphs when a sketch displaying the salient features is perfectly adequate.

The full range of marks was awarded, with a small number of candidates gaining 72 marks.

Comments on Individual Questions

1) This opening question proved straightforward for most candidates, who multiplied both the numerator and denominator by $3+\sqrt{7}$ correctly. However, subsequent cancelling by 2 frequently resulted in a final answer of $6+4\sqrt{7}$ or, in some cases, $12+2\sqrt{7}$.

2) (i) This part question was usually done correctly although candidates from a few centres were clearly unfamiliar with the equation of a circle and could not attempt either part of this question. Of those that answered, errors were rare, the most commonly seen wrong answers being $x^2+y^2=7$ and $x^2+y^2=\sqrt{7}$.

   (ii) For candidates who were familiar with the equation of a circle, this question proved routine. However, there were numerous cases of careless arithmetic leading to 44 instead of 64. Weaker candidates who were unsure what to do simply substituted in the coordinates of the given centre, leading to the statement $-64=0$. They then gave the answer r = 8. These candidates gained no marks. Some candidates completed the square correctly, or tried to use the formula $\sqrt{g^2+f^2-c}$, but arrived at a radius of 2 due to carelessness or confusion with signs.

3) This question proved challenging to many candidates. They had little idea of how to approach the identity, some ending up with a value for x, others with a page of working but no outcomes. Candidates who understood what was required usually started by expanding the right-hand expression and were able to find a and b, although a large number stated that b = 6, despite correct working. A correct value for c was much less common, by far the most popular wrong answer being 1. Weaker candidates often started with the left-hand expression and attempted to complete the square. This required a far higher standard of algebraic manipulation and the correct value of c was rarely found using this approach.

4) (i) While most candidates identified the missing power as –1, there were also plenty of wrong answers, with some candidates unable to offer any value for p. The most commonly seen errors were –2 and $\frac{1}{10}$.

(ii) This question was one of the least successfully tackled questions on the whole paper. It was rather worrying that so many candidates failed to understand the significance of the brackets and wrote 25k = 15 from which there was no recovery. Others tried to square both sides of the initial equation but either failed to get 225 or could not divide 225 by 25 correctly. Only a tiny proportion of the candidates who correctly found 3 remembered to give the negative square root. Among the exceptionally able candidates, this was sometimes the only mark they lost on the paper.

(iii) In general, candidates dealt with the negative fractional power efficiently and most got the correct value for t. The most frequently occurring incorrect answers were $\frac{1}{8}$ and $\sqrt[3]{2}$ but this question proved straightforward for most.

5) It is very pleasing that most candidates now sketch their graphs in the answer booklet rather than on graph paper and there were very few instances of candidates evaluating a set of points and plotting them. In better responses, axes were drawn with a ruler and the curves sketched with care and in pencil.

(i) The shape of a positive cubic graph was well known, although a large proportion of curves had maximum and minimum points. Many sketches showed an x-intercept of 2 or –2, rather than the correct y-intercept. These cubic curves often appeared to have vertical asymptotes but the general shape was widely known.

(ii) The curve $y = 2\sqrt{x}$ was slightly less familiar to candidates but was still sketched correctly by the majority. Some candidates drew the curve for both positive and negative y-values; other sketches had the wrong curvature resulting in a quadratic curve. Some otherwise correct curves had rather long horizontal sections as x increased and a small number of candidates drew the curve $y = \frac{1}{x}$.

(iii) Most candidates recognised this transformation as a stretch (with only a few instances of 'squash' or 'shrink') but few gained all 3 marks because they could not identify the scale factor or muddled the direction. For instance, descriptions like 'in the y-direction, parallel to the x-axis' were not uncommon. The most common wrong factor was 1, but 3 was also frequently seen. A few candidates lost marks because they could not define the transformation as a single stretch or because they described transformations which map the curve $y = \sqrt{x}$ to each of the given curves.

6) This question proved to be a good discriminator of ability. Some candidates did not even attempt it and, of those who did, only the most able managed to gain full marks.

(i) As usual, there were a disappointingly large number of candidates who had not learnt the quadratic formula. Many of those who started correctly and got as far as $\frac{-8 \pm \sqrt{64-40}}{2}$ made mistakes in subsequent working. The most common errors were due to incorrect cancelling, resulting in answers of $-4 \pm 2\sqrt{6}$, $-8 \pm \sqrt{6}$ or $-4 \pm \sqrt{12}$, but there were also many cases of $64 - 40 = 20$ seen. Of those candidates who decided to complete the square, a disappointing number left out the negative square root, thus ending up with a single root.

(ii) As in part (i), it was rare for candidates to score all 3 marks in this part. While nearly all recognised it as a quadratic curve, the most common error was to draw a parabola which was symmetrical about the y-axis, with (0, 10) as its minimum point. A small number of candidates completed the square to find the coordinates of the minimum point and subsequently drew the graph in the correct position with 2 negative real roots. They gained full marks, although their graph was often inconsistent with their answer to part (i).

(iii) The method for solving a quadratic inequality again proved too demanding for all but the most able. Candidates failed to appreciate the link with the earlier parts of the question, many attempting to solve the equation again, often by a completely different method. Some drew a sketch but shaded the incorrect region along the x-axis; others shaded the correct region but then wrote down incorrect inequalities.

7) (i) This mark was earned by most candidates, although there were plenty of alternative answers seen, most commonly $\frac{x}{2}$, $\frac{1}{2}$, 2, -2, 1 and -1.

(ii) The method for finding the equation of a straight line was demonstrated well by nearly all candidates, although some used a gradient perpendicular to that found in part (i). However, many good candidates lost the final mark by being unable to deal with the fraction and the minus sign while rearranging the equation into the required form.

(iii) Nearly all candidates were able to make a start and those who chose to eliminate y usually managed to obtain the correct quadratic equation and solve it to obtain 2 correct x values. Those who decided to eliminate x gave themselves a more difficult route. Firstly, there were unfortunate errors in rearranging $x+2y=4$, with expressions such as $y=2-x$ or $y=4-\frac{1}{2}x$ commonly seen. Then, while struggling to simplify their more complicated expressions, some candidates left out the y term on the other side of their equation and ended up with an incorrect quadratic equation. However, a large proportion of candidates of all abilities completed their solutions correctly, remembered to find the associated y or x values, and scored full marks.

8) Most candidates scored well on this question, regardless of their overall ability, and showed a thorough understanding of how to find stationary points. A very few candidates who did well on other parts of the paper did not attempt this question at all, presumably because they were not familiar with the term ‘stationary points’.

(i) The first 5 marks were readily gained but there were problems in calculating the associated y values. Some candidates who found $\frac{76}{27}$ correctly then made a sign error when substituting $x=-1$ for the other point. A few substituted their x values into their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and hence gave both y-coordinates as zero.

(ii) The second part of this question was also confidently tackled by most, with evaluation of the second derivative being by far the most common method used, although some candidates did investigate the gradient either side of each stationary point. A small number of candidates worked out $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ correctly but then equated their expression to zero and solved for x, thus scoring no marks.

(iii) The concept of a decreasing function proved unfamiliar to many and this question was often left out. Some candidates used 4 and –4 (their values from part (ii)) in inequalities, others stated 'negative values of *x*'. However, a good proportion realised that the *x*-coordinates of the turning points were needed and they gained at least a method mark and often both marks.

9) This question was well done and nearly all candidates gained a good number of marks here.

(i) The gradient was confidently found by most, with only a few instances of $\frac{8}{3}$. This gradient was then correctly substituted into the equation of a line through either *A* or *B*. But, as in Q7(ii), many candidates lost the final mark because they failed to rearrange their equation correctly.

(ii) The vast majority of answers were correct. Only a few candidates used an incorrect formula or miscopied the coordinates of the required points.

(iii) Again, there were many perfect answers, although there were too many candidates who used the wrong points. Most candidates knew the correct formula or drew a sketch and used Pythagoras' theorem. However, as in previous papers, there was a sizeable minority who could not cope with arithmetic involving negative numbers.

(iv) The condition for the perpendicularity of 2 lines was widely known, with most candidates either stating that $m_1 \times m_2 = -1$ or describing the gradients as 'negative reciprocals' of each other. A significant number of candidates misread the question and proceeded to check whether *AB* was perpendicular to *AC*, for which they could only gain 2 marks out of a possible 4. But by far the most common (and unexpected) error was to anticipate a result by cancelling a correct gradient of $-\frac{3}{6}$ to $-\frac{1}{3}$ and to conclude that the lines were perpendicular.

10) (i) As in previous papers, all but the very weakest candidates scored well on this relatively straightforward differentiation. Most understood the notation and differentiated twice, although a sizeable minority stopped at $24x^2 - 3x^{-4}$. In some cases, $\frac{1}{x^3}$ became $x^{\frac{1}{3}}$; other candidates started with x^{-3} but then added 1 to the power. Most solutions, however, were fully correct.

(ii) This final question proved to be the most challenging part of the paper and few candidates scored all 5 marks. Many candidates made no attempt at it while others tried to evaluate f(–9). Some candidates spotted the solution $x = -1$ and a handful managed, by trial and improvement, to find $x = -\frac{1}{2}$.

Attempts to cube root each term individually were seen but only rarely. Able candidates provided a concise and well presented solution, recognising that a substitution for x^3 was appropriate. Those that substituted into the given equation usually went on to rearrange and solve correctly whereas those who rearranged first were more likely to make a mistake. A few good candidates, having reached the penultimate stage of working, lost the final mark by being unable to evaluate $\sqrt[3]{-\frac{1}{8}}$.

4722 Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates, and overall the standard was very good. Candidates seemed well prepared for the paper and familiar with the topics being tested, and there were fewer really poor scripts seen than in previous sessions.

Examiners commented that the overall standard of presentation was much improved, with most showing clear and explicit methods. However, when two or more attempts are made at a question, candidates do need to make it clear which one they wish the examiner to mark. In questions where the candidates were required to prove a given result, sufficient detail was usually shown to be convincing but this was not the case in all of the scripts seen.

Some candidates still struggle to use their calculator efficiently. This was particularly noticeable on questions where an exact answer was required. A number of candidates also lost marks through having their calculator in the wrong mode – on a paper that involves working in both degrees and radians they do need to be able to switch between the two. When using a calculator it is still important that full details are shown – examiners cannot award method marks if little or no method has been shown. Candidates must also ensure that any intermediate values in their working are accurate enough to justify the final answer to the specified degree of accuracy. A significant number failed to gain full marks due to using a rounded value from a previous part of the question.

Whilst most candidates can recall relevant formulae accurately, it is disappointing to see others lose marks by attempting to use an incorrect formula. If a formula is given in the *List of Formulae*, examiners expect to see it quoted accurately and no credit will be given if this is not the case.

Whilst most candidates have a basic understanding of logarithms, and can attempt to use them to solve equations, only the most able candidates can manipulate them accurately. Once again, a number of candidates lost marks through a lack of mastery of basic skills, such as algebraic manipulation, expanding a bracket with a minus sign preceding it, use of indices and solving quadratic equations.

Comments on Individual Questions

1 The majority of candidates made a good attempt at this question, though some seemed unclear on the definition of a segment and just found the area of the sector. Whilst candidates seemed familiar with the formula for the area of the sector, finding the area of the triangle caused more problems, with many long-winded and ultimately unsuccessful methods seen. A number of candidates failed to gain the final mark as premature approximation within the question led to an inaccurate final answer. Some candidates quoted the correct formula but then used their calculator in degree not radian mode, and a few candidates converted radians into degrees, with a resultant loss in accuracy.

2 Most candidates seemed familiar with the trapezium rule and could attempt the question, though errors were common. There were the usual mistakes of using x-coordinates not y-coordinates and using an incorrect value for h. A surprising number did not evaluate the integral between the requested limits; the common errors were using x values of 0, 1, 2 and 3 or 2, 4 and 6. A number of candidates did not use the requested number of strips. This approach gained some credit, as long as their h was consistent with the number of strips, which was rarely the case. A lack of care when writing out the formula often resulted in brackets being omitted. Despite this giving the correct answer, incorrect use of the formula was penalised. A few candidates attempted to integrate the function before using the trapezium rule, which gained no credit, and a small number attempted to use Simpson's rule instead.

3 (i) This was very well answered, with the majority of candidates gaining the mark available. The only errors seen were the occasional 5 not 6, and a failure to evaluate 2×3.

(ii) Whilst candidates seemed familiar with the basic rules of logarithms, they seemed unclear as to which order to apply them in this multi-step problem. All too often there was no attempt to deal with the coefficients by changing them to powers before using the subtraction law for logarithms. Many of those who did manage this step then spoilt their solution by having a log in the denominator as well. Even if this subsequently disappeared, errors in working meant that full credit was not given.

4 (i) This question was generally well answered with most candidates able to make confident use of the sine rule. However, a few failed to get full marks as their calculator was set in radian not degree mode. Some of the candidates attempted to use right-angled triangles throughout the question.

(ii) This was also generally well done, though candidates seemed less familiar with the cosine rule, despite this being the formula that is given in the *List of Formulae*. Some were unable to quote the correct formula, a number struggled to rearrange it, and some substituted the sides in incorrectly thus finding the wrong angle. However, the main reason for failing to gain full marks was using a rounded value from part (i), which led to a loss of accuracy in this part of the question. It was disappointing to see even very able candidates losing a mark in this way.

5 This was a straightforward, routine question that was answered well by many of the candidates. Most realised that integration was required and could make an attempt at this, though dividing by $^3/_2$ caused problems for some. Many then used (4, 50) to find the value of the constant, though some candidates failed to gain the final mark by failing to include y in their final equation. A number of candidates saw the word gradient and attempted to use $y = mx + c$, or an equivalent form. Sometimes this was used to find the constant of integration, but a significant minority tried to treat the entire question in this way, with either an algebraic or numerical gradient. Converting $12\sqrt{x}$ into index form caused fewer problems than in the past, but there were still errors seen.

6 (i) This was an easy start to the question and most candidates gained both of the marks available, though a few treated it as a recurrence relationship not one for an n^{th} term.

(ii) Nearly all the candidates could correctly identify this as an arithmetic sequence, though there were many variations on the actual spelling.

(iii) Despite the rather large hint in part (ii) that this was an arithmetic progression, a number failed to make the connection and attempted to use formulae for a geometric progression. There was also uncertainty over whether to use the formula for the n^{th} term or the sum of n terms. However, many candidates could identify that the sum of an arithmetic progression was required. With the relevant formula given in the *List of Formulae*, examiners expected to see it quoted correctly. Candidates should be encouraged to use the one most appropriate to the given situation – those who attempted to use $^n/_2(a + l)$ rarely made any progress with making a sensible attempt at l. Candidates also struggled to decide what they were summing, and a was often 1, or sometimes 5. There was also uncertainty over whether to use n or S_n as 2200. Whilst candidates generally seemed familiar with the concept of sigma notation, the finer details escaped them. Examiners were also disappointed to see how many candidates struggled to solve the resulting quadratic – some found the correct answer through factorising or, more commonly, use of the formula. However, trial and improvement was also relatively common and a number couldn't even attempt to solve it. It was also quite common to see $n(n + 6) = 2200$, hence $n = 2200$ or $n + 6 = 2200$, which is very worrying at this level.

7 (i) Some candidates gave concise and accurate explanations, referring to 'negative areas', there being areas both above and below the x-axis, or the two areas partially cancelling. However, too many explanations were vague – remarks about 'the line' were not specific enough to gain credit as there are a number of lines on the diagram. A number seemed to believe that areas below the x-axis are not included in integration. Another common misconception was that integration gives the 'area under a curve'; as the area is above the curve for 0 – 3 then this cannot be calculated. This may have implications for the language used when teaching this topic.

(ii) Most candidates could integrate the given expression, though some spoilt it by including 5 or x as well. Most also understood the concept of definite integration and attempted to use limits correctly though, despite the hint in part (i), a surprising number used 0 and 5 as the limits. However, many candidates did split the area into two regions and calculated the area of each separately. Too often these values were then added together without dealing with the negative sign, which suggested that the main point of the question was lost on them despite the hint in part (i). Working in exact values also caused problems for many, with the final answer often given as 13.2. Whilst most candidates gained some credit on this question, only a minority gained full marks.

8 (i) This was very well done, with nearly all candidates able to quote the correct formula and then correctly evaluate it, though a few actually calculated $(ar)^3$ despite the correct expression being quoted.

(ii) Again, this was very well done with only the occasional slip of 19 not 20 in the formula.

(iii) Most candidates quoted the correct formula for the sum to infinity and substituted the relevant values. However, work on rearranging the given inequality was very poor, with a number of manipulation errors. Sign errors were common with the removal of the bracket in S_n and the inequality was often not reversed at the correct point. However, the most common error was for $10\text{x}0.8^n$ to become 8^n before being changed back later. Very few fully convincing solutions were seen. When solving the given inequality, most candidates used logarithms accurately but only the most able reversed the inequality sign when dividing by log(0.8). Very few appreciated that N had to be an integer because of the nature of the question and, of those who did, a number drew the wrong conclusion due to the earlier error with the inequality sign.

9 (i) Candidates could often identify either both x values or both y values, but rarely gave two correct pairs of coordinates. Despite the question being given in degrees, some values were still given in radians.

(ii) Most candidates made a good attempt at this question, though a number failed to get full credit by not simplifying their answers, most commonly by giving $0 - \alpha$ in part (b).

(iii) Most candidates recognised the need to use $\cos^2 x + \sin^2 x \equiv 1$, but a small number of candidates used $\cos x + \sin x \equiv 1$. The removal of brackets caused problems for many, with $-3(1 - \sin^2 x)$ often becoming $-3 - 3\sin^2 x$. Most could then make a reasonable attempt to solve their quadratic, either by factorisation or by formula. Some felt that $-{}^1/_3$ and 1 were the final solutions and made no attempt at the actual angles, an approach that was not helped by using the substitution $x = \sin x$ to solve the quadratic equation. A number did not appreciate that $\sin x = -{}^1/_3$ has two solutions in the given range, and it was also common to see 161° rather than -161°. A number of candidates spoiled their solution by giving -90° as a secondary value following the correct 90°.

10 (i) Most candidates made a good attempt at the binomial expansion, with errors such as finding the sum rather than the product within each term being not common. A few used the binomial coefficients as fractions, but the most common error was an inability to deal with the powers of $2x$ successfully. This has been commented upon in a number of previous Examiners' Reports, yet there has been little or no improvement seen. As in previous sessions, the more successful methods involved effective use of brackets. A small number chose to expand the four brackets, with varying degrees of success. The vast majority of candidates could gain some credit on this question, usually from $1000x + 625$ along with three other terms, but it was a minority that gained full marks.

(ii) This part was generally done well with most candidates recognising the change of sign and writing down a correct expression following their part (i), possibly due to the hint in the question. However, errors in the original expansion prevented full marks being gained here. A few candidates undertook a second full expansion and a few of the most able candidates provided a solution based on the difference of two squares.

(iii) Whilst most candidates attempted f(2) there was often not enough convincing method shown to gain the mark. There was also some confusion between root and factor, with some attempts at f(–2). A number of candidates stopped at this point or attempted to use the quadratic formula on the cubic. However, a pleasing number of candidates continued to make progress through the question, often providing a fully correct solution. Candidates seem to be becoming more proficient with algebraic long division, though a number also used a method based on matching coefficients. Those who had simplified the coefficients of the cubic first tended to make fewer slips, especially when it came to solving the resulting quadratic.

4723 Core Mathematics 3

General Comments

The first six questions of this paper proved reasonably accessible to the vast majority of candidates although the responses to some of the requests, particularly Qs 2(ii), 3(a) and 4(ii), revealed some uncertainty with certain specification items. The final three questions did pose more challenges and the paper proved to be slightly more demanding than those in recent sessions have been. Later questions are designed to assess candidates' ability to choose and apply the techniques appropriate to particular, and possibly unfamiliar, situations. It was therefore very disappointing – and surprising – that so many candidates did not recognise the need in Q7 to use the product rule to differentiate xe^{2x}; surely there are not many simpler expressions where the product rule is required. Most candidates did manage to earn at least some of the marks available in Qs 8 and 9; there were testing parts to these questions and only the most able candidates were able to record full marks on these questions.

In general, the time allowed seemed to be sufficient to enable all candidates to show what they could do. There seemed fewer candidates at this session who scored very low marks. A number of candidates did record full marks on the paper but there were fewer candidates scoring high marks on this paper than has been the case in recent sessions.

Comments on Individual Questions

1) This question was answered competently by the vast majority of candidates. The procedure for composition of functions was well known and the only problem noted with any frequency in part (i) was an inability to evaluate $(-3)^3$ correctly. Part (ii) was also answered very well, most candidates opting for the approach which involved finding an expression for $f^{-1}(x)$ first. Very few candidates recognised that the request was equivalent to asking for the solution of the equation $x^3 + 4 = 12$.

2) Part (i) presented no problem to most candidates. Sufficient detail was provided as requested in the question and the correct conclusion was reached after a suitable number of steps. One mistake which did occur with some frequency was a final answer of 2.876, the result of truncating to 3 decimal places rather than rounding to 3 decimal places.

   Part (ii) presented more problems and a significant number of candidates had no idea how to proceed or thought that 2.877 was somehow involved explictly in the equation. This request assesses that part of the specification that requires candidates to 'understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved'. Many candidates did proceed as expected although many concluded with $x^3 + \frac{5}{2}x - 31 = 0$ rather than with an equation involving integer coefficients.

3) (a) The vast majority of candidates showed the relationship between secant and cosine but many were unable to solve this apparently simple equation accurately. Many did of course find the correct value of α from $\cos\frac{1}{2}\alpha = \frac{1}{4}$ but, often, the correct statement $\frac{1}{2}\alpha = 75.52$ was followed by $\alpha = 37.8$. Others did not realise that $\cos\frac{1}{2}\alpha = \frac{1}{4}$ was a form suitable for finding the value of α and tried to use further identities to find a value of $\cos\alpha$. A disappointingly large number of candidates started with a meaningless statement such as $\frac{1}{\cos}\frac{1}{2}\alpha = 4$ which usually became $\cos\alpha = \frac{1}{8}$ or $\cos\alpha = \frac{1}{2}$.

(b) There was more success with this part although, again, there were many attempts which started with $\tan\beta = \frac{7}{\tan}\beta$. Most candidates reached $\tan^2\beta = 7$ but the next step was usually $\tan\beta = \sqrt{7}$, with the result that the second root was missed. Some candidates opted for an approach involving $\sin\beta$ and $\cos\beta$; this involved more work but was often concluded correctly.

4) Part (i) was generally answered well with most candidates recognising that the chain rule was needed. There were some errors in evaluating the derivative, the most unfortunate of which was the replacement of $3h^5$ by 6^5. In part (ii), many candidates showed a sure grasp of appropriate notation and, recognising the need to combine rates of change, obtained the correct answer without difficulty. Other candidates incorrectly divided the answer from part (i) by 8 or multiplied it by 8. A minority of candidates did not recognise this part as involving connected rates of change; their attempts often did not use the value 8 at all.

5) (a) Most candidates provided an answer involving $\frac{1}{30}(3x+7)^{10}$ although $\frac{3}{10}(3x+7)^{10}$ and $\frac{1}{10}(3x+7)^{10}$ also occurred with some frequency. For this indefinite integral, candidates were expected to include the arbitrary constant in their answer but many failed to do so and lost a mark.

(b) Almost all candidates knew the formula for finding the volume but many were careless when squaring $\frac{1}{2\sqrt{x}}$, and $\frac{1}{2x}$ and $4x^{-1}$ were incorrect versions sometimes noted. Most candidates recognised that the integral involved a natural logarithm but it was common for the integral of $\frac{1}{4x}$ to be $\ln 4x$. The concluding simplification required the use of the relevant logarithm property to deal with $\ln 6 - \ln 3$ or with $\ln 24 - \ln 12$. There was some uncertainty about this and a step involving $\frac{\ln 6}{\ln 3}$ or $\frac{\ln 24}{\ln 12}$ was seen on a number of scripts, with the resulting loss of some credit even though the correct exact answer eventually appeared.

6) A question involving both an inverse trigonometric function and modulus might have been expected to prove challenging for many candidates but most candidates earned at least several of the marks and there were some very good responses.

There are several different possible answers to part (i); the reflection can be in either axis (or indeed in any line parallel to the y-axis) and, with certain choices, the order in which the translation and reflection are carried out is important. A significant number of candidates gave the pair of transformations needed to transform the graph of $y = \sin^{-1} x$ to the given graph; some credit was available for this misreading of the question. Use of the terms 'translation' and 'reflection' was expected and candidates using colloquialisms such as 'shift', 'move' and 'flip' lost marks. Credit was not allowed for candidates stating a stretch in the y-direction with scale factor –1 instead of the appropriate reflection.

Most candidates showed that they understood what was required for the sketch in part (ii) although many sketches were executed carelessly and without due attention to the curvature of the reflected part. Candidates superimposing their answer on a copy of the original sketch did not always make it clear what they intended as their answer; in such cases full credit was not available.

The immediate response of many candidates to the equation in part (iii) was to square both sides and such attempts foundered quickly. Many other candidates did proceed to a statement such as $-\sin^{-1}(x-1) = \pm\frac{1}{3}\pi$ and, in many cases, from there to the correct answers. Sometimes the symmetry of the graph in part (ii) was used to find a second value. Exact answers involving $\sqrt{3}$ were required and solutions such as $1 \pm \sin\frac{1}{3}\pi$ did not receive full credit.

7) An essential skill at A2 level is the ability to assess a piece of mathematics and decide the technique appropriate to the situation. Three such assessments were needed in tackling this question and, in only one of the three, was the correct decision made by the majority of candidates. Differentiation of xe^{2x} needs the product rule but examiners were astonished at the vast number of candidates who did not realise this and gave $2xe^{2x}$ as the derivative. An attempt at the quotient rule was made by almost all candidates. Those who had differentiated xe^{2x} correctly then had little difficulty in confirming the given result. The third assessment was required in part (ii) and again the response was very disappointing; it was a relatively small minority of candidates who appreciated that, for the curve to have one stationary point, the quadratic equation $2x^2 + 2kx + k = 0$ must have equal roots. Use of the discriminant in such a situation is a technique met in Core Mathematics 1 and those candidates proceeding accordingly were usually able to conclude the question successfully. For the majority of candidates, however, little progress was made; there were incorrect factorisations of $2x^2 + 2kx + k$ such as $(2x+k)(x+1)$ or use of the formula to find *x* in terms of *k*. A moment's thought should surely have led more to a realisation that use of the result $b^2 - 4ac = 0$ was appropriate.

8) Part (i) was answered accurately by most candidates and they earned 4 marks with ease. Some candidates associated the coefficients 2 and 4 in Simpson's rule with wrong *y* values and there were a few arithmetic errors such as taking the first *y* value to be zero.

The *List of Formulae* includes the result $e^{x\ln a} = a^x$. Use of this meant that 2^x could immediately be rewritten as $e^{x\ln 2}$. The vast majority of candidates seemed unaware of the presence of this result amongst the given formulae and struggled to make any progress. Some candidates managed to find *k* by considering the equation $2^x = e^{kx}$ but, in many cases, it was hard work. Examiners adopted a tolerant view of the notation $e^{\ln 2x}$ although candidates using this notation were often betrayed when evaluating the integral. It was disappointing to note how often the integral of $e^{x\ln 2}$ was given as $\ln 2\, e^{x\ln 2}$.

For candidates who had answered parts (i) and (ii) correctly, the 2 marks of part (iii) were easily earned. Among those candidates who had made earlier errors, there was seldom any sign of work being checked when the given result in part (iii) could not be confirmed.

9) The majority of candidates earned at least 2 marks in part (i) but clear, accurate, well-constructed proofs were not very common. To be convincing, a proof such as this does need to be approached with care and thought. Some candidates were able to do this but, for others, there were inaccuracies involving signs, exact values of trigonometric ratios and the use of the value 4 on the left-hand side.

The established identity is then needed for the remaining parts of the question and candidates who appreciated this often made good progress with parts (ii) and (iii). Many candidates, however, appeared not to understand the significance of the work they had attempted in part (i), namely that $4\cos(\theta + 60°)\cos(\theta + 30°)$ and $\sqrt{3} - 2\sin 2\theta$ are different versions of the same expression and, moreover, that the latter is the more useful version. So, in part (iii), there were attempts to solve the equation as given in the question rather the equivalent equation $\sqrt{3} - 2\sin 2\theta = 1$. Similarly, in part (iv), some candidates just looked at the equation and decided that –4 and 4 were critical values.

Part (ii) was often completed successfully although many candidates merely quoted a decimal approximation from their calculator. In part (iii), it was disappointing how often only one solution was given. Only a few able candidates met the challenge offered by part (iv) successfully. Many candidates had no idea how to proceed with this final part. An accurate statement of the final answer was expected and the unfortunate $\sqrt{3}-2>k>\sqrt{3}+2$ or $\sqrt{3}+2<k<\sqrt{3}-2$ did not earn full credit. Other candidates showed some idea but offered $\sqrt{3}-2<k<\sqrt{3}+2$, which is the correct answer to a different question.

4724 Core Mathematics 4

General Comments

The complete range of marks, from 72 to 0, was seen in this examination; fortunately there were only a few at the lower end but the number was not insignificant. There seemed to be no problem with the length of the paper.

Comments on Individual Questions

1) This gave a good start for most candidates. The scalar product formula was used nearly all of the time though the cosine rule did appear; there was some mis-reading of vectors and the occasional use of $\sin\theta$ instead of $\cos\theta$.

2) (i) There were very few candidates who did not know the format of the partial fractions though some just started with $A(x+2)+B(x+1)$ and finished with the numerators in the wrong order. It was clear that they were relying on memory and did not know what they were actually doing.

(ii) The integral of $\frac{2}{x+2}$ was sometimes given as $\frac{1}{2}\ln(x+2)$ and the majority of candidates failed to show either modulus notation or a constant or both.

3) Two different methods were seen – either long division or the use of an identity. In general the identity method proved more successful as negative signs proved problematical for some in the division.

4) The principles of implicit differentiation were well known; occasionally $4x^2y$ was differentiated as $8x\frac{dy}{dx}$ but the main errors concerned the use of $\frac{dy}{dx} =$ at the beginning of the differentiation, the concentration waning at the stage of differentiating the 6 and the failure to note that the equation of the normal, and not the tangent, was being requested.

5) (i) This was generally fine; occasionally the wrong vectors were used but most candidates would not have realised the error as these vectors were also perpendicular.

(ii) A more logical approach to the solution of the equations was seen on this occasion; equations (1) and (2) were usually solved for s and t and the non-consistency of these solutions for equation (3) was then demonstrated. There were some candidates solving (1) and (2) for s, substituting this value in (3) to find t and then testing out these values in any of the 3 equations. It seems unlikely that these candidates really understood what they were doing. Having shown that the equations were inconsistent, the deduction of skew lines was usually immediate without any mention of the possibility of parallelism.

6) (i) This was generally done well though $6a^2x^2$ was not uncommon and examiners often had to wait until part (ii) to see the interpretation of $10(ax)^2$.

(ii) The algebraic manipulation in this part was excellent (particularly in comparison to Q9); it was rare to see the wrong basic equations produced. The elimination of b (or a) was soundly effected and the subsequent quadratic equation was solved with a clear reference to the non-acceptability of the negative solutions.

7) This was the bugbear for most candidates. They seemed unable to find a suitable approach to the first part and generally gave up; unfortunately, the majority of those who managed to start the second part failed to understand the simple relevance of the preceding work.

(i) The statement was clearly shown as an identity and suitable values of θ would have rapidly determined *A* and *B*. Some candidates compared coefficients of $\cos\theta$ on each side of the identity and then $\sin\theta$, generally producing correct values of *A* and *B*; whether or not they realised what they were doing and its mathematical significance was another matter. A third group of candidates, generally the very able but also quite a few of the least able, looked at the identity and just said $A = 2$ and $B = -2$.

(ii) This part was very rarely attempted and was usually incorrect when attempted. Quite a few realised that the hint was to change the $4\sin\theta$ in the numerator into $2(\sin\theta + \cos\theta) - 2(\cos\theta - \sin\theta)$ but the following response was usually to say that $\dfrac{2(\sin\theta + \cos\theta) - 2(\cos\theta - \sin\theta)}{\sin\theta + \cos\theta}$ was equivalent to $2 - 2(\cos\theta - \sin\theta)$ as the $(\sin\theta + \cos\theta)$ portions had cancelled out. Many of those who did progress to the stage of integrating $\dfrac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}$ decided to multiply numerator and denominator by either $\sin\theta - \cos\theta$ or $\cos\theta + \sin\theta$, so producing double angle equivalents; these were more complicated than the original, and the work time-consuming, but a surprising number of correct solutions were seen. If only candidates would just sit back when the going gets difficult and try to see the wood instead of the trees, they would make better progress – but so many rush straight into the first idea which hits them.

8) (i) This was generally well answered; common mistakes included the use of $\frac{dt}{dx}$ instead of $\frac{dx}{dt}$ and the omission of the constant of proportionality.

(ii) This was also well answered; on this occasion the use of definite integrals was not particularly suitable and few tried to use them – but errors in the standard method were usually due to carelessness rather than in the method.

9) (i) Those using the format $y - y_1 = m(x - x_1)$ usually sailed through whilst a number using $y = mx + c$ ran into mistakes when clearing the fraction $\frac{2}{3p}$.

(ii) In comparison to the algebra which appeared harder in Q6(ii), the simple change of the equation $21p + 20 = p^3$ into $p^3 - 21p - 20 = 0$ was abysmally performed – it was almost unbelievable how often it became $p^3 - 21p + 20 = 0$. However, either equation was usually solved in a satisfactory way. The other common, though anticipated mistake, involved candidates misinterpreting the question, assuming that the tangent was at the point $(-10, 7)$ instead of going through it. These candidates immediately reached an impasse, so wasting very little time.

10 (i) Most candidates understood the principles involved but the reduction of $(\cos^2\theta)^{3/2}$ to $\cos^3\theta$ defeated many. Many of those obtaining $\dfrac{1}{\cos^2\theta}$ in due course did not spot that it would then be advantageous to change it to $\sec^2\theta$ but moved into the double angle option. The change of limits, when attempted, was often into degrees and not radians (and this was slightly penalised).

(ii) This prompted a much better response than part (i), the main problem being with sign errors. Integration by parts (using the 'sensible' split) was performed well. The very occasional attempt using the substitution $\ln x = t$ was satisfactorily brought to a correct conclusion.

4725 Further Pure Mathematics 1

General Comments

High marks were gained by a good proportion of the candidates. The main areas where marks were lost were in algebraic errors or due to omission of working when the answer was given in the question.

All the questions were accessible to the majority of candidates and there was no evidence of candidates being short of time. Most attempted all questions, usually working sequentially through the paper.

As has been mentioned in previous reports, scales are often omitted from sketches and this leads to a loss of marks.

Comments on Individual Questions

1) (i) The most common errors were to shear with the x-axis invariant or for the image of (1, 0) to be (1, 1). Scales on the axes were often vague or omitted altogether.

(ii) This part was usually answered correctly, with some candidates obtaining the correct matrix, even though their sketch was not correct.

2) Most candidates expressed the given sum as two separate sums and so could obtain the correct value of a. The most frequent error was to use $\sum_{r=1}^{n} b = b$, rather than bn.

3) (i) Most candidates used the substitution given, but a significant number did not clear fractions as required or made algebraic errors.

(ii) Most candidates related the required value to the coefficients of the new cubic or to the symmetric functions of the original cubic correctly, but a small number mixed up the two methods.

4) (i) This part was usually answered correctly.

(ii) The most common error was to use 3 – 3i for z – i.

(iii) Most candidates multiplied by the conjugate, the value of the denominator being 13 rather than 25 being the most common error.

5) (i) This part was usually done correctly.

(ii) A good number of candidates stated that this product could not be evaluated and so lost all 4 marks.

(iii) A good number of candidates gave their answer as a 1 × 3 matrix and those who derived a single value often omitted the matrix brackets.

6) (i) The sketch of C_1 was frequently given as a circle, while C_2 was often either a complete line rather than a half line and often through (0, 2) or (0, 4) rather than the origin.

(ii) An incorrect sketch usually resulted in a complex number that satisfied C_2, but virtually no candidates appeared to check that their answer satisfied C_1.

7) (i) The majority of candidates answered this part correctly.

(ii) Most candidates were able to find an inverse matrix, the omission of the determinant being the most frequent error. Some candidates post-multiplied by the inverse matrix, while some used the value found in part (i) throughout part (ii).

8) (i) Most candidates found the correct terms of the sequence.

(ii) Some candidates simply replaced n with $n - 1$ in the given recurrence relation, rather than suggesting an explicit algebraic form for u_n.

(iii) The induction proof was completed successfully by most candidates.

9) (i) The given result was usually derived correctly.

(ii) Most candidates used part (i) to obtain values for the sum of the new roots and found the product correctly, but then omitted $= 0$ in their final answer for a quadratic equation.

10) (i) The given result was usually derived correctly.

(ii) Many candidates failed to appreciate that the first three terms of the sum need to be expressed using part (i) in order to see which terms cancel. Similarly, many candidates did not look at sufficient terms at the top end of the sum to be able to see which terms remain.

(iii) Most candidates found the sum to infinity of their answer to part (ii).

(iv) A good proportion of candidates did not realise that the required sum is just the answer to part (iii) minus the answer to part (ii). As a result many candidates obtained an equation that did not have a positive integer root.

4726 Further Pure Mathematics 2

General Comments

In general, the candidates answered the questions in the order set and were able to pick up some marks in each question. No question proved to be too difficult. Marks were usually gained at the start of each question, but too often candidates were unable to complete the later parts. It was disappointing that candidates could not take leads from the earlier parts of a question to get into the rest of the question. This was particularly true in Q8 where the majority of candidates resorted to the exponential definition of sinh x in parts (ii) and (iii), instead of using part (i).This occurred in other questions, where candidates looked for a "set" method instead of dealing with the question as given.

It appeared that candidates were not as well prepared as in previous years. It could be that candidates had insufficient time to prepare for the paper, but there was a lack of precision and brevity throughout. There was some evidence that overlong methods and poor presentation led to some candidates rushing towards the end. Marks were often picked up early in the paper, but marks noticeably declined in later questions.

Comments on Individual Questions

1) (i) Most candidates found that this question gave them a sound start, although a substantial minority could not differentiate $\ln(1+\cos x)$. Even so, candidates went on to make a reasonable attempt at $f''(x)$ and to use their answers in a Maclaurin expansion. It was surprising how many candidates attempted to simplify their expression for $f''(x)$, often over many lines, instead of producing $f''(0)$ at once. This was the first of many places in which time was lost.

(ii) Most candidates used the general Maclaurin series correctly with their values of $f(0)$, $f'(0)$ and $f''(0)$. The question as a whole often produced a minimum of 4 marks.

2) (i) It was clear that "verify" was not fully understood. Whilst some credit could be gained by stating "$\cos^{-1}(\frac{1}{2}\sqrt{3}) = \frac{1}{6}\pi$ and $\frac{1}{2}\sin^{-1}(\frac{1}{2}\sqrt{3}) = \frac{1}{6}\pi$", it was expected that more justification would be given, particularly in dealing with the ½ in front of the $\sin^{-1} x$. Candidates who solved $\cos y = \sin 2y$ were on safer ground and were usually successful. As the question referred to a given point, dividing through by $\cos y$ (or ignoring $\cos y = 0$) and taking one solution of $\sin y = \frac{1}{2}$ was acceptable.

(ii) Most candidates could quote the results for the derivatives required (though the ½ was often lost in front of the $\sin^{-1} x$) and derive the correct answers for the given point. However, it took many candidates a number of lines of working to do so, again wasting time. Candidates who differentiated from scratch, produced $dy/dx = -1/\sin y$ (for example) and inserted the y-value of the given point often reached the answer more quickly.

3) (i) This part was generally well done, with only a minority of candidates using the y-values only (for 1 mark) and not relating these values to areas.

(ii) Apart from calculator errors, most candidates produced the answers 3.87(3) and 4.33(1). The majority then believed that $A<4.33(1)$ meant that $A<4.33$. This point has been highlighted in previous reports.

4) (i) This part was generally well done, with candidates using the correct formula and knowing the integrals required. There were some problems relating to the use of the limits, but many candidates gained at least 4 marks.

(ii) With the answer being given, many candidates successfully transformed the polar to the cartesian equation (or vice versa). There were some excellent answers, usually involving the elimination of θ first and then r, with better candidates leaving the replacement of $r = \sqrt{x^2 + y^2}$ to the end. There were some original answers, such as rewriting

$$r\cos\theta = \cos\theta + 2$$
$$r^2\cos\theta = r\cos\theta + 2r$$
$$rx = x + 2r \text{ and so on.}$$

However, other candidates spent an inordinate amount of time on this part, often getting the answer but not recognising it until numerous other steps had been completed.

5) (i) It was disappointing to read answers such as "dy/dx = $e^{-x} - xe^{-x} = 0$ when x =1". Candidates could substitute $x = 1$ into dy/dx to produce 0 or divide by e^{-x} to get the answer given. Most candidates were able to show $x = 1$ in some way or other. But the general presentation of this part was poor.

(ii) Only a minority of candidates was able to describe the Newton-Raphson method in terms of tangents intersecting the x-axis. Some credit was given for a discussion of both $f(x_1)$ and $f'(x_1)$ in the formula, but it was expected that candidates would discuss the process rather than the formula.

(iii) This part was generally well done, although it was evident that some candidates could not use their calculators to produce iterative answers at once, but started from scratch each time. Other candidates believed that x_4 was α. Most candidates scored at least 3 marks on this part.

6) (i) Most candidates produced asymptotes of the form $x = 1$ and $y = ax + b$. Minor algebraic errors leading to the correct $y = 2x - 9$ were condoned. Most candidates divided out for this asymptote, whilst those who equated coefficients spent a longer time for the same result. Overall, the attempts at the oblique asymptote were better this time.

(ii) The first 3 marks were often gained, although candidates did not appear to know why they were considering $b^2 - 4ac \geq 0$. They did not lose marks. Many candidates then believed that getting $y^2 + 14y + 169 \geq 0$ was sufficient and stopped. Others continued by showing $b^2 - 4ac < 0$ for the quadratic in y, but were then not clear as to what this meant. Candidates relating this to a sketch of the quadratic were able to gain full marks if clear. Only a minority completed the square and drew the required conclusion. Candidates are expected to make their methods clear. Candidates choosing a maximum/minimum approach also gained the first 3 marks only.

7) (i) Most candidates attempted integration by parts on the correct product, with the majority producing the correct given answer. Some allowance was made to those candidates who were not entirely clear, particularly in the use of the limits.

(ii) Very few candidates were able to do this part. Candidates should have expected to use part (i) to produce the required reduction formula, as part (i) was not in itself such a formula but did contain $(1 + x^2)^{-n-1}$. However, candidates who attempted this part often used parts again on the given result in part (i).

(iii) Surprisingly few candidates scored well in this part. Attempts to put $n = 1$ in the given result often led to $4I_2 = 2^{-2} + 2I_1$ and too many candidates attempted to find I_0, despite the wording of the question.

8) (i) This part was generally well done, with candidates using the correct definition of sinh x. Most candidates then attempted to cube their expression, although few were able to use the binomial expansion. Again, time was lost by multiplying out three brackets. Most candidates then expanded the right-hand side as well and showed that the two terms were equal in exponential terms. Better candidates used the binomial and then collected together relevant terms to produce the right-hand side, a neater and quicker solution. However, candidates often gained full marks whichever method was used.

(ii) This part was poorly done, with only a minority of candidates using the result in part (i). The majority resorted to the sinh definition and attempted a polynomial in powers of e. This generally led nowhere. Even those who attempted to use part (i) failed to solve for sinh x or $\sinh^2 x$ (or even for k), failing to recognise a simple quadratic. Others using $b^2 - 4ac$ were able to get an answer, although it often involved an equals sign in what should have been a precise inequality. Such candidates could still gain 2 marks.

(iii) As a consequence of part (ii), few candidates attempted this part. The successful candidates quickly gained the marks, although some omitted the ± sign. A few who persevered with the exponential definition gained some credit by producing and attempting to solve a cubic in e^{2x}.

9) (i) Most candidates were able to pick up at least 2 marks. A number of candidates used the logarithmic equivalent, usually successfully.

(ii) It was surprising how few candidates could write the answer down. Those using a substitution were often successful, but the leading ½ was omitted from many answers.

(iii) Again, it was disappointing how few candidates could use the clues of the earlier parts to get a reasonable substitution. Attempts at using trigonometric substitutions came to little, whilst other candidates believed that they could write the answer down at once. Here is an example of where candidates should consider the marks available. The candidates selecting a reasonable hyperbolic substitution usually picked up at least 5 marks. Any reasonable, correct answer in x was acceptable.

4727 Further Pure Mathematics 3

General Comments

As usual in the January session, this paper attracted only a small entry. Most centres require the whole of Year 13 to prepare candidates, and it has to be said that some candidates this time appeared to lack the experience necessary to tackle a variety of questions, even when some of these were fairly standard examples. Nevertheless, there were a few candidates whose work was almost flawless, and whose solutions displayed the qualities expected of first class mathematicians. All questions, with the exception of Q4, contained at least some parts which were answered satisfactorily by most candidates. There did not appear to be any problems with time, as all candidates reached the end of the paper.

Comments on Individual Questions

1) (a) All 3 marks were obtained by many candidates. For part (i) most gave a satisfactory reason, either stating a pair of non-commutative elements, or by noting the absence of symmetry across the leading diagonal. There were a few instances of candidates misreading the order of operation in the table ($ap = r$, not q), but they did not forfeit the mark. The other two parts were answered correctly in most cases, with just a minority omitting the identity element e in their answer to part (iii).

   (b) Some candidates seemed not to know how to answer this, but the majority scored all 3 marks without difficulty.

2) This was a very standard second order differential equation, but only about one third of candidates scored all 7 marks. The complementary function was nearly always correct, but the most common error was then to attempt a particular integral of the form px, rather than $px + q$. Those who tried a quadratic form were not penalised, provided they correctly found the coefficient of x^2 to be zero. Algebraic errors were seen occasionally, but most obtained the final mark for adding the complementary function and particular integral together.

3) This question was not done well, especially parts (ii) and (iii). Many candidates did not appear to appreciate the distinction between the points A, B and P and their position vectors. Although some recognised vectors such as (r – a) and (a – b), they did not always relate them to the points P, A and B and therefore to the loci.

   (i) Of the three parts this was the most successful in terms of marks awarded. It should have been a simple matter to say, in some way, that P was on the line OA, between O and A, but answers were rarely expressed as simply as this. Some stated that the locus was a circle.

   (ii) The 2 marks were awarded for identifying the vectors (r – a) and (r – b) with AP and BP, and for explaining that, because these lines were parallel, then $\sin\theta$ in the vector product definition was 0. The second of these statements was seen quite often, but it was rare for the mark for the first to be awarded.

   (iii) It had been expected that this would be the most demanding part of the question, as the vector equation is not one of those usually used in the numerical type of vector problem. However, as it is not unlike the equation in part (ii), the better candidates realised that the position vector of P was parallel to AB. Only a few also stated that the line passed through O.

4) Despite the clear indication of the method to be used and the stated assumption about the integral of a complex exponential function, this question defeated about a quarter of the candidates. But most of the rest made some progress in the right direction, with quite a number managing at least the integration, the insertion of limits and the multiplication by the complex conjugate of the denominator. The details of the rest of the working were done correctly by the better candidates.

   The two main stumbling blocks for others were the simplification of $e^{\frac{3}{2}\pi i}$ and the separation of real and imaginary parts.

5) This first order differential equation was solved well, with most candidates obtaining most of the marks.

   (i) Almost all answers put the equation into integrable form, usually by finding the integrating factor, but occasionally by multiplying through by x. Integration by parts was then carried out, generally accurately, though with occasional sign errors. Unusually, the most common mistake came at the end, when a number of candidates forgot to divide the arbitrary constant by x when writing the general solution.

   (ii) Almost all answers earned at least the method mark for substitution of the given values of x and y into their solution.

   (iii) There was a pleasing number of correct answers to the request for a function to which y approximates when x is large. Credit could also be earned if the wrong solution had been obtained in part (ii).

6) (i) Although the work for this part was not difficult, many candidates did not realise that the request was essentially to find the perpendicular distance from a point to a plane, and to find the coordinates of the point where the perpendicular meets the plane. As a result many floundered, using (p, q, r) for the point on the plane and writing down a variety of linear equations which may have been correct, but which did not, in most cases, lead to a solution. Of those who did make progress, most attempted the first method shown in the mark scheme which used the parametric form of the equation of AB to find the coordinates of B first. The distance AB was then found easily. A smaller number of solutions found the distance first. They then either followed the procedure shown in the mark scheme or returned to the parametric equation method for the coordinates of B.

   (ii) This part was independent of part (i) and candidates were much more successful. Although a few made no significant attempt, perhaps owing to unfamiliarity with this topic in the specification, those who knew that they needed to find the direction of the normal to the plane ACD had no difficulty in finding an appropriate vector product and using the scalar product of the two normals.

7) Questions of this type have not been done well, but perhaps the structure of this one helped candidates to see their way through, and it was done rather better.

   (i) Part (a) was answered well, by several different methods. These included stating the exact values of $\sin\frac{6}{8}\pi$ and $\sin\frac{2}{8}\pi$, and using the symmetry of the sine curve about $\frac{1}{2}\pi$ or, equivalently, using $\sin(\pi - \theta) = \sin\theta$. Those who attempted part (b) by sketching two curves, as suggested, were usually successful, although some had the wrong number of oscillations for $y = \sin 6\theta$. Others quoted appropriate symmetry of the sine function.

(ii) Candidates are usually confident in proving trigonometric identities by using de Moivre's theorem, and this one was no exception. The binomial expansion of $(\cos\theta + i\sin\theta)^6$ was accurately done, or at least the imaginary part which is all that is necessary in this case. Most then replaced $\sin^2\theta$ by $1-\cos^2\theta$ and then completed the proof, showing sufficient working.

(iii) In order to do the last part it was necessary to realise that using $\sin 6\theta = \sin 2\theta$ reduced the identity in part (ii) to a quadratic equation in $\cos^2\theta$, and many spotted this, but it was very common to see the equation written as $16c^4 - 16c^2 + 3 = 0$ instead of $16c^4 - 16c^2 + 3 = 1$. It was only the best candidates who solved the right quadratic correctly and justified taking the negative sign for the larger root.

8) Many aspects of group theory were tested in this question, and almost all candidates gained some marks.

(i) The identities caused few problems, especially as three of them were 1. For group *D*, some only stated that *m* and *n* had to be equal, rather than giving the identity itself.

(ii) The concept of isomorphism was well known, and most gained at least some marks here. Many constructed all three group tables, from which the isomorphism between *A* and *C*, and the absence of an isomorphism between the other two pairs was easily deduced. Others stated appropriate properties of the groups, such as their being cyclic, or not, or wrote down the orders of the elements. In some cases, however, insufficient detail was given: simple references to not having the same structure were insufficient when they were not supported by evidence.

(iii) The proof of the closure property was usually attempted correctly, with only a minority multiplying two elements which were not distinct or general.

(iv) It seemed that some candidates perhaps misread this final part. It did not ask for a sufficient reason that the set of numbers of the given form did not form a group, but it asked which of the group properties were not satisfied. In this case the only property which does hold is that of associativity, so all of the other three had to be written. Of course, the lack of an identity does imply the lack of inverses, but both of these were required, together with closure which was the one most often omitted.

Chief Examiner Report - Mechanics

The quality of solutions was good for all papers. The principles of mechanics were well understood, and marks were lost for reasons other than a lack of understanding. Reading questions carefully (so ensuring that all parts of a problem are answered) or showing working clearly (so that incorrect answers gain maximum credit) can significantly improve the score on a paper.

Candidates should automatically copy any diagram from the question paper into their answer booklet, so that any annotations can be used in interpreting their answer. Candidates should also be aware that marks are at risk when answers are not given to the specified degree of accuracy (usually 3 significant figures).

4728 Mechanics 1

General Comments

Candidates were well prepared for the examination, and the solutions presented were of a high standard. Negative values were used and interpreted sensibly, and necessary working was almost always shown. It was rare for a question not to be attempted, or for an answer to obtain no marks.

There were no general weaknesses shown, though the last parts of Qs 6 and 7 were found particularly difficult. The most common conceptual problem was regarding the product of mass and acceleration as a force. A feature of many papers was candidates not relating the tariff for a question to the quantity of work to be done or leaving parts of a question unattempted. It remains a concern that candidates risk the loss of many marks by not following the rubric requirement that approximate answers be given correct to 3 significant figures.

The more common errors are described below.

Comments on Individual Questions

1) This question was usually well answered, and candidates who used Newton's Second Law were very successful. More informal methods, involving the separate evaluation of 70×9.8 and 70×0.3, were common, though sometimes spoiled by candidates subtracting 21 from 686.

2) Very few candidates used momentum equals the product of mass and speed, so correct solutions were common. The most common error was not specifying the final speed of the skaters, even when identifying correctly their direction of motion.

3) (i) Invariably candidates obtained the correct magnitude of the resultant. Difficulties were confined to the calculation of the bearing, and two errors were common. Some scripts included diagrams which were consistent with finding the *difference* of two vector quantities. Others showed a confusion of the bearing convention with that for the unit circle used in trigonometric ratios of non-acute angles, so that X and Y were respectively vertical and horizontal.

   (ii) The correct deductions about the magnitude and direction of E, based on the values for R were usually shown.

4) (i) Use of constant acceleration formulae was very rare, and few errors arose from calculus. Invariably candidates showed how the given values of zero would arise from numerical substitution in the relevant formulae. However, partial solutions were common. Whether this was a result of mis-reading the question or faulty logic (the particle is still at O so $v = 0$) was difficult to know.

   (ii) Candidates understood clearly that acceleration is the derivative of velocity. Scripts where no formula for v had been obtained in part (i) usually included its derivation at this stage, and due credit was given.

5) (i) In nearly all scripts the negative value of a arose from the correct work. Sign errors by candidates who considered the car in part (b) the driving force was 1440 N. In part (c) the most common mistake was to find the time, not the distance, needed to achieve the required reduction in speed.

(ii) Most candidates considered the combination of car and trailer, though correct solutions in which Newton's Second Law was applied to the individual components were seen. The common errors were to ignore the components of weight, or to omit *g* when evaluating them. Curiously, such mistakes occurred in part (b) when part (a) had been done correctly.

6 (i) Nearly all candidates gained 2 marks.

(ii) The new value for the normal component of reaction was usually calculated, though sign errors appeared in some scripts. Some candidates continued by equating the new value of the frictional force to the horizontal component of the 4.9 N force, though most solutions for part (a) were fully correct.

In part (b) nearly all candidates evaluated the mass of the particle, and the only common error was to obtain a slightly inaccurate value for the acceleration as a result of prematurely approximating the magnitude of the frictional force.

(iii) Though good candidates appreciated the subtlety of the question, correct solutions were very rare. Most commonly, candidates repeated a calculation of the type done in part (ii) and gave an answer of 3.49 N.

7 (i) Many correct answers were obtained, though some inappropriate rounding of the exact value 0.448 m to an incorrect 0.45 m was seen.

(ii) Solutions were seen in which candidates continued to work with an acceleration of 1.4 ms^{-2}, but most used $a = -9.8$ and were able to obtain high marks. However the request for the *total* time, including the first 0.8 s, was often misunderstood.

(iii) The correct diagram was generally drawn, though a significant number of graphs continued below the *t* axis taking in the time until the string presumably again became taut, and *B* "came suddenly to rest".

(iv) Many correct solutions were seen, with few attempts to apply Newton's second law around the pulley.

(v) Most solutions incorrectly included the masses of *A* and *B* in part (a), but candidates who realised that the slack string would exert no force on the pulley were able to give the correct tension in the chain in part (b)

4729 Mechanics 2

General Comments

The paper demonstrated a broad spread of ability and level of preparation for this early January examination. There were many excellent scripts which showed thorough understanding and some of which scored full marks. On the other hand, a small number of candidates showed very little knowledge of the techniques required. As is often the case, the clarity of diagrams was often poor and this frequently led to misunderstanding, particularly with Qs 3 and 8. On a positive note, there was very little confusion between speed and angular speed in Q6. However, many candidates could improve their performance by taking greater care over presentation, notation and their diagrams. It is, of course, possible that candidates use the diagrams on the question paper, but this is not helpful to the examiner seeking to understand an incorrect solution. Most candidates appeared to have had sufficient time to complete the paper.

Comments on Individual Questions

1) (i) Well answered.

   (ii) Very few candidates failed to gain the 2 available marks, particularly as it included a follow through from part (i).

2) Candidates preferred to use Newton's Law rather than energy. However both approaches often led to errors as many candidates were not sure how to deal with the friction and change in height. A fairly safe approach is to look at potential and kinetic energy transfer and to introduce the energy lost due to the work done against friction. Some candidates omitted g and some incorrectly stated $R\cos 30° = mg$.

3) Many candidates were not clear about the direction of the reaction between the rod and the smooth wall. Examiners were sometimes faced with diagrams too small to be seen clearly. Another criticism was that moments equations were often started but the second half often omitted a distance. There were different options for solving this problem but the simplest was to take moments about A having realised that the reaction force was perpendicular to the wall. Perhaps it helps to realise that with a smooth wall, it is not possible for there to be a component of force parallel to the wall.

4) (i) This was well answered, perhaps helped by the fact that the answer was given.

   (ii) Most candidates (understanding power, force, velocity and acceleration) created good solutions.

   (iii) This was also well answered.

5) (i) Most candidates correctly executed the equation for conservation of linear momentum and coped with the different directions of motion. The speed of Q was often correctly found. However, candidates found it more difficult to interpret the range of possible values of k; $k \leq 1$ was not acceptable as the direction of motion was reversed.

   (ii) In calculating the impulse, candidates were less good at coping with the change of direction. Many gave the impulse as mu and answers of $-3mu$ lost one mark as the magnitude was requested.

	(iii)	Most candidates knew what to do but there were errors in calculation, particularly in finding the speed of *Q* after the impact.
6)	(i)(a)	Well answered, perhaps helped by the fact that the answer was given.
	(i)(b)	Poorly answered. In resolving horizontally, many candidates ignored the tension in the section *PB*.
	(ii)(a)	Candidates had a better understanding of the two tensions than they did in part (i)(b). Some could not get very far as they thought that the tensions were different in the sections *PB* and *RB*.
	(ii)(b)	There were many correct solutions.
7)	(i)	This part was well answered.
	(ii)	A large variety of techniques were used in creating good solutions.
	(iii)	A disappointing number of candidates interpreted the speed on entry into the cloud to be simply the vertical component and did not consider the horizontal component of velocity.
8)	(i)	Well answered, perhaps helped by the fact that the answer was given, although some candidates used α in degrees.
	(ii)(a)	Most candidates coped with the combination of rectangular and semicircular laminae and scored full marks.
	(ii)(b)	Many candidates were successful here perhaps helped by the lead given in part (ii)(a).
	(iii)	This final part question was found to be the most difficult on the paper. In taking moments about *A*, many candidates ignored the component of weight in the direction parallel to *DA*. There were a small number of excellent solutions where candidates correctly calculated the perpendicular distance between *A* and the line of action of the weight of the board.

4730 Mechanics 3

General Comments

Candidates were generally well prepared for examination and only a few scored very low marks. However a smaller than usual number of candidates scored very high marks. One reason for this was not completing the paper. It was particularly unfortunate that some candidates did not have the opportunity to attempt Q7, where parts (i) and (iii) were generally found to be straightforward by those who did answer the question.

Another reason why very high marks were not reached may be that three particular elements of work, each worth two marks, defeated a higher proportion of candidates than might have been expected. These elements are in Qs 4(i), 7(ii) and 7(iv) and are referred to below.

Comments on Individual Questions

1) (i) This part of the question was well attempted, although some candidates considered an impulse/momentum triangle, calculating angles but making no progress towards the required answers. Other candidates found the components of momentum instead of velocity, but usually proceeded to obtain the magnitude of the velocity after first finding the magnitude of the momentum. A special ruling in the mark scheme applies to the work of such candidates.

   (ii) Many candidates were able to write down the correct answer in this part of the question, but some omitted the minus sign.

2) All three parts of this question were very well attempted.

3) (i) Almost all candidates recognised the need to use both the principle of conservation of momentum and Newton's experimental law, and there were very many correct answers.

   (ii) Most candidates were aware of the fact, and of its significance, that the component of the velocity of *A* remains unchanged as a result of the collision. However a significant number of candidates failed to obtain the correct answer because accuracy had not been sustained throughout both parts of the question.

4) (i) The reason for requiring candidates to obtain a differential equation in a particular form was to assist them in solving the equation, as required in the second part of the question. Almost all candidates were able to obtain the equation in the form $v\frac{dv}{dx} = g - 0.49v$ or $\frac{v(dv/dx)}{g - 0.49v} = 1$, but many candidates were unable to proceed to the required form of the equation.

   (ii) This part of the question was well attempted, although many candidates made an error on integrating $\frac{20}{20 - v}$. A considerable proportion of such candidates obtained the factor $\frac{-1}{20}$ instead of –20. Omission of the minus sign was also common.

5) (i) Almost all candidates recognised that the maximum speed occurs when the acceleration is zero, and used $a = 0$ accordingly in applying Newton's second law.

(ii) Most candidates used the principle of conservation of energy in both parts of the question.
(iii) There were many correct answers in both parts, although some candidates had an inconsistency between the height of *O* above *P*, as used in the expression for GPE, and the extension as used in the EPE. Some candidates found the extension in part (iii) as 2.4m, without giving the required answer for the distance *OP*.

Some candidates used Newton's second law in both parts, and were generally less successful than those who used energy. Some such candidates attempted to find an expression for v^2 in terms of the displacement of *P* from its equilibrium position, or the extension in the string, or the distance OP, by solving the differential equation obtained using $a = v\frac{dv}{dx}$. Other candidates used $a = \frac{d^2x}{dt^2}$, with x as the displacement of *P* from its equilibrium position, to show that *P* moves with SHM, which justifies the use of the formula $v^2 = \omega^2(a^2 - x^2)$.

6) (i) This part of the question was well attempted. However a significant minority found the required speed but did not find the tension. Answers in part (ii) suggest the omission was accidental in almost all such cases.

(ii) Many candidates scored all the available method marks, but in some such cases there were errors of accuracy, notably the absence of a minus sign with the weight component term of the equation obtained by using Newton's second law.

A significant minority of candidates thought the instant the string becomes slack is when the velocity is zero.

7) (i) Most candidates used Newton's second law with $a = \frac{d^2x}{dt^2}$, but very many made an error in writing down T, notwithstanding the definition of x in the question, and many others omitted the weight of *P*.

(ii) Although some candidates used $\omega = 7$ instead of 3.5 in $T = 2\pi/\omega$, most candidates obtained T = 1.8. However hardly any candidates satisfactorily made the connection between the period and the required time interval.

(iii) This part of the question was very well answered by the relatively few candidates who attempted it.

(iv) There were relatively few attempts at this part of the question and most were either confused or appeared to be executed in haste. However a significant minority made the relevant calculation $-3.5(0.08)\sin(3.5\times0.25) = -0.215$, but most such candidates attached the label v to this, and endorsed their mistaken belief that they had reached the required answer by attaching the units ms^{-1}. Almost all of those who did realise they had found the *angular* velocity, had no idea how to proceed to the required answer.

Chief Examiner's Report - Statistics

Centres are particularly asked to note the following points which have been agreed by the examiners responsible for these papers in order to encourage good practice.

1. Answers given to an excessive number of significant figures (such as "probability = 0.11853315"), which have not in the past lost marks, may in future be penalised.

2. Hypothesis tests are likely no longer to include the explicit instruction "stating your hypotheses clearly"; any answer to a question involving hypothesis tests should include a statement of hypotheses unless they are already given in the question.

3. Likewise, questions that involve critical regions (particularly for significance tests using discrete distributions) may not ask explicitly for the relevant probabilities to be quoted from tables, but candidates should always write down the values of these probabilities.

4. Conclusions to hypothesis tests should be stated in terms that acknowledge the uncertainty involved. Thus "the mean height is 1.8" is too assertive and may not gain full credit. A statement such as "there is insufficient evidence that the mean height is not 1.8" is much to be preferred.

It would seem worth reminding candidates that modelling assumptions should always be stated in the context of the question. Thus "events occur at constant average rate" will usually not gain credit. Such an answer should be written, for example, "house sales occur at constant average rate".

4732 Probability & Statistics 1

General Comments

Most candidates showed a good understanding of many of the topics in this paper. In Q8(iii) most candidates used far more algebra that was necessary and in so doing showed a lack of algebraic facility. This was also the case in Q7(ii). There were several questions that required an interpretation to be given in words, and these were answered less well than has often been the case in the past. Some candidates lost marks unnecessarily because their answers did not refer to the context.

Very few candidates rounded their answers to fewer than three significant figures, thereby losing marks. However, in a few cases, marks were lost through premature rounding of intermediate answers.

Very few candidates scored full marks. This was due mainly to the difficulties found by candidates in the interpretative parts and in Qs 4, 7(ii) and 8(iv). A small number appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination, centres should refer to the published mark scheme.

Use of statistical formulae and tables

The *List of Formulae and Statistical Tables*, MF1, was useful in Qs 6(i) and 9(ii) (and in Qs 5 and 7(ii) for binomial tables and formula). However, a few candidates appeared to be unaware of the existence of MF1. Some candidates tried to use the given formulae, but clearly did not understand how to do so properly (e.g. $\Sigma x^2 p$ was misinterpreted as Σxp^2). Some candidates found Σxp or $\Sigma x^2 p$ correctly but then divided by 3 or 6. For the variance, some candidates omitted to subtract μ^2 from $\Sigma x^2 p$, or subtracted just μ. A few candidates used the less convenient version, $\Sigma (x - \mu)^2 p$, from MF1, leading to arithmetical errors in most cases. A few candidates quoted the formulae for r and/or b wrongly. In Q3(i) some candidates quoted their own formulae for r, rather than using the one in MF1. Usually these were incorrect.

Although this is relevant only for a small minority of candidates, it is worth noting yet again, that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1. They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Some candidates' use of the binomial tables showed misunderstanding. Others used the binomial formula rather than the tables where the latter were clearly more appropriate (in Q5(i)(b)). Perhaps some centres advise students to use the formula for all binomial calculations, since it is always applicable, whereas the tables can only be used for those values that are included therein. This is bad advice. On the other hand, in Q5(ii), a few candidates attempted to use the tables when they were not appropriate. Because there is no table for $n = 15$, these candidates averaged the values from the tables for $n = 14$ and $n = 16$. This is not an acceptable method.

Comments on Individual Questions

1) (i)(a) This question was well answered, with only a few candidates attempting to use a combination.

(i)(b) Many candidates recognised the need to regard AB as one item, but then omitted to consider BA as well as AB. A large minority divided by 5!.

(ii) Many candidates attempted to use one or both of their answers to parts (i)(a) and (i)(b). This is a blind alley. Those who understood that a new start had to be made sometimes treated it as a "with replacement" example or omitted to consider both AB and BA.

2) A few candidates treated the geometric variable as a binomial variable.

(i) This part was well answered, with just a few candidates finding $(^4/_5)^4 \times (^1/_5)$.

(ii) Many candidates found $(^4/_5)^4$ but then either multiplied by $^1/_5$ or subtracted from 1. As usual, a significant minority attempted the long method, often either omitting a term or adding an extra one, either $(^4/_5)^4 \times (^1/_5)$ or $(^4/_5)^{-1} \times (^1/_5)$. This latter would appear to come from a mechanical application of the formula for the probabilities from a geometric distribution. In fact, the best way to approach questions on the geometric distribution is probably by common sense or by native wit rather than by using the formula.

(iii) Some candidates tried to use *np*. Others gave an answer of $^1/_5$. Some tried to use Σxp. A few even attempted to use integration.

3) (i) This was generally well answered, although the errors noted above were seen.

(ii) Most candidates appeared to be unaware that the definition of r_s is the product moment correlation coefficient of the ranks.

(iii) Many candidates doubled or halved one or both of r and r_s. A value greater than 1 was common.

4) Candidates who tried to use (conditional) probability usually failed to score any marks. Most wrote: (i) 0.4 × 0.12 = 0.048, (ii) 0.4 × 0.88 = 0.352. Those who employed a more intuitive approach were often more successful.

This question tests understanding of the idea of conditional probability. Most candidates appeared not to be able to distinguish between "given that" and "AND". Others, however, knew more than was good for them about conditional probability and attempted to use the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$. This led to a long method in part (i), often incorrect. In fact this formula is not required for this module; it appears only in the specification for module S4. Whenever conditional probability is tested in S1, as in this question, it will only involve an understanding of the idea of conditionality, not any formal treatment.

(i) Most candidates multiplied 0.4 × 0.12. A few seemed to spot instinctively that $^{12}/_{40}$ was involved, but then went on to multiply this by 0.4.

(ii) Some candidates correctly found $1 - {}^{12}/_{40}$ (even though they may have mutiplied 0.12 × 0.4 in part (i)) and some went on to multiply this by 0.4. Most, however, just found 0.4 × (1 – 0.12).

5) (i)(a) This part was well answered. A few candidates subtracted from 1. Some found P($U \le 4$). A small minority added several values from the table.

(i)(b) 1 – 0.7046 (ie 1 – P($U \le 3$)) was common. A significant minority of candidates attempted to use the formula rather than the table. Most made arithmetical errors or omitted terms.

(ii) This part was well answered, although some candidates used some or all of the data from part (i).

6) (i) This part was well answered on the whole although the errors outlined above were not uncommon. Some candidates showed confusion between the formulae for a probability distribution and for a frequency distribution.

(ii) This part was answered well by many candidates. Some included only (1, 2) and not (2, 1). Some candidates just counted pairs, ignoring the associated probabilities.

(iii) Few candidates identified precisely the five pairs needed. Many omitted (2, 2). Some included some pairs twice. Some candidates just counted pairs, ignoring the associated probabilities. Sadly, almost none used a short-cut method such as P($Y = 2$) + (P($Y = 1$) + (P($Y = 3$)) × P($Z = 2$)

7) (i)(a) The two conditions required are independence and constant probability, both of which must be stated with reference to the context, i.e. mentioning matches or winning. Some candidates gave these two conditions in general terms and gained no marks. Many others gave general conditions for a binomial distribution stating e.g. "Two possible outcomes or "A fixed number of trials". Others simply restated facts given in the question, e.g. "He plays 10 matches" or "He does not draw any matches".

(i)(b) Most candidates gave answers such as $X \sim B(10, {}^1/_2)$ or "whether Andrew wins or loses". The few candidates who attempted a definition in words usually failed to include the crucial phrase "The number of . . ."

(ii) Some candidates used the formula to give a correct expression for P($X = 10$) and/or P($X = 9$) but failed to write an equation. Others had an expression involving p^{21}. Others gave correct combinations but incorrect powers of p and/or q, not adding to 21. Some found the correct equation and correctly evaluated the combinatuions but did not cancel them. Many could not handle the simple algebra involved in sorting out the powers of p and q (or $(1 - p)$). Some thought that $(1 - p)^{11} = 1 - p^{11}$. Others used $p - 1$ instead of $1 - p$. A few attempted to use logs.

8) (i) Any correct method for finding the quartiles was acceptable. However, surprisingly few candidates carried out a consistently correct method. For the quartiles, interpolation was common, but many candidates used ${}^n/_4$, ${}^n/_2$ and ${}^{3n}/_4$ instead of ${}^{(n+1)}/_4$, ${}^{(n+1)}/_2$ and ${}^{3(n+1)}/_4$. Some of those who used the correct fractions nevertheless rounded up or down apparently at random. Centres might wish to note that candidates generally find the simplest method for quartiles to be the following: the quartiles are the medians of the lower and upper halves of the data. (If n is odd, first remove the original median). This method yielded 22, 39 in this question. Alternative correct methods gave 21.5, 40 or 21.75, 39.5. Many candidates obtained a mixture with one quartile from one list and the other from another. These lost a mark.

(ii) This was fairly well answered, although some candidates wrote sentences whose meaning was so unclear that marks could not be awarded.

(iii) This was well answered. Candidates who referred to "anomalies" rather than "outliers" did not score the mark. Some stated that the median was preferable because it is the age of an actual person in the list, which in this particular case is not true.

(iv) Most candidates showed no familiarity with the use of coding or an assumed mean. Some just used the given totals as if they referred to x rather than (x – 200). These (perhaps by luck) gained 5 out of 6 marks. Many tried to obtain Σx and Σx^2 from the given totals. Some of these were successful in finding the mean, but not the standard deviation. A common error was (245 + 200) ÷ 49 (= 9.08) followed by $\sqrt{\frac{9849+200}{49}-9.08^2}$. For the few who understood coding, this was an easy six marks.

9) (i) Most candidates wrote about x being "independent" without justification or "controlled" which is incorrect. Discussion of the context was essential here.

(ii) This part was well answered although many candidates ignored the instruction about three significant figures.

(iii) This part was well answered.

There was confusion in parts (iv), (v) and (vi) about the relationship between the correlation coefficient and the reliability of an estimate using a regression line. The notion that extrapolation is not a good idea was understood – but often to the exclusion of everything else.

(iv) Most candidates referred to the low value of r_Q, although some referred to "small positive correlation" which rather misses the point. Others pointed to the huge variety in y values for $x = 8.5$. This answer was accepted. Some candidates stated (incorrectly) that as the pH value rises, so does the area covered.

(v)(a) A majority of candidates lost the mark because they referred only to the fact that 8 lies within the range of the original data and omitted to mention the high value of r.

(v)(b) This was well answered.

(vi) Most candidates referred (correctly) to the low value of r_Q, although a significant number of candidates stated that the regression line would provide a reliable estimate because 8 lies within the range of the given data.

4733 Probability & Statistics 2

General Comments

There were many very good scripts seen on this paper. Clearly a substantial proportion of candidates had a confident grasp of the material and many scored around 60 out of 72 with little difficulty. Presumably many of them were Further Mathematics candidates. There was also very little evidence of candidates running out of time. However, the paper also revealed a good deal of misunderstanding. Many candidates floundered whenever faced with anything slightly different from previous questions, provoking strong suspicions of rote learning. This was particularly apparent in Qs 3, 4(iii) and 8(ii). There are also a number of widespread misunderstandings of particular topics, notably the Central Limit Theorem and also anything to do with continuous random variables that went beyond routine integration.

It is particularly requested that Centres take note of the comments on Q6(iii).

Centres are reminded of changes in future examinations and mark schemes. Questions on hypothesis testing will not necessarily ask explicitly for hypotheses to be stated; stating of hypotheses should be considered automatically part of any such test. Such statements should be couched in terms of letters (representing parameter values) that candidates should define – for example, "where μ denotes the population mean concentration". Likewise, it should be assumed without explicit instruction that all relevant probabilities should be given to support answers such as critical regions or minimum sample sizes.

Comments on Individual Questions

1) (i) This was well done by many. The most usual error was omitting the sign of 0.674.
It is surprising that so many candidates attempt to solve the simultaneous equations by substitution, when the absolutely standard method, to write them in the form "$80 - \mu = 1.645\sigma$, $50 - \mu = -0.674\sigma$" and subtract, produces the answers so much more quickly.

2) (i) Most realised that the relevant distribution was B(12, 5/6). Some used "sampling without replacement", which is not correct for random numbers, but they could still get 2 marks out of 3.

(ii) Many realised that the *method* was unbiased, despite the unrepresentative *outcome*.

3) (i) Some candidates answered this easily and confidently. Others did some very strange things, apparently trying to force the question into one that they had met before – for example, finding both P(≤ 1) and P(≤ 2) and deciding which was below 5%. Having found P(≤ 1) and P(≥ 9), many did not realise that all they had to do was add the answers. Perhaps they thought that significance levels could only be things like 10% or 15%.

(ii) Most realised that they had to find $P(2 \le G \le 8)$ from Po(5.5), but there were frequent mistakes with end-points and tails.

4) (i) Almost all candidates got this correct.

(ii) Also very often correct, although there were also attempts to divide σ^2 by 40 or to use 59 instead of 60.

(iii) However, here almost all candidates thought that the Central Limit Theorem (or, according to a lot of candidates, "Central Limit *Theory*") had to be applied. This would have been relevant to part (ii), but not to part (i) where of course the formulae for the estimates have nothing to do with the distribution.

5) (i) Many candidates performed the significance test well. The common mistakes, all standard, were omission of the $\sqrt{4}$ factor, failure to state the conclusion in context, and (rarely, it is good to record) confusion of the roles of the population mean (500) and the sample mean (435).

(ii) Again poorly answered. The correct answer is that you can't perform the significance test without knowing the distribution of $\overline{X}$ and as n is small you cannot invoke the Central Limit Theorem. Some said that "the distributions had to be the same before and afterwards", but the distribution before prices were introduced is irrelevant. Some seem to think that the Central Limit Theorem includes the statement "$X \sim$ Normal implies that $\overline{X} \sim$ Normal", but this is not as serious a mistake as such complete failures of logic as "we need to assume it is normal so we can use the Central Limit Theorem". Unfortunately such answers were not all that rare, which is cause for concern.

It is also worth mentioning that there were a number of answers along the lines of "it can't be binomial or Poisson so it must be normal".

6) (i)
(ii) Most candidates scored at least 6 out of 9 in parts (i) and (ii), the utterly predictable common mistakes being "$P(> 4) = 1 - P(\leq 3)$" in part (i)(a), and the continuity correction wrong, or omitted, in part (ii).

(iii) Answers for the modelling assumptions were disappointing. First, such answers never gain marks unless given in the context of the question; thus "*events* occur at constant rate" does not get credit. Second, it is frustrating to find many still believing that "events occur singly" is a relevant modelling condition for the Poisson distribution. This is not true. "Events occur singly" is usually nothing more than part of the "independence" condition, as here. If not, it is either so trivially false that no one would think of applying the Poisson distribution to it (for instance, points scored in rugby), or meaningless, or simply wrong. Certainly in this context there is no obvious reason why the sale of more than one house at a time should by itself falsify the Poisson model. Many also wrote that "house sales have to occur randomly", which could at a pinch be taken to mean "independently" but really means nothing more than "they do not occur at exactly fixed intervals", with no implication about distributions. Candidates would be well advised to avoid the words "randomly" and "events" in answering this type of question. These comments have been made in many previous Reports and it is disappointing to find such errors recurring. Those who said "each house sale is equally likely" had confused the conditions with those for a binomial distribution. Specimen correct answers are:

(a) The sale of one house does not affect the sale of any other houses.
(b) House sales occur at a constant *average* rate.

Strictly (b) should not be just "constant *rate*" as this implies no random variation, but this distinction is probably pedantic and was ignored the examiners.

7) The disappointment with this question was the utter casualness with which the limits on the value of x were so often ignored. Many candidates drew lines that went well beyond 0 and 2 (in part (ii)) or 1 and 3 (in part (iv)) as if the statement "$f(x) = 0$ elsewhere" was meaningless. Even when this led to PDFs taking negative values it seemed to cause them no concern. In general, parts (i) and (iii) were found very easy, but in part (iv) few made it clear that the graph was translated 1 to the right, rather than 1 down. (Perhaps they had not realised that the non-zero range was now [1, 3].) Nevertheless some realised that, in part (v), the mean increased by 1 and the variance was

unchanged. Those who attempted to do part (v) by integration rarely got the right answers, mainly because they did not appreciate what the limits were (many said that both the mean and the variance "equalled ∞").

8) (i) This was probably the hardest question. As usual, a hypothesis test for a discrete random variable was poorly done, with many attempting to find $P(R = 6)$. Some attempted to use a normal approximation, but as this is not valid, they gained few marks. The particular wording of the scenario ("claims that at least 65% are in favour") caused some confusion, but it is easy to see what to do by realising that 6 out of 12 is below the expected value, so that one needs to find $P(\leq 6)$ from $B(12, 0.65)$. However, those who knew what to do did it well, and those who used a two-tail test could gain 10 of the 13 marks.

(ii) Unexpectedly, this was rarely answered correctly. Because $|0.65 - 0.5| = |0.35 - 0.5|$, the outcome of a significance test with $R = 6$ is the same for $H_1: p < 0.65$ as for $H_1 : p > 0.35$. Many took the common-sense view that if a test showed it was reasonable to claim that p was at least 0.65, then $p = 0.35$ was "clearly wrong", which is not the case. A large number misunderstood the scenario and thought that the residents' group formed a new sample, or something, saying that "they were biased".

(iii) By contrast, many answered this correctly and efficiently, although it requires some virtuoso work with statistical tables. Of course the values of relevant probabilities should, as always, be shown. A normal approximation is valid here, although it is difficult to justify its use by a non-circular argument and the working is intricate.

4734 Probability & Statistics 3

General Comments

There was a significantly larger entry than in January 2007 and although there were several excellent scripts, there were more candidates who had clearly not spent enough time practising the required techniques.

Most candidates are familiar with the procedures of hypothesis tests but some are using statements that are not to be encouraged. When not rejecting the null hypothesis some candidates conclude with, for example, "there is evidence at the 5% significance level to show that the proportions are equal", rather than " there is insufficient evidence to show that proportion of … is greater than…". Others make an over-assertive statement, e.g. "the proportions are not equal". Most candidates must know that we cannot prove a result using a statistical test.

As has been stated on previous occasions, candidates should make clear why a null hypothesis is (or is not) rejected. The examiners would like to see statements like "1.381<1.645, therefore do not reject H_o".

Comments on Individual Questions

1) (i) This did not turn out to be as straightforward as was hoped. Many candidates missed the divisors and some used a pooled estimate of s^2 (which was accepted).

(ii) The procedure for obtaining the confidence interval was usually known, but a correct variance was given only by more able candidates.

(iii) Most candidates were aware that the large sample sizes justified the use of a *z*- value.

2) (i) Since the standard deviation was given in the question, the confidence interval required a *z*-value. About 50% of the candidates used *t*.

(ii) A binomial distribution was usually recognised, but was not always carried out correctly. One candidate found the answer by calculating P(≤ 3) for a Geo(0.02), a method not seen before by the examiners.

(iii) Candidates often knew the differences in procedure if σ was unknown, but many forgot to state that an unbiased estimate of variance would be required.

3) (i) An obvious notation for the population proportions of the trains is p_W and p_N but several different others were seen. Most candidates are aware that μ should not be used. If *p* is not used, then the notation must be clearly defined by the candidate, and in the context of proportions (where the notation does not distinguish clearly between population and sample) it is always sensible to explain what the symbols used mean. In any case, it is good practice.

Many knew which variance estimate to use, but significant numbers used $p_1q_1/n_1+p_2q_2/n_2$ and this did not receive full credit.

(ii) Those that knew the required variance estimate in part (i) usually could give the one required here. Several referred merely to "pooled estimate", without stating what this was.

4) (i) The candidates who had met this type of problem usually coped well, although some had difficulty relating the *L* and S_1+S_2 to 1. Many used $L - 2S$, which is incorrect, but calculated the variance appropriately and this was accepted.

(ii) This was often confused with part (i) and gained very little credit.

5) (i) The required assumption for the paired-sample test is for the population of differences of the pairs to have a normal distribution. Each variable having a normal distribution is a sufficient condition, but is not necessary so this statement did not receive credit. The actual test was usually carried out well.

(ii) The data were clearly paired and correlated since the same gardener mowed the same area of lawn using the different mowers. It would be wrong to use the two-sample test with correlated pairs. A minority of candidates could express at least one of these ideas. Some thought the reason to be that the sample sizes were equal.

6) (i) A majority of candidates knew how to calculate the given value, but since this was given, a value to at least 3 decimal places should have been shown.

(ii) Most candidates could carry out the test accurately, including finding the value of υ, but some gave a null hypothesis of non-independence. The null hypothesis was actually given in the question, and this should have been used. Some merely stated "Independence", and this was not acceptable. Some candidates used "uncorrelated" rather than "independent", but uncorrelated variables are not always independent.

(iii) This part required a goodness-of-fit test, so those using the 2×3 table earned very little. Using the goodness-of-fit test often scored high marks, but the significance level was often given as 97.5%.

7) (i) Most candidates could obtain the required CDF, but in many cases it was not defined over the complete range of T.

(ii) This was the most searching question on the paper and only the most able candidates could obtain the given p.d.f. correctly. The key to success is starting from $P(H \leq h)$ and those that did this often were unable to convert $P(1/T^4 \leq h)$ to $P(T \geq h^{-\frac{1}{4}})$. The interval for h was very often wrong.

(iii) There were three main methods of obtaining the required value, and all of these were seen. Those that had a zero in their range of integration of $f(h)$ were only able to earn 2 of the 3 for this part. Some obtained full marks by converting to t, even with very little earned in part (ii).

4736 Decision Mathematics 1

General Comments

Most candidates were able to achieve reasonable marks on this paper, with rather fewer extreme marks than in some previous sessions. The candidates were, in general, well prepared and were able to show what they knew.

Some candidates threw away marks by not following the instructions in the questions and some resorted to assuming that the questions were all of standard types rather than trying to think for themselves.

Several candidates did not use graph paper or drew a freehand graph without ruling the lines. Some candidates used correcting fluid, or a variety of coloured inks rather than just using blue ink for written work.

Centres are asked to use treasury tags to *loosely* join the sheets when the paper has an insert or graph paper, rather than just placing the insert inside the answer booklet or tagging the sheets together using other means.

Comments on Individual Questions

1) Most candidates were able to apply the methods, although some misunderstood the order in which the boxes were stacked and a few did not go back to try to fit the first bin once they had begun on the second. In part (iii), several candidates were concerned that the weights were not evenly spread or that both methods used three bins so there was no advantage in sorting first, rather than discussing the practicalities of extracting the heaviest box from the bottom of the stack or taking the stack down to rearrange it.

2) (i) Some candidates omitted to label the vertices of their graph. Most candidates who had correct graph structures, even if they were unlabelled, were able to state that four moves were needed to get from position 1 to position 9.

(ii) Many candidates gave the definitions of Eulerian and semi-Eulerian but most then went on to state that neither of these applied to this particular graph. Candidates should note that the term ‘non-Eulerian’ has no meaning and was not an appropriate answer to the question.

3) (i) Several candidates gave no evidence of having used the table to apply Kruskal’s algorithm, and indeed quite a few specifically showed that they had used Prim’s algorithm on the network.

(ii) Most candidates were able to use their minimum spanning tree from part (i) to find the weight of the minimum spanning tree for the reduced network. For some candidates this was all they did, others remembered that they needed to add the two shortest arcs from *G* but chose *GE* and *GF* rather than *GB* and *GE*.

(iii) Most candidates were able to apply the nearest neighbour method, at least as far as visiting every node, but some did not close their cycle by returning to the start point and quite a few did not write down their cycle.

4) (i) Nearly all the candidates were able to extract the required information and label the arc weights appropriately. There were quite a few slips in the subsequent application of Dijkstra's algorithm, in particular candidates who wrote down extra temporary labels (temporary labels should only be recorded if they are an improvement on current values) or deleted their temporary labels (values that have been obliterated cannot be marked) or those who did not complete the permanent label or order of labelling boxes. Most candidates found the route *J*-*A*-*G*-*U*-*M*.

(ii) Some candidates described what the order of labelling value represents, while others gave an answer that was devoid of context. Most were able to state that the value of the permanent label at *M* showed the shortest possible journey time in minutes.

(iii) There were many interesting suggestions for why Jenny might prefer to choose another route. The most popular answers were that flying would be expensive and that she may want to visit her friend on the way. Some candidates identified reasons relating to the quantity being minimised (she might want the cheapest route or the shortest route rather than the quickest route), some gave reasons why she may not want to use the chosen route (environmental issues, planes may be delayed or not at the right time) and others gave reasons for preferring other routes (if it were raining she may prefer to take the taxi rather than walk, she may want to break her journey at her friend's house and continue with a shorter journey on the morning of the meeting). Any sensible suggestions were accepted, provided they related to taking or rejecting the route identified.

5) (i) Most candidates identified x as panelling, y as paint and z as pinboard, but several did not specify what was being recorded (the area covered, in square metres).

(ii) The two marks for this part were for identifying that the inequality came from the cost constraint and then stating that this gave $8x + 4y + 10z \leq 150$, from which the given inequality followed.

(iii) Some candidates did not read back to identify that Mark wants to complete the job in the shortest time possible, instead they opted for trying to minimise the cost.

(iv) This part just involved substituting $24 - x - y$ for z in the objective and the four inequality constraints (who of which were trivial since they did not involve z). Several candidates did not rewrite the objective function in terms of x and y only, or just ignored the z term completely, and quite a few only replaced z in the inequality $4x + 2y + 5z \leq 75$ but did nothing about the constraint $z \leq 2$.

The candidates who did try to substitute for z often ended up with their inequality signs reversed, for example achieving $-x - 3y \leq -45$, but then claiming that $x + 3y \leq 45$.

(v) Candidates who did not use graph paper scored no marks. Several candidates ignored the suggestion to only use values from 9 to 15 on each axis and consequently ended up with a very tiny feasible region. Candidates should use ruled lines not draw in graph lines freehand.

When a range is suggested in the question it should be used, even if it results in the candidates needing to calculate points on the lines that are not on the axes.

Some candidates did not label their axes, in these cases markers assumed that the horizontal axis was x. A small, but worrying, number of candidates could not draw the line $x = 10$ correctly, or 'hedged their bets' by drawing $x = 10$, $y = 10$ and $x + y = 10$ on the basis that one of them must be right.

6) (i) Most candidates were able to formulate an appropriate initial Simplex tableau, a few had signs reversed in the objective row (the objective row represents $P - 25x - 14y + 32z = 0$, so the values 25 and 14 should be negative and 32 should be positive in the objective row).

(ii) Candidates generally knew that the pivot column needed to have a negative entry in the objective row, but often assumed that it must be the column with the most negative entry, or sometimes the first column with a negative entry. Whilst it is often the case that choosing the column with the most negative entry in the objective row will result in the biggest improvements in the value of P, this is not always the case and it is certainly not necessary to always pivot on the column with the most negative entry in the objective row (although most computer programs use this criterion).

It tended to only be the candidates who had realised that the y column was also a potential pivot column that went on to explain why the y column could not be used and hence the pivot had to be from the x column.

Most candidates showed both ratio calculations ($24 \div 6 = 4$ and $15 \div 5 = 3$) although several left the choice of the least non-negative ratio to be implied from their pivoting in the next tableau. This did not incur any loss of marks.

(iii) The iteration was often performed correctly, although there were a few candidates who, despite having identified 5 as the pivot in part (ii) then used 6 as the pivot here. Some candidates omitted to divide the pivot row through by the value in the pivot cell or did this but did not describe this operation.

Most candidates gave valid explanations of how they had calculated the other rows by adding or subtracting an appropriate multiple of the new pivot row, it was quite acceptable to give these in an encoded form (e.g. r1+25×npr).

The resulting tableau should have the appropriate number of basis columns, consisting of 0s and a single 1, should have non-negative entries in the column representing the right-hand side and should have a P value that is no smaller than in the previous tableau.

Some candidates did not read off the values of x, y, z and P from their tableau, or did not do so correctly.

(iv) The issue here was that there was no limit to how large y could be and hence no limit to the size of P. Some candidates just stated that the problem was unbounded with no indication of how they knew this, and several referred to the negative entry in the y column of the objective row but did not go on to explain that the y column was the only potential pivot but that a pivot entry could not be chosen because there were no positive entries in the y column. Some candidates thought that there was a problem with both y and z, apparently assuming that the optimal value could only be achieved after pivoting on x, y and z.

7) (i) Most candidates were able to work through the algorithm correctly, although some wasted time by writing out each row instead of just recording the values of F, G, H, C and N each time Step 9 is reached, as instructed in the question.

Some candidates thought that INT(2.5) = 3, despite the example given in the question, and some just messed up their arithmetic, for example calculating $B \div N$ instead of $N \div B$.

(ii) Candidates who worked through the algorithm using the new values were usually able to see what was happening and achieve full marks. Those who tried to describe what would happen often assumed, incorrectly, that the results would just be the negative of those from part (i).

(iii) Most candidates correctly applied the algorithm but only a few were able to explain what it was actually achieving. There were some excellent answers in which candidates not only said what the algorithm was achieving when $B = 10$ but also described it for a general base B.

4737 Decision Mathematics 2

General Comments

Most candidates were able to achieve reasonable marks on this paper. The candidates were, in general, well prepared and were able to show what they knew.

Comments on Individual Questions

1) (i) Almost all the candidates were able to draw a correctly labelled bipartite graph.

(ii) Nearly all the candidates identified Faye as being the star whose requirements had not been met and most drew a bipartite graph showing the incomplete matching. A few candidates included *F*-6 in their bipartite graph.

(iii) Some candidates did not show their alternating path clearly, a listing is preferred to trying to show the path on a diagram. Similarly, the resulting allocation should be listed not shown on a diagram.

(iv) Many candidates were able to apply the Hungarian algorithm accurately and efficiently. Some candidates only reduced rows before trying to augment (‘reducing rows first’ does not mean ‘reducing rows only’). Even candidates who had made small slips were usually able to write down an appropriate allocation and to name Arnie as being the star whose requirements were now not satisfied.

A small number of candidates assumed that this was a maximisation problem and subtracted all the entries from 6 before starting; this inevitably led to the loss of several marks.

2) (i) Some candidates assumed that the table given represented a zero-sum game, while others confused ‘number of hits scored’ with ‘number of times they were hit’.

(ii) Often the explanations referred only to a specific instance rather than a general case. For every pair of captains the sum of the number of hits scored by R and the number of hits scored by C is 10, so subtracting 5 from each makes the total zero in each case.

(iii) Most candidates were able to find the play-safe strategies, although some used the original game rather than the zero-sum game and some used row maxima or column minima in error. Some candidates indicated the row maximin value and column minimax value but did not state what the play-safe strategies were. There continues to be confusion between a play-safe strategy (e.g. Philip) and the corresponding minimax/maximin value (e.g. –1).

It was not sufficient to say that the game was not stable because there is no saddle point, the issue needed to be demonstrated or shown using the row maximin and column minimax values.

If team R play safe then team C should choose Liam, several candidates chose Nicola, having forgotten that the pay-offs for C are the negatives of those for R (or 10 minus the score for R in the original table).

(iv) Just stating that this is in the play-safe row and not in the play-safe column does not constitute a ‘careful explanation’. Candidates needed to explicitly consider the effect of increasing this entry on the row minima and column maxima.

(v) Most candidates realised that this was an issue of dominance, but did not always identify which row dominated row S, and even when row P was identified they did not always show that for *each* column the entry in row P is strictly greater than the corresponding entry in row S.

(vi) Most candidates were able to show how the expression $6 - 2p$ had been obtained and nearly all candidates were also able to obtain the appropriate expressions for the other two columns.

(vii) Candidates who used graph paper were usually more successful than those who made a sketch. The axes of the graph need to be scaled and labelled and should not take unnecessary values, in particular the p axis should not extend beyond the range 0 to 1.

Most candidates who had reasonable graphs were able to use them to calculate the optimal value for p and hence the minimum number of hits that R could expect to score.

3) (i) The majority of candidates completed the table correctly. A few made silly arithmetic slips in finding the maxima or minimax values and some did not transfer the suboptimal minimax values correctly from one stage to the next. A few candidates had clearly drawn the network first and then tried to create the table from it, with varying degrees of success.

(ii) The candidates with correct answers to part (i) were usually able to state the minimax value but often they could not trace the route from the table. At each stage, the action value for the state that gives the suboptimal value corresponds to the state label in the previous stage.

(iii) Surprisingly many candidates missed out on at least one aspect of reconstructing the network from the table.

4) (i) Most candidates were able to add a supersource and a supersink, a few candidates did not give sufficiently large capacities to the new arcs.

(ii) Some candidates gave incorrect explanations involving the flow through A and the flow through B and some idea of needing to split the flow between all the pipes leaving these two vertices. Most candidates identified that the problem arose at E where, if both AE and BE were full to capacity, there would 50 gallons per hour entering but at most 40 gallons per hour leaving.

(iii) Few candidates failed to calculate the capacity of the marked cut correctly.

(iv) Despite drawing the cut correctly on the diagram, several candidates did not include the arc DC in their calculation. There seemed to be an assumption with some candidates that only arcs where there could be flow from left to right should be included, rather than thinking about whether the flow was from the source side to the sink side of the cut or vice versa.

(v) When candidates are asked to show a flow they should mark each arc with an arrow showing how much is flowing through it, not try to use a labelling procedure. Most candidates had achieved a flow of 200 gallons per hour, but a few had exceeded some of the pipe capacities or had vertices where the flow in did not match the flow out. The cut $\{S_1, S_2\}$, $\{A, B, C, D, E, F, G, T_1, T_2\}$ showed that this flow was maximal. Some candidates did not appreciate the difference between the actual flow across a cut and the maximum possible flow across a cut.

(vi) Some candidates knew how to show a vertex restriction and did this part well, others merely reinstated the vertex *C*, sometimes trying to incorporate the restriction into the capacities of the arcs either entering or leaving *C*. Several candidates were able to state the maximum flow even if they could not show the restriction properly.

5) (i) Most candidates were able to complete the table to show the precedences.

(ii) The dummy activities caused problems for some candidates, particularly with the backward pass. A dummy activity has duration 0, on the forward pass when two paths merge the later time must be used, on the backward pass the earlier time must be chosen. There should only be one early time and one late time at each event, some candidates seemed to have made multiple passes through the network and then tried to extract the critical cases from them.

Most candidates with correct forward and backward passes were able to state the minimum project duration and list the critical activities.

(iii) When candidates are instructed to use graph paper, a sketch on lined paper will score no credit. Some candidates are still leaving holes in their resource histograms, presumably in an attempt to keep each activity as a single block, rather than plotting a histogram showing number of workers against days.

(iv) Some descriptions were complete and unambiguous, often accompanied by a diagram or graph, most were rather vague and sketchy, or just referred to ‘changing the start times for the critical activities’ or similar.

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
January 2008 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	58	50	42	35	28	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	60	52	45	38	31	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	51	44	37	31	25	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	57	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	56	49	42	36	30	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	49	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	59	52	45	38	31	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	57	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	50	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	52	45	38	31	25	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	57	51	45	40	35	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	59	52	45	39	33	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	25.5	49.6	70.9	84.3	96.0	100	478
3892	28.6	71.4	100	100	100	100	7
7890	33.0	58.3	79.1	92.2	97.4	100	115
7892	11.1	44.4	100	100	100	100	9

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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