

ADVANCED GCE

4753/01

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 2 JUNE 2008

Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):

Answer Booklet (8 pages)

Graph paper

MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

- 1 Solve the inequality $|2x - 1| \leq 3$. [4]
- 2 Find $\int xe^{3x} dx$. [4]
- 3 (i) State the algebraic condition for the function $f(x)$ to be an even function.
What geometrical property does the graph of an even function have? [2]
- (ii) State whether the following functions are odd, even or neither.
(A) $f(x) = x^2 - 3$
(B) $g(x) = \sin x + \cos x$
(C) $h(x) = \frac{1}{x + x^3}$ [3]
- 4 Show that $\int_1^4 \frac{x}{x^2 + 2} dx = \frac{1}{2} \ln 6$. [4]
- 5 Show that the curve $y = x^2 \ln x$ has a stationary point when $x = \frac{1}{\sqrt{e}}$. [6]
- 6 In a chemical reaction, the mass m grams of a chemical after t minutes is modelled by the equation
$$m = 20 + 30e^{-0.1t}.$$

(i) Find the initial mass of the chemical.
What is the mass of chemical in the long term? [3]
- (ii) Find the time when the mass is 30 grams. [3]
- (iii) Sketch the graph of m against t . [2]
- 7 Given that $x^2 + xy + y^2 = 12$, find $\frac{dy}{dx}$ in terms of x and y . [5]

Section B (36 marks)

8 Fig. 8 shows the curve $y = f(x)$, where $f(x) = \frac{1}{1 + \cos x}$, for $0 \leq x \leq \frac{1}{2}\pi$.

P is the point on the curve with x -coordinate $\frac{1}{3}\pi$.

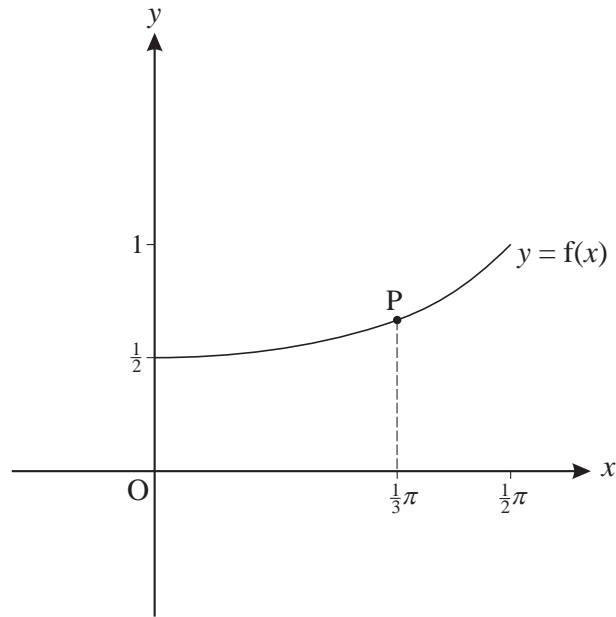


Fig. 8

- (i) Find the y -coordinate of P. [1]
- (ii) Find $f'(x)$. Hence find the gradient of the curve at the point P. [5]
- (iii) Show that the derivative of $\frac{\sin x}{1 + \cos x}$ is $\frac{1}{1 + \cos x}$. Hence find the exact area of the region enclosed by the curve $y = f(x)$, the x -axis, the y -axis and the line $x = \frac{1}{3}\pi$. [7]
- (iv) Show that $f^{-1}(x) = \arccos\left(\frac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = f^{-1}(x)$ to a copy of Fig. 8. [5]

[Question 9 is printed overleaf.]

9 The function $f(x)$ is defined by $f(x) = \sqrt{4 - x^2}$ for $-2 \leq x \leq 2$.

- (i) Show that the curve $y = \sqrt{4 - x^2}$ is a semicircle of radius 2, and explain why it is not the whole of this circle. [3]

Fig. 9 shows a point $P(a, b)$ on the semicircle. The tangent at P is shown.

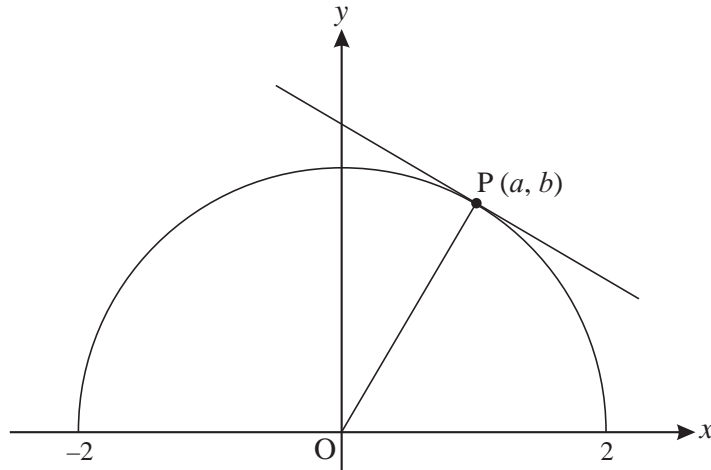


Fig. 9

- (ii) (A) Use the gradient of OP to find the gradient of the tangent at P in terms of a and b .

(B) Differentiate $\sqrt{4 - x^2}$ and deduce the value of $f'(a)$.

(C) Show that your answers to parts (A) and (B) are equivalent. [6]

The function $g(x)$ is defined by $g(x) = 3f(x - 2)$, for $0 \leq x \leq 4$.

- (iii) Describe a sequence of two transformations that would map the curve $y = f(x)$ onto the curve $y = g(x)$.

Hence sketch the curve $y = g(x)$. [6]

- (iv) Show that if $y = g(x)$ then $9x^2 + y^2 = 36x$. [3]