RECOGNISING ACHIEVEMENT

**ADVANCED GCE** **4754/01A**

**MATHEMATICS (MEI)**

Applications of Advanced Mathematics (C4) Paper A

**TUESDAY 22 JANUARY 2008** Afternoon

Time: 1 hour 30 minutes

**Additional materials:** Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

**INSTRUCTIONS TO CANDIDATES**

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

**NOTE**

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **4** printed pages.



**[Turn over**

**Section A** (36 marks)

**1** Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $3\cos\theta + 4\sin\theta = 2$ for $-\pi \leqslant \theta \leqslant \pi$. [7]

**2** **(i)** Find the first three terms in the binomial expansion of $\dfrac{1}{\sqrt{1-2x}}$. State the set of values of $x$ for which the expansion is valid. [5]

**(ii)** Hence find the first three terms in the series expansion of $\dfrac{1+2x}{\sqrt{1-2x}}$. [3]

**3** Fig. 3 shows part of the curve $y = 1 + x^2$, together with the line $y = 2$.

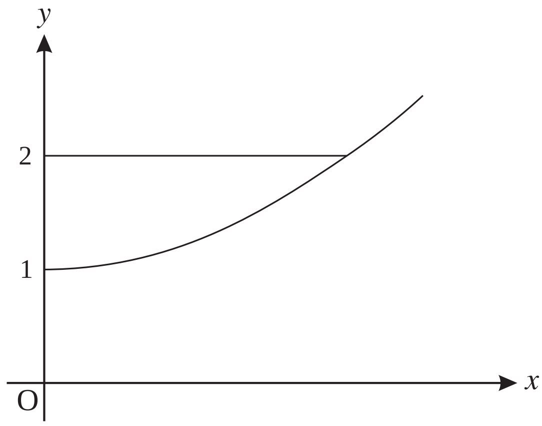


**Fig. 3**

The region enclosed by the curve, the $y$-axis and the line $y = 2$ is rotated through 360° about the $y$-axis. Find the volume of the solid generated, giving your answer in terms of $\pi$. [5]

**4** The angle $\theta$ satisfies the equation $\sin(\theta + 45°) = \cos\theta$.

**(i)** Using the exact values of $\sin 45°$ and $\cos 45°$, show that $\tan\theta = \sqrt{2} - 1$. [5]

**(ii)** Find the values of $\theta$ for $0° < \theta < 360°$. [2]

**5** Express $\dfrac{4}{x(x^2+4)}$ in partial fractions. [6]

**6** Solve the equation $\operatorname{cosec}\theta = 3$, for $0° < \theta < 360°$. [3]

## **Section B** (36 marks)

**7** A glass ornament OABCDEFG is a truncated pyramid on a rectangular base (see Fig. 7). All dimensions are in centimetres.

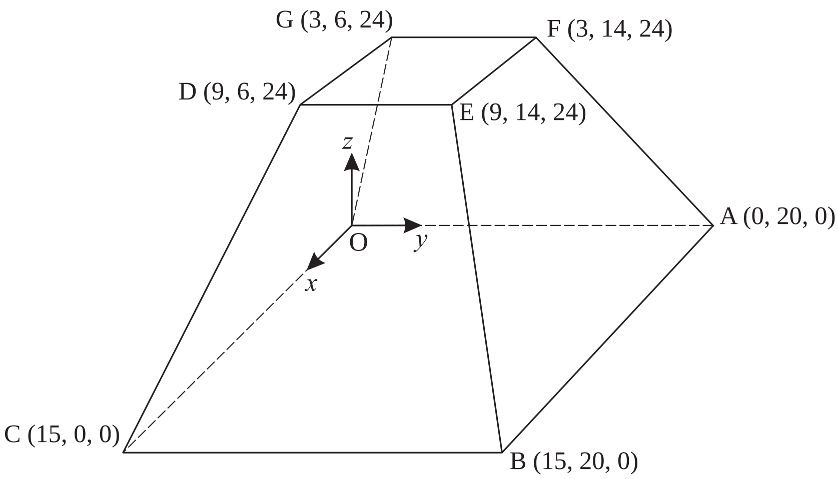


**Fig. 7**

**(i)** Write down the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. [2]

**(ii)** Find the length of the edge CD. [2]

**(iii)** Show that the vector $4\mathbf{i} + \mathbf{k}$ is perpendicular to the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. Hence find the cartesian equation of the plane BCDE. [5]

**(iv)** Write down vector equations for the lines OG and AF.

Show that they meet at the point P with coordinates $(5, 10, 40)$. [5]

You may assume that the lines CD and BE also meet at the point P.

The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ height.

**(v)** Find the volumes of the pyramids POABC and PDEFG.

Hence find the volume of the ornament. [4]

**8** A curve has equation

$$x^2 + 4y^2 = k^2,$$

where $k$ is a positive constant.

**(i)** Verify that

$$x = k\cos\theta, \qquad y = \tfrac{1}{2}k\sin\theta,$$

are parametric equations for the curve. [3]

**(ii)** Hence or otherwise show that $\dfrac{dy}{dx} = -\dfrac{x}{4y}$. [3]

**(iii)** Fig. 8 illustrates the curve for a particular value of $k$. Write down this value of $k$. [1]

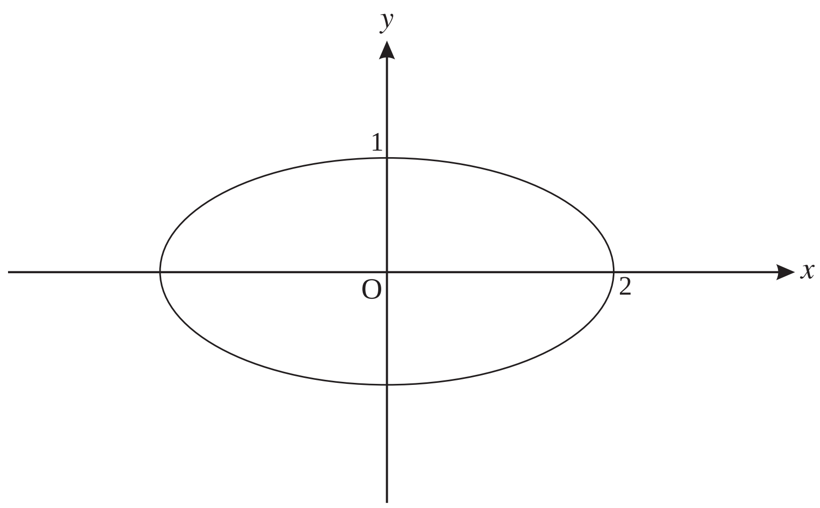


**Fig. 8**

**(iv)** Copy Fig. 8 and on the same axes sketch the curves for $k = 1$, $k = 3$ and $k = 4$. [3]

On a map, the curves represent the contours of a mountain. A stream flows down the mountain. Its path on the map is always at right angles to the contour it is crossing.

**(v)** Explain why the path of the stream is modelled by the differential equation

$$\frac{dy}{dx} = \frac{4y}{x}.$$

[2]

**(vi)** Solve this differential equation.

Given that the path of the stream passes through the point $(2, 1)$, show that its equation is $y = \dfrac{x^4}{16}$. [6]