

**ADVANCED GCE**

**MATHEMATICS (MEI)**

Applications of Advanced Mathematics (C4) Paper A

**4754A**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Friday 15 January 2010**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**NOTE**

- This paper will be followed by **Paper B: Comprehension**.

## Section A (36 marks)

- 1 Find the first three terms in the binomial expansion of  $\frac{1+2x}{(1-2x)^2}$  in ascending powers of  $x$ . State the set of values of  $x$  for which the expansion is valid. [7]

- 2 Show that  $\cot 2\theta = \frac{1 - \tan^2 \theta}{2 \tan \theta}$ .

Hence solve the equation

$$\cot 2\theta = 1 + \tan \theta \quad \text{for } 0^\circ < \theta < 360^\circ. \quad [7]$$

- 3 A curve has parametric equations

$$x = e^{2t}, \quad y = \frac{2t}{1+t}.$$

- (i) Find the gradient of the curve at the point where  $t = 0$ . [6]

- (ii) Find  $y$  in terms of  $x$ . [2]

- 4 The points A, B and C have coordinates  $(1, 3, -2)$ ,  $(-1, 2, -3)$  and  $(0, -8, 1)$  respectively.

- (i) Find the vectors  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ . [2]

- (ii) Show that the vector  $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$  is perpendicular to the plane ABC. Hence find the equation of the plane ABC. [5]

- 5 (i) Verify that the lines  $\mathbf{r} = \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$  and  $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$  meet at the point  $(1, 3, 2)$ . [3]

- (ii) Find the acute angle between the lines. [4]

## Section B (36 marks)

- 6 In Fig. 6, OAB is a thin bent rod, with  $OA = a$  metres,  $AB = b$  metres and angle  $OAB = 120^\circ$ . The bent rod lies in a vertical plane. OA makes an angle  $\theta$  above the horizontal. The vertical height BD of B above O is  $h$  metres. The horizontal through A meets BD at C and the vertical through A meets OD at E.

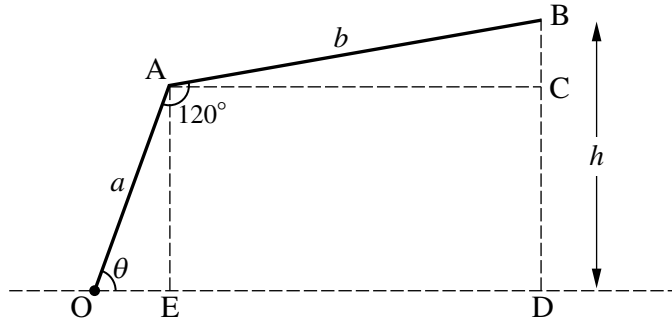


Fig. 6

- (i) Find angle BAC in terms of  $\theta$ . Hence show that

$$h = a \sin \theta + b \sin(\theta - 60^\circ). \quad [3]$$

- (ii) Hence show that  $h = (a + \frac{1}{2}b) \sin \theta - \frac{\sqrt{3}}{2}b \cos \theta$ . [3]

The rod now rotates about O, so that  $\theta$  varies. You may assume that the formulae for  $h$  in parts (i) and (ii) remain valid.

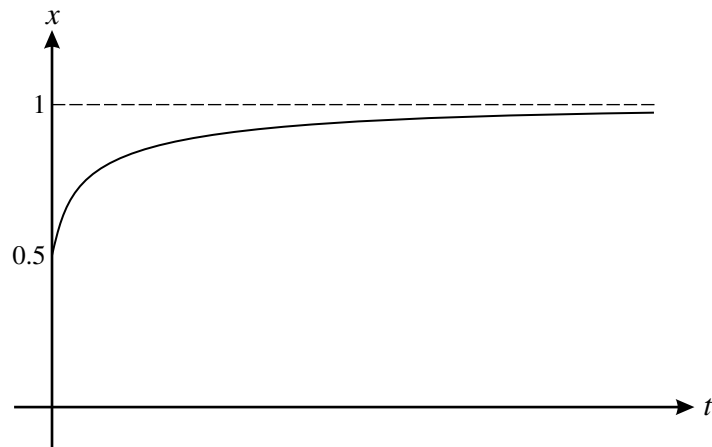
- (iii) Show that OB is horizontal when  $\tan \theta = \frac{\sqrt{3}b}{2a + b}$ . [3]

In the case when  $a = 1$  and  $b = 2$ ,  $h = 2 \sin \theta - \sqrt{3} \cos \theta$ .

- (iv) Express  $2 \sin \theta - \sqrt{3} \cos \theta$  in the form  $R \sin(\theta - \alpha)$ . Hence, for this case, write down the maximum value of  $h$  and the corresponding value of  $\theta$ . [7]

[Question 7 is printed overleaf.]

- 7 Fig. 7 illustrates the growth of a population with time. The proportion of the ultimate (long term) population is denoted by  $x$ , and the time in years by  $t$ . When  $t = 0$ ,  $x = 0.5$ , and as  $t$  increases,  $x$  approaches 1.



**Fig. 7**

One model for this situation is given by the differential equation

$$\frac{dx}{dt} = x(1 - x).$$

- (i) Verify that  $x = \frac{1}{1 + e^{-t}}$  satisfies this differential equation, including the initial condition. [6]
- (ii) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. [3]

An alternative model for this situation is given by the differential equation

$$\frac{dx}{dt} = x^2(1 - x),$$

with  $x = 0.5$  when  $t = 0$  as before.

- (iii) Find constants  $A$ ,  $B$  and  $C$  such that  $\frac{1}{x^2(1-x)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{1-x}$ . [4]
- (iv) Hence show that  $t = 2 + \ln\left(\frac{x}{1-x}\right) - \frac{1}{x}$ . [5]
- (v) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. [2]

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.