OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 15 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Find the first three terms in the binomial expansion of $\dfrac{1+2x}{(1-2x)^2}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[7]**

2 Show that $\cot 2\theta = \dfrac{1-\tan^2\theta}{2\tan\theta}$.

Hence solve the equation

$$\cot 2\theta = 1 + \tan\theta \quad \text{for } 0^\circ < \theta < 360^\circ.$$ **[7]**

3 A curve has parametric equations

$$x = e^{2t}, \quad y = \frac{2t}{1+t}.$$

(i) Find the gradient of the curve at the point where $t = 0$. **[6]**

(ii) Find y in terms of x. **[2]**

4 The points A, B and C have coordinates (1, 3, −2), (−1, 2, −3) and (0, −8, 1) respectively.

(i) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **[2]**

(ii) Show that the vector $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ is perpendicular to the plane ABC. Hence find the equation of the plane ABC. **[5]**

5 **(i)** Verify that the lines $\mathbf{r} = \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ meet at the point (1, 3, 2). **[3]**

(ii) Find the acute angle between the lines. **[4]**

Section B (36 marks)

6 In Fig. 6, OAB is a thin bent rod, with OA = a metres, AB = b metres and angle OAB = 120°. The bent rod lies in a vertical plane. OA makes an angle θ above the horizontal. The vertical height BD of B above O is h metres. The horizontal through A meets BD at C and the vertical through A meets OD at E.

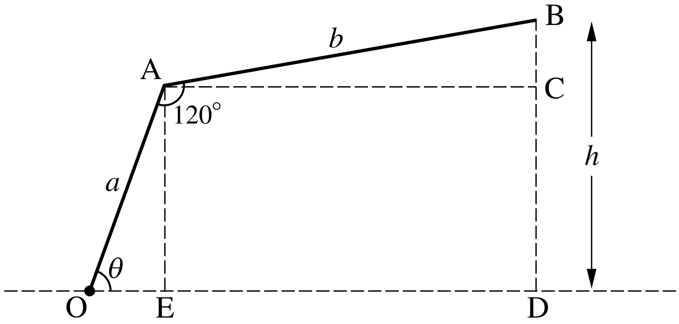

Fig. 6

(i) Find angle BAC in terms of θ. Hence show that

$$h = a\sin\theta + b\sin(\theta - 60°).$$ **[3]**

(ii) Hence show that $h = (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta$. **[3]**

The rod now rotates about O, so that θ varies. You may assume that the formulae for h in parts **(i)** and **(ii)** remain valid.

(iii) Show that OB is horizontal when $\tan\theta = \dfrac{\sqrt{3}b}{2a + b}$. **[3]**

In the case when $a = 1$ and $b = 2$, $h = 2\sin\theta - \sqrt{3}\cos\theta$.

(iv) Express $2\sin\theta - \sqrt{3}\cos\theta$ in the form $R\sin(\theta - \alpha)$. Hence, for this case, write down the maximum value of h and the corresponding value of θ. **[7]**

[Question 7 is printed overleaf.]

7 Fig. 7 illustrates the growth of a population with time. The proportion of the ultimate (long term) population is denoted by x, and the time in years by t. When $t = 0$, $x = 0.5$, and as t increases, x approaches 1.

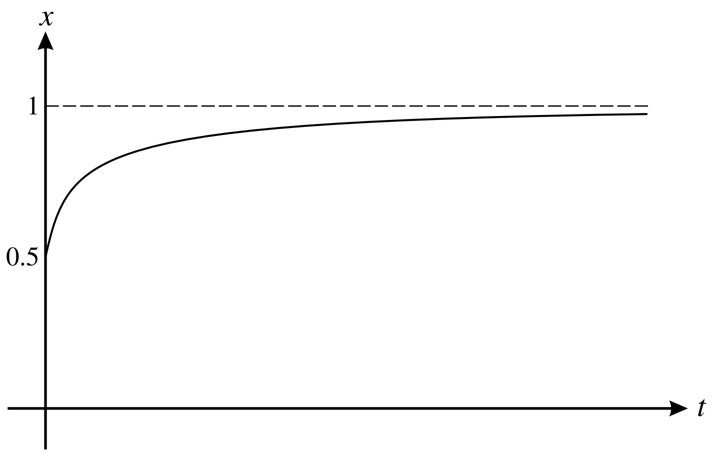

Fig. 7

One model for this situation is given by the differential equation

$$\frac{dx}{dt} = x(1 - x).$$

(i) Verify that $x = \frac{1}{1 + e^{-t}}$ satisfies this differential equation, including the initial condition. **[6]**

(ii) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. **[3]**

An alternative model for this situation is given by the differential equation

$$\frac{dx}{dt} = x^2(1 - x),$$

with $x = 0.5$ when $t = 0$ as before.

(iii) Find constants A, B and C such that $\frac{1}{x^2(1-x)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{1-x}$. **[4]**

(iv) Hence show that $t = 2 + \ln\left(\frac{x}{1-x}\right) - \frac{1}{x}$. **[5]**

(v) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. **[2]**
