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RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4735**

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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1(i)	Var(2A – 3B) = 4Var(A) + 9Var(B) – 12Cov(A,B) $\Rightarrow$ 18 = 36 + 54 – 12Cov(A,B) $\Rightarrow$ Cov(A ,B) = 6	M1 A1 A1 **3**	Correct formula**.** Allow one error Substitute relevant values CAO
(ii)	Since Cov(A, B) ≠ 0, A and B are not independent	B1 ft **1** **(4)**	Must have a reason. ft Cov≠ 0
2(i)	G′(t) = $8te^{4t^2} \big/ e^4$ E(X) = G′(1) = 8	M1A1 A1 **3**	M1 for ct^2/e^4
(ii)	EITHER: G(t) = e^{-4}(1 + 4t^2+…) P(X=2) = coefficient of t^2 = 4e^{-4} or 4/e^4 or 0.0733 OR G″(t) = (8+64t^2) e^{4t^2-4} P(X=2) = $\frac{1}{2}$ G″(0) = 4e^{-4} or 4/e^4 or 0.0733	M1A1 A1 **3** M1A1 A1 **(6)**	Expand in powers of t M1 for reasonable attempt at M"(t)
3(i)	Number of different rankings $^{11}C_5$ =462 For R ≤ 17: 1+2+3+4+5 = 15 1+2+3+4+6=16 1+2+3+5+6=17 1+2+3+4+7=17 P(R ≤ 17) = 4/462 = 2/231 AG	M1 A1 B2 A1 **5**	Number of selections of 5 from 11 B1 for 2 or 3 correct
(ii)	W = 17 P(W ≤ 17) = $\frac{2}{231}$ Smallest SL = $\frac{400}{231}$ %	M1 A1ft **2** **(7)**	Allow $\frac{4}{231}$; ft $\frac{2}{231}$, but must be exact
4(i)	EITHER**:** (α) M′(t) = n(1 – 2t)$^{-½n-1}$ E(Y) = M'(0)=n M″(t) = n(n+2)(1 – 2t)$^{-½n-2}$ Var(Y) = n(n+2) – n^2 = 2n OR: M(t) = 1 + nt + $\frac{1}{2}$ n(n+2)t^2 E(Y) = n Var(Y) = n(n+2) – n^2 = 2n	M1 A1 A1 M1 A1 **5** M1A1A1 A1 A1 **5**	Correct form for M1 Ft similar M′(t) M″(0) – (M′(0))^2
(ii)	MGF = (1 – 2t)$^{-30}$ χ^2 distribution with 60 d.f.	B1 B1 **2**	From [(1 – 2t)$^{-1/2}$]^{60}
(iii)	E(S) = 60, Var(S) = 120 Using CLT, Probability =1 – Φ(10/√120) = 0.181	B1ft M1 A1 **3** **(10)**	From **(i)** Correct tail: allow cc

5(i)	Assumes salaries symmetrically distributed H_0: m(edian) = 19.5, H_1: m(edian)$\neq$ 19.5 P = 867 (or 408) Using normal approximation $\mu = \frac{1}{4} \times 50 \times 51$ (= 637.5) $\sigma^2 = 50 \times 51 \times 101/24$ (= 10731.25) $z = (a - 637.5)/\sqrt{10731.25}$ Use a = 866.5 = 2.211, or 2.215 or 2.220 (– from 408) Compare their z with 1.96 and reject H_0 There is sufficient evidence at the 5% SL that the median salary differs from £19 500	B1 B1 M1 A1 A1 M1 A1 A1 M1 A1 ft **10**	In context For both ; not μ ; accept words a=866.5, 867, 867.5 (or 408.5, 408, 407.5) Or p-value rounding to 0.026 or 0.027 Compare with 0.05 or equivalent ft z Or find critical region
(ii)	Use sign test when salary distribution is skewed	B1 **1** **(11)**	
6(i)	$\begin{array}{ccccc} & & & N & \\ & & 0 & 1 & 2 \\ & 0 & 0 & c & 2c \\ R & 1 & 2c & 3c & 4c \\ & 2 & 4c & 5c & 6c \end{array}$ Total $27c = 1$ $c = \frac{1}{27}$	B1 M1 A1 **3**	Calculate 9 probs in terms of c
(ii)	$9c/27c$ $= \frac{1}{3}$	M1 A1 ft **2**	Marginal probability AEF; ft c
(iii)	$P(N + R > 2)$ $= 15c/27c = \frac{5}{9}$	M1 A1 ft **2**	AEF; ft c
(iv)	$P(R=2) = \frac{15}{27}$ $P(N \mid R=2)$: $p_0 = \frac{4}{15}$, $p_1 = \frac{1}{3}$, $p_2 = \frac{2}{5}$ $E(N \mid R=2) = 1 \times \frac{1}{3} + 2 \times \frac{2}{5}$ $= \frac{17}{15}$	M1 A1 ft A1 ft A1 **4**	Using conditional probabilities One value; ft values in **(i)** All values Or 1.13
(v)	Eg P(N = 0 and R = 0) = 0 $P(N=0) \times P(R=0) = \frac{6}{27} \times \frac{3}{27} \neq 0$ So N and R are not independent	M1 A1 **2** **(13)**	Or from conditional probs M0 from N=1 with R=1 or 2 All correct

7(i)	$\int_0^{2\theta} \frac{x^{n+1}}{2\theta^2}\,\mathrm{d}x = \left[\frac{x^{n+2}}{2(n+2)\theta^2}\right]$	M1		Correct integral
	$= 2^{n+1}\theta^n/(n+2)$	A1		AEF
	$E(X) = 4\theta/3$	B1 ft	3	B0 if not ‘deduced’
	$Var(X) = 2\theta^2 – (4\theta/3)^2 = 2\theta^2/9$	M1A1ft		
(ii)	$Var(X^2) = E(X^4) – (E(X))^2$			ft **(i)** with no *n*
	$= 16\theta^4/3 – 4\theta^4 = 4\theta^4/3$	M1A1ft	4	
				ft **(i)** with no *n*
	$E(\sum X_i) = 3\times 4\theta/3$	M1		
	$= 4\theta$	A1 ft		
(iii)	$T_1 = \frac{1}{4}\Sigma X_i$	A1 ft		ft with no *n*
	$E(\sum X_i^2) = 3\times 2\theta^2$	M1		ft with no *n* or *θ*
	$=6\theta^2$	A1 ft		
	$T_2 = (\sum X_i^{\ 2})/27$	A1 ft	6	ft with no *n*
				ft with no *n* or *θ*
		M1		
(iv)	$Var(T_2) = 1/27^2 \times 3 \times Var(X^2)$	A1	2	
	$= 4\theta^4/729$			
		(15)		
				CAO
8(i)	$P(L \cap M) = P(L \mid M)P(M) = 0.12$ and	*M1*		
	$P(L) = P(M \cap L)/P(M \mid L)$ = 0.12/0.4 = 0.3	A1		
	$P(L' \cup M') = P\left[(L \cap M)'\right]$			
	$= 1 - P(L \cap M)$			
	= 1 – 0.2 × 0.6 = 0.88	B1	3	
(ii)	$P(N \mid L \cap M) = 0.3$	M1		
	$\Rightarrow P(N \cap L \cap M) = 0.3 \times 0.12$	A1		
	=0.036	A1	3	
	$P(L' \cup M' \cup N') = 1 - 0.036 = 0.964$		**[6]**	

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