

Friday 22 October 2021 – Afternoon

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Time allowed: 1 hour 30 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- · a scientific or graphical calculator



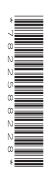
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the Printed Answer
 Booklet. If you need extra space use the lined pages at the end of the Printed Answer
 Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer all the guestions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \, \text{m} \, \text{s}^{-2}$. When a numerical value is needed use g = 9.8 unless a different value is specified in the question.
- Do not send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION

- The total mark for this paper is 75.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

Read each question carefully before you start your answer.



Answer **all** the questions.

1 In this question you must show detailed reasoning.

Express the number 41723_{10} in hexadecimal (base 16). [3]

2 The following Cayley table is for G, a group of order 6. The identity element is e and the group is generated by the elements a and b.

G	e	а	a^2	b	ab	a^2b
e	e	а	a^2	b	ab	a^2b
а	а	a^2	e	ab	a^2b	b
a^2	a^2	e	а	a^2b	b	ab
b	b	a^2b	ab	e	a^2	а
ab	ab	b	a^2b	а	e	a^2
a^2b	a^2b	ab	b	a^2	а	e

- (a) List all the proper subgroups of G. [4]
- (b) State another group of order 6 to which G is isomorphic. [1]
- 3 The points P, Q and R have position vectors $\mathbf{p} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k}$, $\mathbf{q} = \mathbf{i} \mathbf{j} + \mathbf{k}$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + t\mathbf{k}$ respectively, relative to the origin O.

Determine the value(s) of t in each of the following cases.

(a) The line
$$OR$$
 is parallel to $\mathbf{p} \times \mathbf{q}$. [2]

- **(b)** The volume of tetrahedron *OPQR* is 13. [4]
- 4 Solve the simultaneous linear congruences $x \equiv 1 \pmod{3}$, $x \equiv 5 \pmod{11}$, $2x \equiv 5 \pmod{17}$. [6]
- 5 The surface S has equation $x^2 + y^2 + z^2 = xyz 1$.

(a) Show that
$$(2z - xy)\left(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right) = 2(1 + z^2)$$
. [6]

(b) Deduce that S has no stationary point. [2]

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6 The binary operation \Diamond is defined on the set \mathbb{C} of complex numbers by

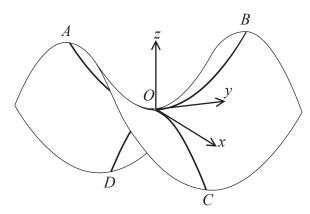
$$(a+ib) \Diamond (c+id) = ac+i(b+ad)$$

where a, b, c and d are real numbers.

- (a) Is \mathbb{C} closed under \lozenge ? Justify your answer. [1]
- (b) Prove that \Diamond is associative on \mathbb{C} .
- (c) Determine the identity element of \mathbb{C} under \Diamond .
- (d) Determine the largest subset S of \mathbb{C} such that (S, \diamond) is a group. [3]
- 7 Let $I_n = \int_0^{\frac{1}{2}\pi} \cos^n x \, dx$ for integers $n \ge 0$.
 - (a) Show that, for $n \ge 2$, $nI_n = (n-1)I_{n-2}$. [4]
 - (b) Use this reduction formula to deduce the exact value of I_8 . [2]
 - (c) Use the results of parts (a) and (b) to determine the exact value of $\int_0^{\frac{1}{2}\pi} \cos^6 x \sin^2 x \, dx$. [2]
- 8 (a) Solve the second-order recurrence system $H_{n+2} = 5H_{n+1} 4H_n$ with $H_0 = 3$, $H_1 = 7$ for $n \ge 0$.
 - (b) (i) Write down the quadratic residues modulo 10. [1]
 - (ii) By considering the sequence $\{H_n\}$ modulo 10, prove that H_n is never a perfect square.
- 9 For each value of k the sequence of real numbers $\{u_n\}$ is given by $u_1 = 2$ and $u_{n+1} = \frac{k}{6+u_n}$. For each of the following cases, either determine a value of k or prove that one does not exist.

(a)
$$\{u_n\}$$
 is constant. [2]

- (b) $\{u_n\}$ is periodic, with period 2. [3]
- (c) $\{u_n\}$ is periodic, with period 4. [5]



A student wishes to model the saddle of a horse. They use a surface described by a function of the form z = f(x, y) with a saddle point at the origin O. The z-axis is vertically upwards. The x- and y-axes lie in a horizontal plane, with the x-axis across the horse and the y-axis along the length of the horse (see diagram).

The arc *AOB* is part of a parabola which lies in the *yz*-plane. The arc *COD* is part of a parabola which lies in the *xz*-plane. The saddle is symmetric in both the *xz*-plane and *yz*-plane.

The length of the saddle, the distance AB, is to be 0.6 m with both A and B at a height of 0.27 m above O. The width of the saddle, the distance CD, is to be 0.5 m with both C and D at a depth of 0.4 m below O.

- (a) On separate diagrams, sketch the sections x = 0 and y = 0.
- (b) Determine a function f that describes the saddle. [You do not need to state the domain of function f.] [5]

END OF QUESTION PAPER



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