Oxford Cambridge and RSA

GCE

Further Mathematics A

Y534/01: Discrete Mathematics

Advanced Subsidiary GCE

Mark Scheme for Autumn 2021

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓ and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Question			Answer	Marks	AO	Guidance
1	(a)		5 partitions into a set of size 1 and a set of size 4 {X | X, X, X, X} 10 partitions into a set of size 2 and a set of size 3 {X, X | X, X, X} because there are $^5C_2$ choices for the set of size 2	B1 B1	1.1 2.5	5 where smaller set has size 1 or $^5C_1 = 5$ 10 where smaller set has size 2, with an explanation of why it is 10 (note the total of 15 is given in the question) e.g. $^5C_2 = 10$ or $(5\times4) \div 2 = 10$ or $4 + 3 + 2 + 1 = 10$
			Alternative solution {A}, {B, C, D, E} {B}, {A, C, D, E} {C}, {A, B, D, E} {D}, {A, B, C, E} {E}, {A, B, C, D}	B1		List (or any equivalent) that has exactly 5 distinct cases where smaller set has size 1 May just list one set, e.g. {A}, {B}, {C}, {D}, {E}
			{A, B}, {C, D, E} {A, C}, {B, D, E} {A, D}, {B. C. E} {A, E}, {B, C, D} {B, C}, {A, D, E} {B, D}, {A, C, E} {B, E}, {A, C, D} {C, D}, {A, B, E} {C, E}, {A, B, D} {D, E}, {A, B, C}	B1		List (or any equivalent) that has 10 distinct cases sets where smaller set has size 2 May just list one set, e.g. {A, B}, {A, C} {A, D}, {A, E}, {B, C}, {B, D}, {B, E}, {C, D}, {C, E}, {D, E}
				[2]		
1	(b)		Partitions into sets of sizes 1, 1 and 3 $5 \times 4 \div 2 = 10$ partitions of this type Partitions into sets of sizes 1, 2 and 2 $5 \times (^4C_2 \div 2) = 5 \times 3 = 15$ partitions of this type	M1 A1 M1 A1 [4]	1.1 2.1 1.1 2.1	Considering cases where set sizes are 1, 1, 3 Explanation of why there are 10 of these e.g. $^5C_3 = 10$ or $5 \times 4 \div 2 = 10$ or a list of the cases Considering cases where set sizes are 1, 2, 2 Explaining why there are 15 of these e.g a relevant calculation or list of cases
1	(c)		10 partitions into sets of sizes 1, 1, 1, 2 1 partition into sets of sizes 1, 1, 1, 1, 1 $15 + 25 + 10 + 1 = 51$	M1 A1 [2]	2.1 1.1	Trying to deal with the cases when there are more than 3 subsets May be implied from answer 51 51
1	(d)		Number line is split into 6 pieces But there are 8 numbers Hence result by the pigeonhole principle	B1 B1 [2]	2.1 2.2a	6 pieces Using pigeonhole, or explaining why there must be at least one piece with two or more numbers

Question			Answer	Marks	AO	Guidance
2	(a)	(i)	Next-fit method Bin 1: 12 Bin 2: 23 Bin 3: 15 Bin 4: 18, 8 Bin 5: 7, 5	M1 A1 [2]	1.1 1.1	Bins 1 and 2 correct All correct
2	(a)	(ii)	First-fit method Bin 1: 12, 15 Bin 2: 23, 7 Bin 3: 18, 8 Bin 4: 5 Bin 5:	M1 A1 [2]	1.1 1.1	Bins 1 and 2 correct All correct
2	(a)	(iii)	23 18 15 12 8 7 5 First-fit decreasing method Bin 1: 23, 7 Bin 2: 18, 12 Bin 3: 15, 8, 5 Bin 4: Bin 5:	 M1 A1 [2]	 1.1 1.1	Ordered list may be seen Bins 1 and 2 correct All correct
2	(b)		With 'online' lists the items are presented one at a time and the whole list is not known until the end. With next-fit and first-fit the items are placed in the order they appear in the list, so these methods can be used 'offline' or 'online'. However, for first-fit decreasing the whole list needs to be known before it can be sorted, so first-fit decreasing can only be used for an 'offline' list.	B1 B1 [2]	1.2 2.3	Evidence of understanding what 'online' means Evidence of realising that ffd cannot be used with an online list (or implied from an appropriate statement about next-fit and first-fit)

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>2</td><td>(c)</td><td></td><td>$88 \div 4 = 22$, so M is at least 22
But it is not possible to fill 4 bins of capacity 22
Since 22 – 18 = 4 which is less than 5
So the 23 would have to be split as 4 and 19
And then there is no 3 to go with the 19
$M = 23$ is possible
e.g. $23 - x$ and x, 18 + 5, 15 + 8, 12 + 7
Hence, least M is 23</td><td>M1
A1

B1
[3]</td><td>1.1
2.4

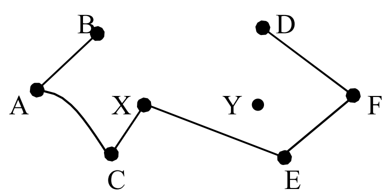
2.2a</td><td>Identifying that M must be at least 22
Showing that $M = 22$ is not possible
Fully correct explanation

Showing that $M = 23$ is possible</td></tr>
<tr><td>3</td><td>(a)</td><td></td><td>ABXE</td><td>B1
[1]</td><td>1.1</td><td></td></tr>
<tr><td>3</td><td>(b)</td><td></td><td><table>
<tr><td>A</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>0.6</td><td>B</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1.1</td><td>1.7</td><td>C</td><td></td><td></td><td></td></tr>
<tr><td>2.7</td><td>2.1</td><td>2.5</td><td>D</td><td></td><td></td></tr>
<tr><td>2.8</td><td>2.2</td><td>1.8</td><td>1.2</td><td>E</td><td></td></tr>
<tr><td>3.3</td><td>2.7</td><td>2.5</td><td>0.6</td><td>0.7</td><td>F</td></tr>
</table></td><td>M1
A1

M1
A1

[4]</td><td>1.1
1.1

1.1
1.1</td><td>AB = 0.6, AC = 1.1
AD = 2.7, AE = 2.8

DF = 0.6, EF = 0.7
BF = 2.7, CF = 2.5
AF = 3.3 or ft from other values</td></tr>
<tr><td>3</td><td>(c)</td><td></td><td>AB = 0.6
AC = 1.1
CE = 1.8
EF = 0.7
DF = <u>0.6</u>
4.8

</td><td>M1

A1

B1 ft
[3]</td><td>3.1b

3.2a

1.1</td><td>A graph that connects {A, B, C, D, E, F} with or without X and/or Y
Correct tree drawn or arcs listed, including CX and XE

4.8 (km) or total for their tree</td></tr>
<tr><td>3</td><td>(d)</td><td></td><td>Adapting the answer to part (c)

B – A – C – X – Y – E – F – D</td><td>M1

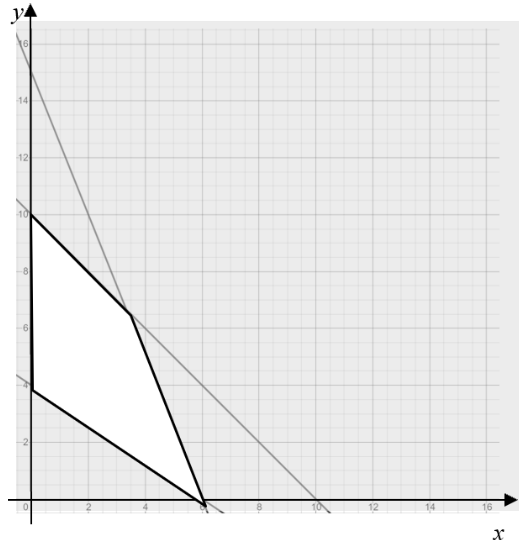
A1</td><td>3.1b

1.1</td><td>Any walk or cycle that starts at B and uses every vertex at least once, including X and Y
cao</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[2]</td><td></td><td></td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>4</td><td>(a)</td><td>(i)</td><td>
<table>
<tr><td>Points won by Mia</td><td></td><td colspan="3">Mia</td></tr>
<tr><td></td><td></td><td>X</td><td>Y</td><td>Z</td></tr>
<tr><td rowspan="3">Li</td><td>X</td><td>4</td><td>15</td><td>9</td></tr>
<tr><td>Y</td><td>11</td><td>6</td><td>5</td></tr>
<tr><td>Z</td><td>10</td><td>5</td><td>1</td></tr>
</table>
</td><td>B1
[1]</td><td>1.1</td><td>15, 9 and 6 all correct</td></tr>
<tr><td>4</td><td>(a)</td><td>(ii)</td><td>
<table>
<tr><td>Points won by Li</td><td></td><td colspan="3">Mia</td></tr>
<tr><td></td><td></td><td>X</td><td>Y</td><td>Z</td></tr>
<tr><td rowspan="3">Li</td><td>X</td><td>0.5</td><td>-10.5</td><td>-4.5</td></tr>
<tr><td>Y</td><td>-6.5</td><td>-1.5</td><td>-0.5</td></tr>
<tr><td>Z</td><td>-5.5</td><td>-0.5</td><td>3.5</td></tr>
</table>
</td><td>B1</td><td>1.1</td><td>Subtract 4.5 from each value in the table showing points won by Li
All correct
Or any positive multiple of this table</td></tr>
<tr><td></td><td></td><td></td><td>
Alternative solution
<table>
<tr><td>Points won by Li</td><td></td><td colspan="3">Mia</td></tr>
<tr><td></td><td></td><td>X</td><td>Y</td><td>Z</td></tr>
<tr><td rowspan="3">Li</td><td>X</td><td>1</td><td>-21</td><td>-9</td></tr>
<tr><td>Y</td><td>-13</td><td>-3</td><td>-1</td></tr>
<tr><td>Z</td><td>-11</td><td>-1</td><td>7</td></tr>
</table>
</td><td>B1
[1]</td><td></td><td>Points won by Li minus points won by Mia
All correct (not follow through from (a))</td></tr>
<tr><td>4</td><td>(b)</td><td></td><td>
Row Z dominates row Y
Column Y dominates column Z
<table>
<tr><td>Points won by Li</td><td></td><td colspan="2">Mia</td></tr>
<tr><td></td><td></td><td>X</td><td>Y</td></tr>
<tr><td rowspan="2">Li</td><td>X</td><td>0.5</td><td>-10.5</td></tr>
<tr><td>Z</td><td>-5.5</td><td>-0.5</td></tr>
</table>
</td><td>B1
B1
[2]</td><td>1.1
1.1</td><td>
Row Y removed, seen in resulting 2×2 table
Column Z removed, seen in resulting 2×2 table
Follow through their table from part (a)(ii) or using original table
<table>
<tr><td>e.g.</td><td>X</td><td>Y</td></tr>
<tr><td>X</td><td>1</td><td>-21</td></tr>
<tr><td>Z</td><td>-11</td><td>-1</td></tr>
</table>
<table>
<tr><td>e.g.</td><td>X</td><td>Y</td></tr>
<tr><td>X</td><td>5</td><td>-6</td></tr>
<tr><td>Z</td><td>-1</td><td>4</td></tr>
</table>
</td></tr>
<tr><td>4</td><td>(c)</td><td></td><td>Row minima are –10.5 and –5.5
Row maximin = –5.5
Li’s play-safe is strategy Z
Mia should play strategy X</td><td>M1

A1 ft
B1 ft
[3]</td><td>1.1

1.1
1.1</td><td>Follow through their table from part (b) or original tables
Working may be seen on table from part (b)
Seen or implied from row minima, or identifying play-safe for Li
Or indicated in table, or implied from their row maximin value stated
Correct choice for their table from part (b)</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>5</td><td>(a)</td><td>(i)</td><td>
</td><td>
M1

M1

M1

A1

[4]</td><td>
1.1

1.1

1.1

1.1</td><td>Ignore any extra lines (e.g. profit lines) or working for parts (b), (c)

Line $2x + 3y = 12$
through (6, 0) and (0, 4)

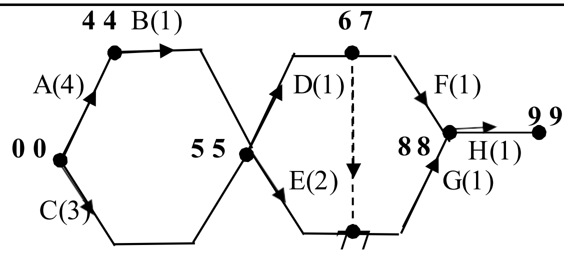
Line $x + y = 10$
through at least two of (10, 0), (2, 8), (4, 6), (6, 4), (8, 2) and (0, 10)

Line $5x + 2y = 30$
through at least two of (6, 0), (4, 5), (2, 10) and (0, 15)

Feasible region identified and correct</td></tr>
<tr><td>5</td><td>(a)</td><td>(ii)</td><td><table><tr><th>x</th><th>y</th><th>$P = 4x - y$</th></tr><tr><td>0</td><td>4</td><td>–4</td></tr><tr><td>0</td><td>10</td><td>–10</td></tr><tr><td>3.33</td><td>6.67</td><td>6.67</td></tr><tr><td>6</td><td>0</td><td>24</td></tr></table>Maximum $P = 24$</td><td>M1

A1
[2]</td><td>3.1a

1.1</td><td>'Determine' so method must be seen, not implied
Checking at least two of their vertices
or sliding a profit line (a line of gradient 4 anywhere on graph or indicating the vertex (6, 0))
24</td></tr>
<tr><td>5</td><td>(b)</td><td></td><td>FR has boundaries $x = 0$, $x + y = k$, $2x + 3y = 12$
$x + y = k$ and $2x + 3y = 12$ or $4x - y = 3$
Profit line $4x - y = 3$ cuts $2x + 3y = 12$ at (1.5, 3)
$k = 4.5$</td><td>
M1
M1
A1</td><td>
3.4
3.1a
2.2a</td><td>Not graphical
Vertex where $2x + 3y = 12$ and $x + y = k$ or profit on line $x + y = k$
Calculate where profit = 3 on boundary $2x + 3y = 12$ or (1.5, 3)
4.5</td></tr>
<tr><td></td><td></td><td></td><td>Alternative solution
$4x - (k - x) = 3 \Rightarrow 3x - k = 3$
and $2x + 3(k - x) = 1 \Rightarrow 3k - x = 12$
$k = 4.5$</td><td>
M1
M1
A1
[3]</td><td></td><td>
Use $x + y = k$ to substitute for y (or x) in $4x - y = 3$
Form a second simultaneous equation in the same unknowns
4.5</td></tr>
</table>

Question			Answer	Marks	AO	Guidance
5	(c)		Profit line $4x - y = 3$ cuts $5x + 2y = 30$ at $\frac{3}{13}$, $\frac{1}{13}$ $k = \frac{141}{13}$	 M1 A1 [2]	 3.1a 2.2a	Not graphical Or $2\frac{1}{13}$, $8\frac{1}{13}$ or (2.7 to 2.8, 8.0 to 8.2) Or $10\frac{11}{13}$ or 10.8 to 10.9
6	(a)	(i)	 Minimum time = 9 hours A, B, E, G, H have no float	M1 A1 M1 M1 A1 A1 [6]	3.3 1.1 3.4 3.4 1.1 1.1	Activity network with A, B and C correct D, E, F, G, H and dummy correct (accept directions missing) Forward pass attempted, or implied from min duration correct Backward pass attempted, or implied from critical activities correct 9 A, B, E, G, H (in any order) and no others
6	(a)	(ii)	Assuming that there are enough workers for each activity Resourcing may restrict <u>how many</u> activities can happen together	B1 [1]	3.5b	A reason why it may not always be possible to do all the activities that are needed at the same time NOT an assumption about the durations or immediate predecessors or that would delay the start time of an activity (e.g. weather or delays in arrival of materials)
6	(b)		Earliest time that E can start is 5 hours from start If there are not enough workers then A, B, C may need to be done one after another, taking 8 hours. And E could also be delayed until after D and F, giving a latest start time for E of 10 hours	B1 M1 A1 [3]	1.1 3.5a 2.2b	5 (all the activities that *must* be done before E have min completion time 5) Recognising that tasks may be done sequentially (or implied from answer 8, 9 or 10) 10 (all the activities that *can* be done before E have total duration 10 – starting E after 10 would be an unecessary delay)
6	(c)		Extend the duration of D to 3 hours	B1 [1]	3.5c	Or add an activity immediately after D of duration 2 hours

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