Oxford Cambridge and RSA

## Thursday 7 October 2021 - Afternoon

## AS Level Further Mathematics B (MEI)

## Y411/01 Mechanics a

## Time allowed: 1 hour 15 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator


## INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer all the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $\mathrm{gm} \mathrm{s}^{-2}$. When a numerical value is needed use $g=9.8$ unless a different value is specified in the question.
- Do not send this Question Paper for marking. Keep it in the centre or recycle it.


## INFORMATION

- The total mark for this paper is $\mathbf{6 0}$.
- The marks for each question are shown in brackets [ ].
- This document has 8 pages.


## ADVICE

- Read each question carefully before you start your answer.


## Answer all the questions.

1 The specific energy of a substance has SI unit $\mathrm{Jkg}^{-1}$ (joule per kilogram).
(a) Determine the dimensions of specific energy.

A particular brand of protein powder contains approximately 345 Calories (Cal) per 4 ounce (oz) serving. An athlete is recommended to take 40 grams of the powder each day.

You are given that $1 \mathrm{oz}=28.35$ grams and $1 \mathrm{Cal}=4184 \mathrm{~J}$.
(b) Determine, in joules, the amount of energy in the athlete's recommended daily serving of the protein powder.

2 The vertices of a triangular lamina, which is in the $x-y$ plane, are at the origin O and the points $\mathrm{A}(4,0)$ and $\mathrm{B}(0,3)$. Forces, of magnitude $T_{1} \mathrm{~N}, T_{2} \mathrm{~N}$ and 10 N , whose lines of action are in the $x-y$ plane, are applied to the lamina at $\mathrm{O}, \mathrm{A}$ and B respectively, as shown in the diagram.

(a) (i) Show that $\sin \alpha=0.6$.
(ii) Write down the value of $\cos \alpha$.

The lamina is in equilibrium.
(b) Determine the values of $T_{1}, T_{2}$ and $\theta$.

3 Three small uniform spheres A, B and C have masses $2 \mathrm{~kg}, 3 \mathrm{~kg}$ and 5 kg respectively. The spheres move in the same straight line on a smooth horizontal table, with B between A and C. Sphere A moves towards B with speed $7 \mathrm{~m} \mathrm{~s}^{-1}, \mathrm{~B}$ is at rest and C moves towards B with speed $u \mathrm{~m} \mathrm{~s}^{-1}$, as shown in the diagram.


Spheres A and B collide. Collisions between A and B can be modelled as perfectly elastic.
(a) Determine the magnitude of the impulse of A on B in this collision.
(b) Use this collision to verify that in a perfectly elastic collision no kinetic energy is lost.

After the collision between A and B, sphere B subsequently collides with C. The coefficient of restitution between $B$ and $C$ is $\frac{1}{4}$.
(c) Show that, after the collision between B and C, B has a speed of $(1.225-0.78125 u) \mathrm{m} \mathrm{s}^{-1}$ towards C.
(d) Determine the range of values for $u$ for there to be a second collision between A and B .

4 The diagram shows the path of a particle P of mass 2 kg as it moves from the origin O to C via A and B . The lengths of the sections $\mathrm{OA}, \mathrm{AB}$ and BC are given in the diagram. The units of the axes are metres.


P , starting from O , moves along the path indicated in the diagram to C under the action of a constant force of magnitude $T \mathrm{~N}$ acting in the positive $x$-direction. As P moves, it does $R \mathrm{~J}$ of work for every metre travelled against resistances to motion.

It is given that

- the speed of P at O is $3 \mathrm{~m} \mathrm{~s}^{-1}$,
- the speed of P at A is $11 \mathrm{~m} \mathrm{~s}^{-1}$,
- the speed of P at C is $15 \mathrm{~m} \mathrm{~s}^{-1}$.

You should assume that both $x$ - and $y$-axes lie in a horizontal plane.
(a) By considering the entire path of P from O to C , show that

$$
20 T-30 R=108 .
$$

(b) By formulating a second equation, determine the values of $T$ and $R$.
(c) It is now given that the $x$-axis is horizontal, and the $y$-axis is directed vertically upwards. By considering the kinetic energy of P at B , show that the motion as described above is impossible.

5 A car of mass 1600 kg is travelling uphill along a straight road inclined at $4.7^{\circ}$ to the horizontal. The power developed by the car is constant and equal to 120 kW . The car is towing a caravan and together they have a maximum speed of $32 \mathrm{~m} \mathrm{~s}^{-1}$ uphill.

In this question you may model any resistances to motion as negligible.
(a) Determine the mass of the caravan.

The caravan is now detached from the car. Continuing up the same road, the car passes a point A at a speed of $20 \mathrm{~m} \mathrm{~s}^{-1}$. The car later passes through a point $B$ on the same road such that $\mathrm{AB}=80 \mathrm{~m}$ and the car takes 3.54 seconds to travel from A to B . The power developed by the car while travelling from A to B is constant and equal to 80 kW .
(b) Determine the speed of the car at B.
(c) State one possible refinement to the model used in parts (a) and (b).

6 Fig. 6.1 shows a cross-section through a block of mass 5 kg which is on top of a trolley of mass 11 kg . The trolley is on top of a smooth horizontal surface. The coefficient of friction between the block and the trolley is 0.3 .

Throughout this question you may assume that there are no other resistances to motion on either the block or the trolley.


Fig. 6.1
Initially, both the block and trolley are at rest. A constant force of magnitude 50 N is now applied horizontally to the trolley, as shown in Fig. 6.1.
(a) Show that in the subsequent motion the block will slide.
(b) Find the acceleration of
(i) the block,
(ii) the trolley.

The same block and trolley are again at rest. An obstruction, in the form of a fixed horizontal pole, is placed in front of the block, the pole is 91 cm above the trolley and the width of the block is 56 cm as shown in Fig. 6.2, as well as the forward direction of motion.


Fig. 6.2
It is given that the block is uniform and that the contact between the pole and the block is smooth.
A small horizontal force is now applied to the trolley in the forward direction of motion and gradually increased.
(c) Determine whether the block will topple or slide.

7 The vertices of a uniform triangular lamina, which is in the $x-y$ plane, are at the origin and the points $(20,60)$ and $(100,0)$.
(a) Determine the coordinates of the lamina's centre of mass.

Fig. 7.1 shows a uniform lamina consisting of a triangular section and two identical rectangular sections. The coordinates of some of the vertices of the lamina are given in Fig. 7.1.

The rectangular sections are then folded at right-angles to the triangular section, to give the rigid three-dimensional object illustrated in Fig. 7.2. Two of the edges, $E_{1}$ and $E_{2}$, are marked on both figures.


Fig. 7.1


Fig. 7.2
(b) Show that the $x$-coordinate of the centre of mass of the folded object is 43.6, and determine the $y$ - and $z$-coordinates.
(c) Determine whether it is possible for the folded object to rest in equilibrium with edges $E_{1}$ and $E_{2}$ in contact with a horizontal surface.

## END OF QUESTION PAPER

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