OCR
Oxford Cambridge and RSA

Wednesday 6 October 2021 – Afternoon

Level 3 Certificate Core Maths A (MEI)

H868/01 Introduction to Quantitative Reasoning

Time allowed: 2 hours

You must have:
- the Insert (inside this document)

You can use:
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number						Candidate number				

First name(s) ______________________________

Last name ______________________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working.
- Give your final answers to a degree of accuracy that is appropriate to the context.

INFORMATION
- The total mark for this paper is **72**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 This question refers to article A in the pre-release material, "The mark and recapture method". You can find the article on the insert accompanying this paper.

(a) In Aracaju, a city in Brazil, there are children living on the street.
They are called street children.

A charity carried out two surveys collecting the names and ages of a sample of street children in the city.

- There were 295 street children in the first survey (the marked).
- In the second survey, conducted at the same time of day and in the same areas, the names of 257 street children were collected.
- The names of 32 children appeared in both lists (the recaptured).

Use the mark and recapture method to estimate the number of street children in Aracaju.
Give your answer to a sensible accuracy. **[3]**

1(a)	

(b) According to the local government there are 526 street children in Aracaju.

A local politician suggests taking the mean of the two values: the charity's estimated number in part **(a)** and the local government's number.

Does the mean give a realistic summary of the number of street children in Aracaju?
Explain your reasoning and show any calculations involved. **[2]**

1(b)	

(c)
- The population of Aracaju is 640 000 people.
- 25% of these are children.

A newspaper states that the local government's figure of 526 street children suggests that fewer than 1 in 10 000 of Aracaju's children are street children.

(i) Calculate how many children there are in Aracaju. **[2]**

(ii) Is the newspaper's statement correct?
Show your reasoning. **[2]**

1(c)(i)	
1(c)(ii)	

2 Sarah makes jewellery to sell online.
She is planning a new item, the stone pendant with a silver wire and chain shown in **Fig. 2.1**.

Fig. 2.1

Each pendant costs Sarah £9.50 to make. She needs to find a suitable selling price.

She emails this question to a representative sample of her customers:
"Would you buy one of my new pendants and, if so, what is the most you would be prepared to pay for it: £10, £15, £20, £25 or £30?"

(a) She uses the positive responses to produce the table in **Fig. 2.2**.

Complete the table and use it to find the selling price which would maximise Sarah's total profit. **[3]**

2(a)

Selling price (£)	Cost to make (£)	Profit per pendant (£)	Estimated number sold	Total profit (£)
10.00	9.50	0.50	87	43.50
15.00	9.50	5.50	71	
20.00	9.50	10.50	39	409.50
25.00	9.50	15.50	21	325.50
30.00	9.50	20.50		61.50

Fig. 2.2

The selling price which gives the maximum profit is ....

(b) Changes in the cost of materials may affect the cost of making a pendant.

Sarah begins to construct a spreadsheet to model the most profitable selling price.

She enters the cost of making a pendant into cell E1.
In the example in **Fig. 2.3** it is £10.

	A	B	C	D	E
1				Cost to make a pendant (£)	10.00
2					
3	Selling price (£)	Cost to make (£)	Profit per pendant (£)	Estimated number sold	Total profit (£)
4	10.00	10.00	0.00	87	0.00
5	15.00	10.00	5.00	71	
6	20.00	10.00	10.00	39	
7	25.00	10.00	15.00	21	

Fig. 2.3

(i) Sarah wants to enter the cost just once, in cell E1, rather than key in the cost four times in cells B4 to B7.

Write down the formula that should be in each of the cells B4 to B7. **[1]**

2(b)(i)	

(ii) She will copy the formula in cell E4 down as far as cell E7.

Write down the formula that should be in cell E4. **[1]**

2(b)(ii)	

(iii) The results of some of Sarah's modelling are shown in **Fig. 2.4**.

Cost to make a pendant (£)	10	11	12	13	14	15	16
Best selling price (£)	20	20	20	20	20	25	25
Total profit (£)	390	351	312	273	234	210	189

Fig. 2.4

She states:
"If the cost to make a pendant increases by 50% the total profit decreases by 50%."

Check her statement in the case when it costs £10 to make a pendant. **[3]**

2(b)(iii)	
	So her statement is ....

(c) At the time when it costs Sarah £10 to make a pendant, silver makes up 80% of this amount. The price of silver then increases by 12% but all her other costs stay the same.

Find the increase in Sarah's costs, as a percentage of £10.00. **[4]**

2(c)	

3 This question refers to article B in the pre-release material, "Crowd size measurement". You can find the article on the insert accompanying this paper.

(a) A large demonstration is observed from a helicopter.
The number of people is thought to be approximately

$$(9 \times 4.25 \times 400) + (11 \times 4.75 \times 400).$$

Estimate this number, without using a calculator.
Show the approximate numbers you work with and give your answer to a sensible degree of accuracy. **[3]**

3(a)	

(b) In 2010, a politician held a rally in the triangular park shown shaded in **Fig. 3.1**.
Each small square represents $400\,m^2$.

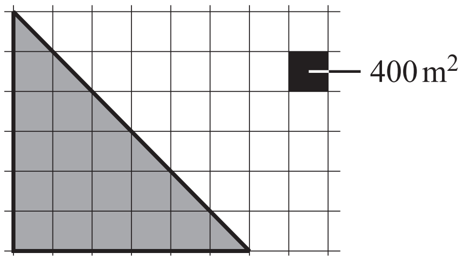


Fig. 3.1

(i) Calculate the area of the triangular park. **[2]**

3(b)(i)	

(ii) It was claimed that 200 000 people attended the rally.

Decide whether this was a reasonable claim.
Make clear any calculations and assumptions you use. **[3]**

3(b)(ii)	

(c) In an experiment, the mobile phone activity of spectators at football matches played in Milan's San Siro stadium was monitored for eight matches.

Fig. 3.2 shows the mobile phone activity and match crowd numbers, rounded to the nearest thousand, for each of eight matches in November and December 2013.

Match crowd (thousands)	40	49	35	43	13	34	38	62
Phone activity (units)	98	123	79	106	44	75	99	113

Fig. 3.2

The first 7 of these pairs are shown plotted on the scatter diagram in **Fig. 3.3**.

(i) Plot the result for the last match in the table in **Fig. 3.2** to complete the scatter diagram in **Fig. 3.3**. **[1]**

(ii) Draw a line of best fit on the scatter diagram in **Fig. 3.3**. **[1]**

3(c)(i) and 3(c)(ii)	

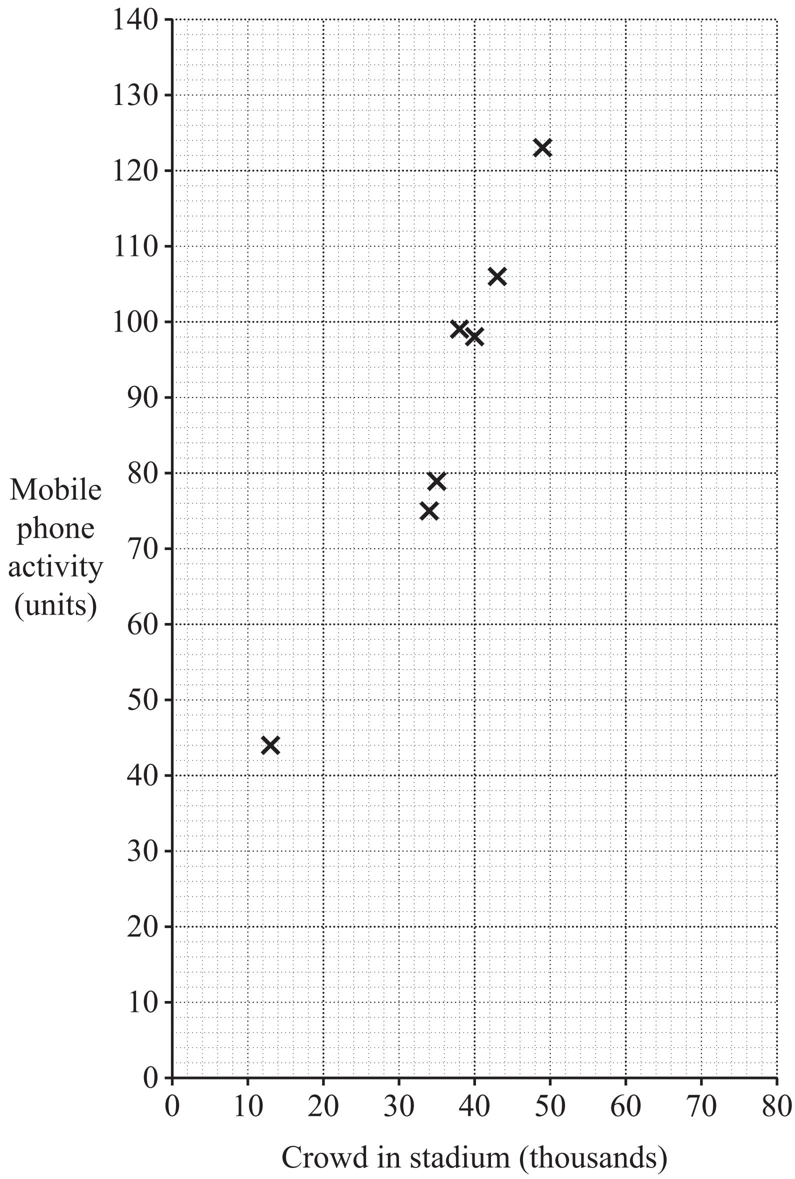


Fig. 3.3

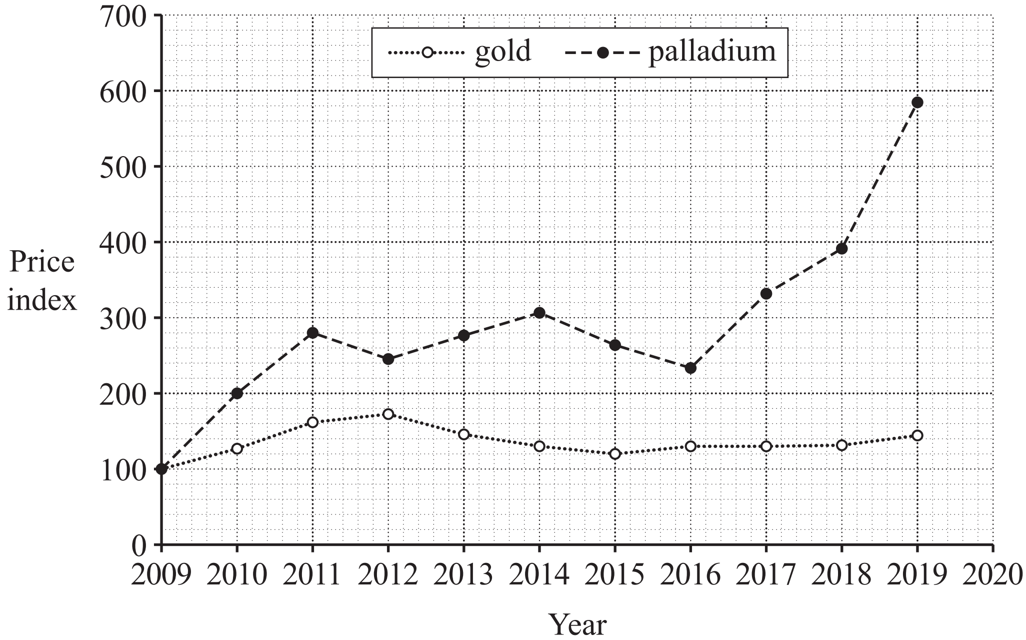
(iii) Estimate the mobile phone activity on a non-match day. **[1]**

3(c)(iii)	

4 Gold and palladium are expensive metals. Palladium is used in pollution control devices.

The graph shows the price index for each of gold and palladium with the 2009 values taken as 100.

- In 2009 the average price of gold was £25.12 per gram.
- In 2009 the average price of palladium was £6.80 per gram.



(a) **(i)** Describe briefly how gold prices changed from 2009 to 2019. **[2]**

4(a)(i)	

(ii) In which year was the price of palladium first slightly more than three times its price in 2009? **[1]**

4(a)(ii)	

(b) In 2018 the price of palladium was £27.05 per gram.

Which was the higher price in 2018, palladium or gold?
Show your working. **[3]**

4(b)	

(c) In January 2020 palladium was one of the most expensive precious metals.
It had a price of £70.03 per gram.

Calculate the side length of a cube of palladium worth £1 million at its January 2020 cost.
The density of palladium is 12 grams per cubic centimetre. **[5]**

4(c)	

5 The Desert Locust is an insect. When they form swarms, as shown in **Fig. 5.1**, they are the most destructive pests on the planet. These swarms can eat hundreds of square kilometres of vegetation in a very short time.

Fig. 5.1

(a) In 1875 in the Western United States there was a locust swarm of area 495 000 square miles. It was estimated that on average each square mile of this swarm held 130 million locusts.

According to these figures, calculate how many locusts there were in the swarm in total. Give your answer to the nearest trillion locusts. A trillion is 10^{12}. **[3]**

5(a)	

(b) An officer of the U.N. Food and Agriculture Organisation said, 'A swarm the size of Manhattan can, in a single day, eat the same amount of food as all the people in New York and California combined.'

Use the information in **Fig. 5.2** to answer the questions below.

- Area of Manhattan = $59.1\,km^2$.
- Population of New York = 19.5 million.
- Population of California = 39.5 million.
- The average American eats approximately the equivalent of 600 grams of wheat a day.
- Locusts can eat their own body weight of wheat each day.
- A typical locust weighs 2 grams.
- A square kilometre of a locust swarm has up to 80 million locusts in it.

Fig. 5.2

(i) Find the combined population of New York and California. **[1]**

(ii) Calculate the equivalent weight of wheat eaten in a day by all the people in New York and California combined. **[1]**

(iii) Is the statement made by the U.N. Food and Agriculture Organisation officer correct? Assume that locusts and Americans only eat wheat.
Show your working. **[3]**

5(b)(i)	
	.... million people
5(b)(ii)	
	.... million grams
5(b)(iii)	

(c) Under certain conditions, the population of even a small group or swarm of locusts can grow exponentially with its age. Locusts breed every three months. A graph showing the growth of a swarm is shown in **Fig. 5.3**.

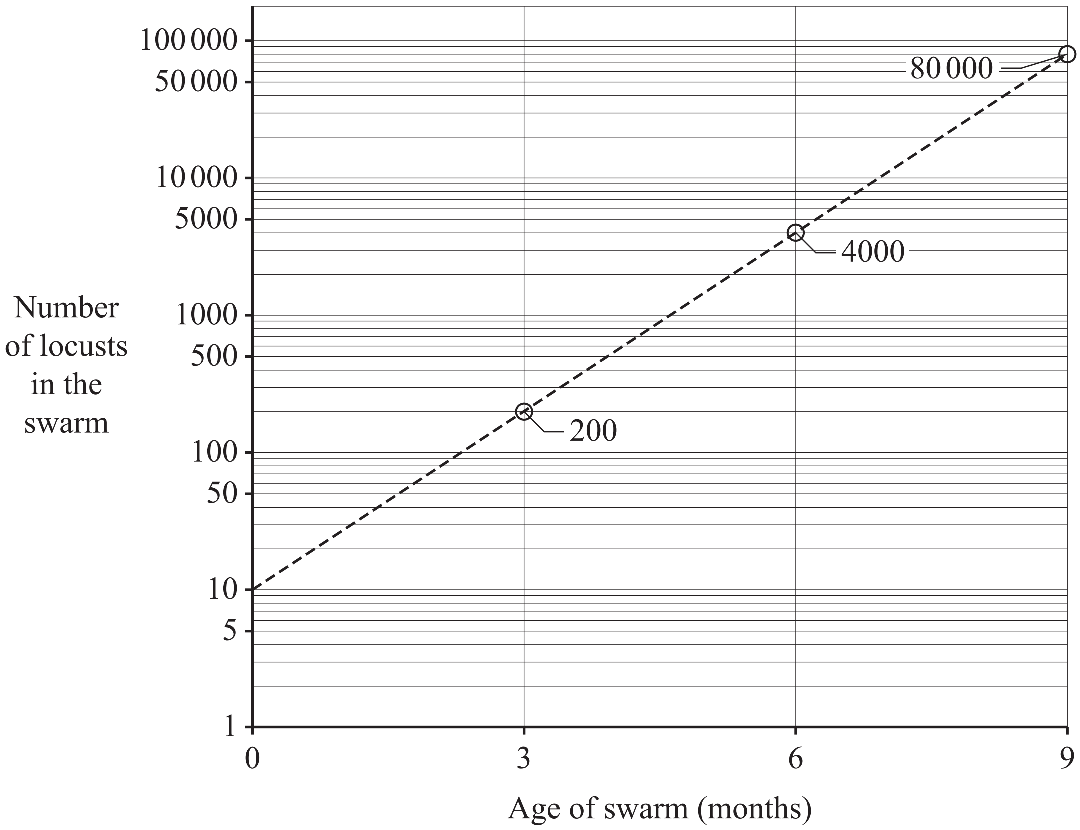


Fig. 5.3

(i) How many locusts were there in the swarm at first? **[1]**

5(c)(i)	

(ii) By what factor does the population of the swarm increase every 3 months? **[1]**

5(c)(ii)	

(iii) Spraying a swarm at 3 months can reduce its population. However, the remaining locusts still reproduce at the same rate for each subsequent 3-month period.

Use your answer to part **(ii)** to complete the table shown in **Fig. 5.4**. **[1]**

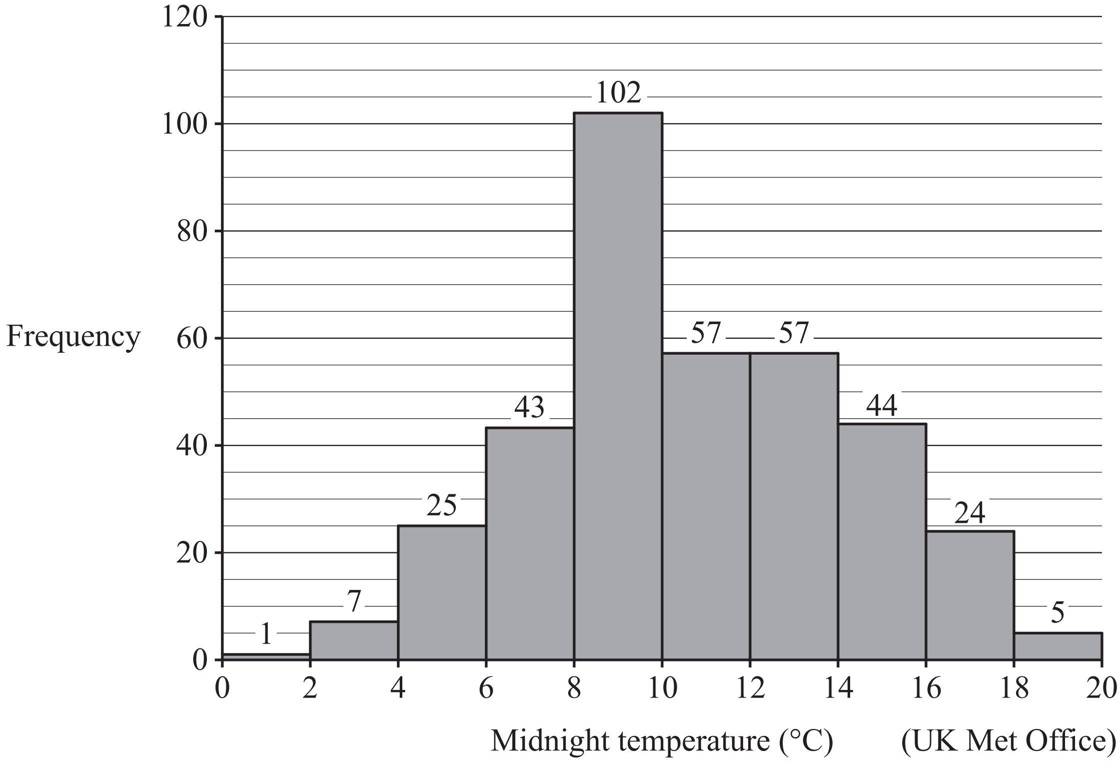
5(c)(iii) How swarm population varies with time whether sprayed at 3 months or not

Age of swarm (months)	3	6	9
No spraying	200	4000	80 000
Spraying at 3 months	20		

Fig. 5.4

6 The frequency chart below illustrates the midnight temperature, in °C, each night of 2019 in Camborne, Cornwall. The width of each interval is 2 °C.

Temperatures were below 20 °C at midnight for all 365 nights, but below 2 °C for just one night.



(a) Would it be reasonable to model the distribution by a Normal distribution? Support your answer with a reason. **[1]**

6(a)	.... because

(b) **(i)** How many nights was the midnight temperature 18 °C or more? **[1]**

6(b)(i)	

(ii) Show that 4 out of every 5 nights have a midnight temperature below 14 °C. **[3]**

6(b)(ii)	

(c) A farmer, near Camborne, grows orchids all year round in her heated greenhouse. When the outside temperature falls below –8 °C all her orchids die.

According to meteorologists a night with a temperature below –8 °C occurring near Camborne is a once in 25 years event.

The farmer assumes the meteorologists are right and calculates the risk of a temperature below –8 °C during the next night to be about $\frac{1}{365 \times 25}$. She ignores leap years.

(i) What additional assumption has she made? **[1]**

6(c)(i)	

(ii) Is her assumption likely to be correct?
Give a reason for your answer. **[1]**

6(c)(ii)	

7 Compactness describes how tightly packed objects are.
Circles are the most compact shapes.
The three shapes in **Fig. 7.1** are arranged in order of their compactness.
R is more compact than S and S is more compact than T.

Fig. 7.1

(a) A student thinks that a measure of compactness of a shape might be

$$\frac{\text{Area of the shape}}{\text{Perimeter of the shape}}.$$

Fig. 7.2 shows three shapes A, B, and C drawn on a centimetre square grid.

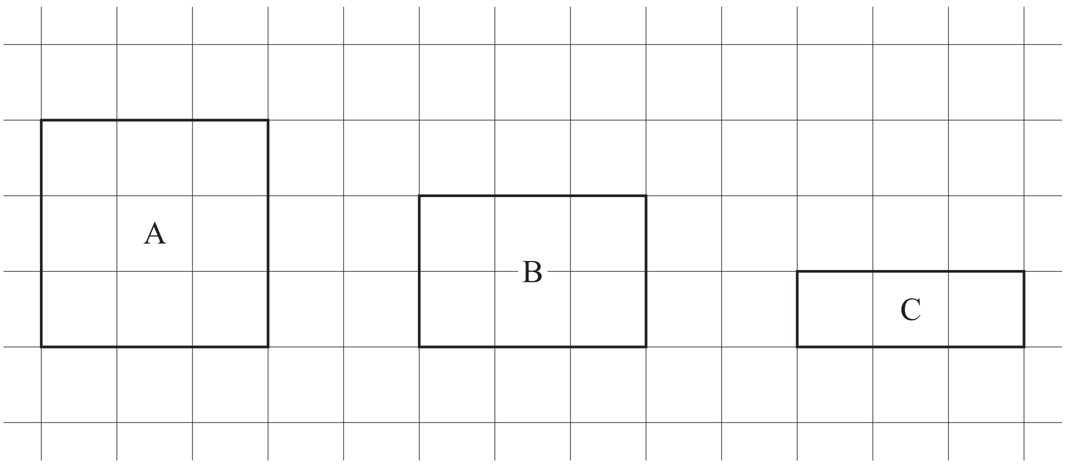


Fig. 7.2

Use the student's suggested method to complete the table in **Fig. 7.3**. **[2]**

7(a)

Shape	Area (cm^2)	Perimeter (cm)	Compactness (student's measure)
A	9	12	0.75
B			
C			

Fig. 7.3

(b) The two squares in **Fig. 7.4** are equally compact.
They are drawn on a centimetre square grid.

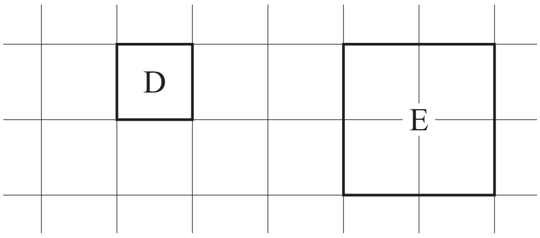


Fig. 7.4

(i) Does using the student's formula

$$\frac{\text{Area of the shape}}{\text{Perimeter of the shape}}$$

to measure compactness show them as equally compact?
Explain your answer fully. **[3]**

7(b)(i)	

(ii) This alternative measure of compactness is suggested by another student.

$$\frac{\text{Area of the shape}}{(\text{Perimeter of the shape})^2}$$

Show that the compactness of both squares D and E is the same using this measure. **[2]**

7(b)(ii)	

(c) The expression in part **(b)(ii)** when used by architects and geographers is multiplied by 4π to give the compactness, C, of a shape as

$$C = \frac{4\pi \times (\text{Area of the shape})}{(\text{Perimeter of the shape})^2}.$$

(Remember the circle is the most compact shape.)

(i) Calculate the value of C for a circle of radius 2 cm. **[3]**

7(c)(i)	

(ii) Why is it useful to multiply the expression in part **(b)(ii)** by 4π, particularly when dealing with circles? **[1]**

7(c)(ii)	

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).