OCR
RECOGNISING ACHIEVEMENT

LEVEL 3 CERTIFICATE

MATHEMATICS FOR ENGINEERING

H860/01

Paper 1

Candidates answer on the answer booklet.

OCR supplied materials:
- 16 page answer booklet (sent with general stationery)
- Graph paper
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Friday 20 May 2011
Afternoon

Duration: 2 hours

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H860001

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **60**.
- This document consists of **8** pages. Any blank pages are indicated.

1 A small engineering company specialises in the manufacture of reproduction classic motorcycles. Three models are produced, Type B, Type S and Type A. The profit made by selling one of each type of motorcycle is £600, £400 and £450 respectively. The company needs to establish the number of motorcycles of each type to produce each month in order to maximise its profit.

Each motorcycle has to pass through four workshops during its manufacture. The number of hours needed for each motorcycle in each workshop is given in Table 1a.

		Workshop			
		1	**2**	**3**	**4**
Model type	**B**	10	12	20	20
	S	20	12	10	5
	A	15	10	10	15

Table 1a

The company employs workshop mechanics who provide a maximum number of hours in each workshop each month as shown in Table 1b.

	Workshop			
	1	**2**	**3**	**4**
Hours available each month	480	320	360	360

Table 1b

The company has a contract that requires it to produce at least 6 motorcycles of each type every month.

(a) (i) If b, s and a are variables that represent the numbers of motorcycles of each type, show that the constraint imposed by the hours available in workshop 1 is

$$10b + 20s + 15a \leqslant 480\,. \quad \textbf{[1]}$$

(ii) Formulate the problem as a linear programme involving six further constraints and the objective function which maximises the profit. **[3]**

(b) In one particular month, the company decides to produce exactly 8 motorcycles of type A.

(i) Reformulate the problem as a linear programme involving two variables. **[2]**

(ii) Represent the constraints on a graph using appropriate scales. **[3]**

(iii) Indicate the feasible region on the graph. **[1]**

(iv) Determine graphically the number of motorcycles of types B and S that the company must produce in order to maximise its profit. **[1]**

2 A precision engineering company produces measuring devices that have been calibrated to a high degree of accuracy. The quality control department has found that 20% of all such devices have to be recalibrated before they can be sold.

(a) Use the binomial distribution to calculate the probability that, in a random sample of ten devices, two or more will have to be recalibrated. **[3]**

(b) In a random sample of 100 devices, the distribution of the number of devices that need to be recalibrated may be approximated by the Normal distribution.

(i) Find the mean and variance of this distribution. **[1]**

(ii) Estimate the probability that, in a random sample of 100 devices, between 18 and 24 inclusive will have to be recalibrated. **[3]**

(c) The company has developed a new manufacturing technique which has reduced the proportion of devices that will have to be recalibrated to 5%.

In a random sample of 100 such devices, the distribution of the number of devices that need to be recalibrated may be approximated by a Poisson distribution with parameter 5.

Estimate the probability that no more than two devices will have to be recalibrated. **[3]**

3 A canvas marquee has a width of 6 m and a wall height of 2 m. The cross-section of the marquee, together with the x-axis and y-axis, is shown in Fig. 3a. The marquee is symmetrical about the centre line. The profile of the left-hand side of the roof section is a curve with equation $y = (e^{\frac{1}{2}x} + e^{-\frac{1}{2}x})$.

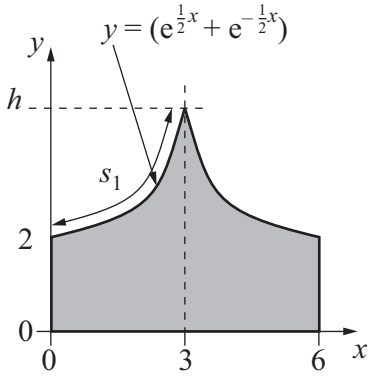

Fig. 3a

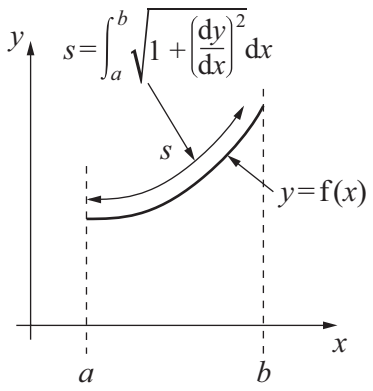

Fig. 3b

(a) Calculate the height, h m, of the marquee. **[1]**

(b) Calculate the cross-section area, shown shaded in Fig. 3a. **[4]**

(c) The length, s, of a continuous line with equation $y = f(x)$ between $x = a$ and $x = b$, as shown in Fig. 3b, is given by

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

(i) Given $y = (e^{\frac{1}{2}x} + e^{-\frac{1}{2}x})$, show that

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{1}{2}(e^{\frac{1}{2}x} + e^{-\frac{1}{2}x}).$$

[3]

(ii) Use this formula to calculate the length of the curve s_1, as shown in Fig. 3a. **[1]**

4 The output, R, of a particular system is given by

$$R = \mathrm{f}(T) = \frac{T^2 - 2}{2(T^2 - 4)}$$

where T is a continuous real variable.

(a) Express $\mathrm{f}(T)$ in the form $A + \frac{B}{T-2} + \frac{C}{T+2}$

where A, B and C are constants to be determined. **[2]**

(b) **(i)** Show that the function has one turning point and determine its coordinates. **[2]**

(ii) Calculate the coordinates of any poles and zeros. **[2]**

(iii) Sketch a graph of $\mathrm{f}(T)$ against T and include labels showing the coordinates of the key features. **[2]**

(c) Rearrange $R = \mathrm{f}(T)$ into the form $T = \mathrm{g}(R)$ and state the range of values of R for which T is real. **[3]**

5 A quadrilateral roof section of a building with corners A, B, C and D is shown in Fig. 5 together with the x-axis, y-axis and z-axis.

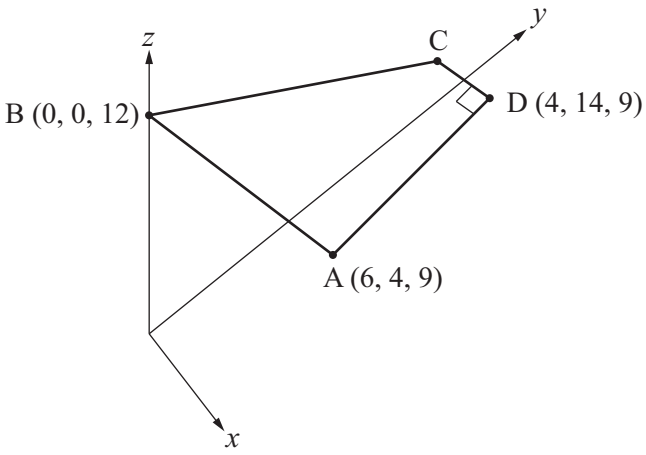

Fig. 5

The corners A, B, C and D have coordinates (x, y, z) and all lie in the same plane. Corners A, B and D have the following coordinates.

A: (6, 4, 9)
B: (0, 0, 12)
D: (4, 14, 9)

(a) (i) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$ in component form. **[1]**

(ii) Calculate the angle BAD. **[2]**

(b) The standard cartesian equation for the plane is

$$ax + by + cz = d$$

where a, b, c and d are constants.

Find the values of a, b, c and d. **[5]**

(c) The angle between edges DA and DC is 90°. It is known that the x-coordinate of corner C is equal to 2.

Calculate the y- and z-coordinates of corner C correct to 3 significant figures. **[4]**

6 In digital communication systems, a device known as a *vector modulator* is used for frequency conversion. A diagram of such a device, which consists of two mixers and a combiner, is shown in Fig. 6. A mixer produces an output which is equal to the product of its inputs. The combiner produces an output which is equal to the sum of its inputs.

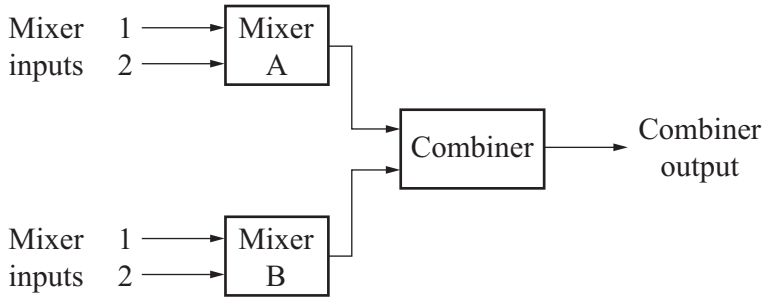

Fig. 6

The inputs to mixer A are of the forms $\cos(\omega_c t)$ and $\cos(\omega_o t)$ and the inputs to mixer B are of the forms $\sin(\omega_c t)$ and $\sin(\omega_o t)$.

(a) Express the output of the combiner in the form $\cos(kt)$ where k is to be determined. **[2]**

(b) You are given the two following standard trigonometric relationships.

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Show that the output of mixer A may be written as

$$\tfrac{1}{2}\cos(\omega_c + \omega_o)t + \tfrac{1}{2}\cos(\omega_c - \omega_o)t$$

and find a similar expression for the output of mixer B. **[3]**

(c) The first input to mixer B now contains a phase error, ϕ, and has the form $\sin(\omega_c t + \phi)$.

Show that the output of mixer B may be written as

$$\sin(\omega_c t)\sin(\omega_o t)\cos\phi + \cos(\omega_c t)\sin(\omega_o t)\sin\phi.$$ **[2]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.