Oxford Cambridge and RSA

Cambridge Technicals
Engineering

Unit 23: Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering
05823 - 05825

Mark Scheme for January 2022

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
DM1	Method mark dependent on previous M mark
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
λ	Omission sign
Other abbreviations in mark scheme	**Meaning**
oe	Or equivalent
Soi	Seen or implied
www	Without wrong working
ecf	Error carried forward

Subject specific marking instructions

Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. These annotations must be in the body of the work and **not** anywhere near the right hand margin of each page.
Mark in using a red pen.

Put the mark for each subquestion near to and to the right of the mark for the question. Total all marks for the question and put this total in a ring at the bottom right of each question.

Transfer these marks to the box on the front page.

Total the marks for the paper. I suggest that all unringed marks are then totalled to make sure that the final mark is correct.
An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

DM
A method mark which is dependent on a previous method mark.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

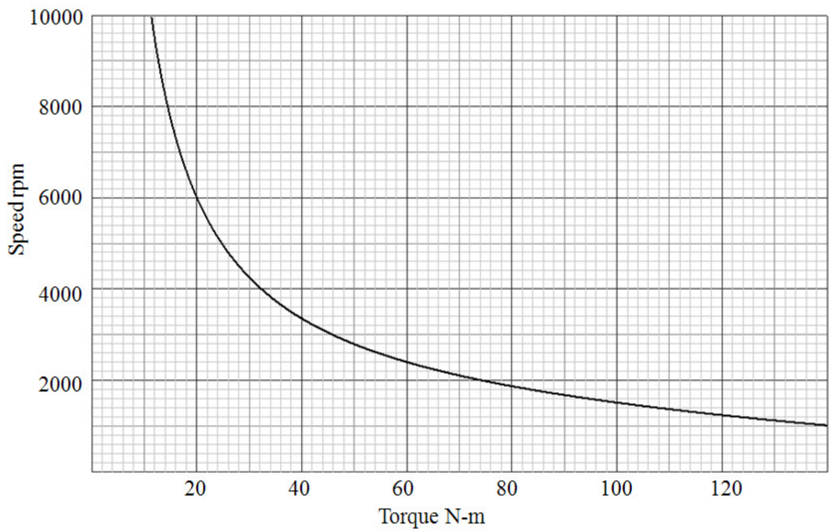
When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Question			Answer	Marks	Guidance
1	(a)		$\frac{H}{d-D}=\frac{h}{d}$ OR $\frac{h-H}{D}=\frac{h}{d}$ OE $h=\frac{dH}{d-D}$ **AG**	B1	Using similar triangles Allow other correct methods Must be convincing
				[1]	
1	(b)	(i)	$h=\frac{0.8\times0.5}{0.8-0.6}=2$	M1 A1	
			r = 0.4 and R =0.3 $V=\frac{\pi}{3}\left(r^2h-R^2(h-H)\right)$	B1	
			$V=\frac{\pi}{3}\left((0.4)^2 2-(0.3)^2(1.5)\right)$	M1	Allow two volumes to be expressed separately
			0.19(37) (m^3)	A1	Accept exact equivalent $\frac{37}{600}\pi$
				[5]	
1	(b)	(ii)	$l=\sqrt{2^2+0.4^2}$ OR $L=\sqrt{(2-0.5)^2+0.3^2}$ $=\sqrt{4.16}$ oe AND $=\sqrt{2.34}$ oe	M1 A1	Appropriate calculation for slant height of either cone Both slant heights correct soi
			$S=\pi rl$	B1	Use of $S=\pi rl$ SOI
			$\pi(0.4)l-\pi(0.3)L$	M1	Attempt at curved surface area of frustum using their slant heights
			$S=\pi(0.4\times\sqrt{4.16}-0.3\times\sqrt{2.34}+0.3^2)$ $=1.4(04)$ (m^2)	A1	
				[5]	
				[11]	

Question			Answer	Marks	Guidance
2	(i)		$6000 = \frac{a}{20} + 20b + 1000$ AND $1500 = \frac{a}{100} + 100b + 1000$	M1	Form simultaneous equations
			$\frac{a}{20} + 20b = 5000$ $\frac{a}{100} + 100b = 500$ $a + 400b = 100000$ $a + 10000b = 50000$ $9600b = -50000$	M1	Solve their equations by any valid method
			$b = \frac{-50000}{9600} = -5.208..$	A1	-5.21 or better
			$a = -400b + 100000 = 102083.333$	A1	102 000 or better
				[4]	
2	(ii)			B1 B1	Graph must have the approximate correct curved shape Graph required over domain [20,100] and must pass very close to (20, 6000) and (100, 1500)
				[2]	

Question			Answer	Marks	Guidance
2	(iii)		$\omega = (\frac{a}{\tau} + b\tau + 1000)\frac{2\pi}{60}$	B1	Evidence of rpm to rads/sec
			$P = \omega\tau = (a + b\tau^2 + 1000\tau)\frac{2\pi}{60}$	M1	Form an expression for Power by multiplying their ω by τ
			for max/min $\frac{dP}{d\tau} = 0 = (2b\tau + 1000)\frac{2\pi}{60} \Rightarrow \tau = \frac{-1000}{2b} \approx 96$	DM1	Attempt to differentiate their $P = f(\tau)$ and equate to zero
				A1FT	Correct τ following through their b from (i)
			$P = \omega\tau = (102000 - 5.21 \times 96^2 + 1000 \times 96)\frac{2\pi}{60} = 15706.35$ (kW)	A1	CAO Accept AWRT 15700 (W) or 15.7 kW
				[5]	
				[11]	

Question			Answer	Marks	Guidance
3	(i)		$\frac{mv^2}{r} \geq 1.25mg$ $v \geq \sqrt{1.25gr}$ $v \geq \sqrt{1.25 \times 9.8 \times 8}$ $= 9.9$ (ms^{-1})	M1 A1	Rearrange to make v subject Accept anything that rounds to 9.9
				[2]	
3	(ii)		$mgh = mg(2 \times 8) + \frac{1}{2}m(9.9)^2$ $h = 16 + \frac{9.9^2}{2g}$ $= 21$ (m)	M1 M1 A1FT	Equate initial energy with energy at top of loop, using their v Solve AWRT 21 FT *their* v from part (i)
				[3]	
3	(iii)		$mg(21) = \frac{1}{2}m(v_b)^2$ $(v_b) = \sqrt{21 \times 2g} = 20.29$ (m s^{-1})	M1 A1FT	Equate initial PE with KE at bottom of loop 3sf or better FT their h from part (ii)
				[2]	
3	(iv)		Force $= \frac{20.29^2 \times 1000}{8} + 9.8 \times 1000$ $= 61250$ (N)	M1 A1FT	AWRT 61300 FT their v_b from part (iii)
				[2]	

Question			Answer	Marks	Guidance
3	(v)		Drag and other frictional forces **would slow down the car** leading to smaller forces acting on the track. Slower speed at the top of the loop may **invalidate safety rules**.	**B1** **B1**	Any relevant modelling observation related to frictional forces Any relevant practical implication for safety of the roller-coaster
				[2]	
				[11]	

Question			Answer	Marks	Guidance
4	(i)		$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix}$	B1	All elements must be shown as exact values but isw if decimal values are subsequently shown
				[1]	
4	(ii)		$\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$	B1 B1	Correct multiplication SOI
				[2]	
4	(iii)		$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ $= \begin{bmatrix} \sqrt{\frac{3}{2}} - \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{2}} \end{bmatrix}$ $= \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{3} - 1 \\ \sqrt{3} + 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}-1}{\sqrt{2}} \\ \frac{\sqrt{3}+1}{\sqrt{2}} \end{bmatrix}$	B1 M1 A1	Matrix $\boldsymbol{A}$ correct Multiplication: *their* matrix $A \times$ *their* $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ Any exact equivalent simplified form that is clearly $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ but isw if decimal values are subsequently shown.
				[3]	
4	(iv)		$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ $= \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 - \sqrt{3} & -(\sqrt{3} + 1) \\ \sqrt{3} + 1 & 1 - \sqrt{3} \end{bmatrix}$	B1 B2	$\boldsymbol{B}$ formed with all elements correct Allow equivalent answers but isw if decimal values are subsequently shown.
				[3]	
				[9]	

Question			Answer	Marks	Guidance
5	(i)		$G(j\omega)=\dfrac{1}{(j\omega)^2+2j\omega+4}=\dfrac{1}{4-\omega^2+2j\omega}$	M1	Evidence of $j^2=-1$
			$=\dfrac{4-\omega^2-2j\omega}{(4-\omega^2+2j\omega)(4-\omega^2-2j\omega)}=\dfrac{4-\omega^2-2j\omega}{(4-\omega^2)^2+4\omega^2}$	M1	Rationalise and simplify
			$a=\dfrac{4-\omega^2}{(4-\omega^2)^2+4\omega^2}$ and $b=\dfrac{-2\omega}{(4-\omega^2)^2+4\omega^2}$ **AG**	A1	Must be convincing
				[3]	
5	(ii)		$a^2+b^2=\left(\dfrac{4-\omega^2}{(4-\omega^2)^2+4\omega^2}\right)^2+\left(\dfrac{-2\omega}{(4-\omega^2)^2+4\omega^2}\right)^2$	M1	Form squares
			$=\dfrac{(4-\omega^2)^2+4\omega^2}{((4-\omega^2)^2+4\omega^2)^2}=\dfrac{1}{(4-\omega^2)^2+4\omega^2}=\dfrac{1}{16-8\omega^2+\omega^4+4\omega^2}$	M1	Simplify
			$=\dfrac{1}{\omega^4-4\omega^2+16}$	A1	
			$A=\dfrac{1}{\sqrt{\omega^4-4\omega^2+16}}$	A1	
				[4]	
5	(iii)		$\dfrac{dA}{d\omega}=-\frac{1}{2}(\omega^4-4\omega^2+16)^{-3/2}(4\omega^3-8\omega)$	M1 M1	Attempt to differentiate using chain rule Leading to the correct form
			$=0\Rightarrow\omega(4\omega^2-8)=0$ Disregard $\omega=0$ $\omega=-\sqrt{2}$ Final answer $\omega=\sqrt{2}$ **only**	M1 A1	Equate to zero and solve for ω
				[4]	

Question			Answer	Marks	Guidance
5	(iv)		$a = \dfrac{4-\omega^2}{(4-\omega^2)^2+4\omega^2}$ $b = \dfrac{-2\omega}{(4-\omega^2)^2+4\omega^2}$		
			$\dfrac{b}{a} = \dfrac{-2\omega}{4-\omega^2}$ $\quad \dfrac{b}{a} = \dfrac{-2\sqrt{2}}{4-2} = -\sqrt{2}$	M1	Division *b/a* (accept unsimplified) and sub *their* ω
			$\alpha = \tan^{-1}(-\sqrt{2})$	M1	Evaluate α
			$= \mathbf{-0.96}$ (-54.7°)	A1	Allow degrees or radians Allow -0.96 or 2.2 Allow -55° or 125°
				[3]	
				[14]	

Question			Answer	Marks	Guidance
6	(i)		Let $X = e^{x/2}$ $5 = X + X^{-1} + 1$ $X + X^{-1} = 4$ $X^2 - 4X + 1 = 0$ X=3.732 also 0.268 $x = 2\ln 3.732 = 2.634$ AND $x = 2\ln 0.268 = -2.634$	B1 B1 M1 A1	 Correct quadratic oe Solve for X as decimal or surd $2 \pm \sqrt{3}$ Both correct to 1dp or better
				[4]	
6	(ii)	(*A*)	$y = (e^{x/2} + e^{-x/2}) + 1$; $\frac{dy}{dx} = \frac{1}{2}(e^{x/2} - e^{-x/2})$ $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(e^x - 2 + e^{-x})$ oe $1 + \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(4 + e^x - 2 + e^{-x})$ $= \frac{1}{4}(e^x + 2 + e^{-x}) = \frac{1}{4}(e^{x/2} + e^{-x/2})^2$ $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{1}{2}(e^{x/2} + e^{-x/2})$ **AG**	M1 A1 M1 A1	Attempt to differentiate Obtain unsimplified expression for $\left(\frac{dy}{dx}\right)^2$ Attempt to present as a perfect square Convincing completion
				[4]	

Question			Answer	Marks	Guidance
6	(ii)	(*B*)	$S = 2\int_0^4 \frac{1}{2}(e^{x/2} + e^{-x/2})dx$	M1	Substitute expression
			$2\left[e^{x/2} - e^{-x/2}\right]_0^4$	M1	Integrate
			$2\left[(e^2 - e^{-2}) - (1-1)\right]$	A1	Correct
			$2\left[7.3891 - 0.1353\right] = 14.5074$	M1	Apply limits $F[4] - F[0]$
				A1	Accept anything that rounds to 14.5
					Full marks for answer www
				[5]	
				[13]	

Question			Answer	Marks	Guidance
7	(i)		$EI\dfrac{d^2y}{dx^2} = -\dfrac{W(L-x)^2}{2L} = -\dfrac{W(L^2-2Lx+x^2)}{2L}$ $EI\dfrac{dy}{dx} = -\dfrac{W(L^2x - Lx^2 + (x^3/3)}{2L}$ $EIy = -\dfrac{W/12(6L^2x^2 - 4Lx^3 + x^4)}{2L}$ $y = -\dfrac{Wx^2(6L^2-4Lx+x^2)}{24EIL}$ AG	M1 M1 M1 A1	Expand bracket First integration Second integration
				[4]	
7	(ii)		$y_{\max} = -\dfrac{WL^2(6L^2-4L^2+L^2)}{24EIL}$ $y_{\max} = -\dfrac{WL^2(3L^2)}{24EIL} = -\dfrac{WL^3}{8EI}$	M1 A1	Substitute $x = L$ Simplify
				[2]	
7	(iii)		$y = -\dfrac{1600\times10^3}{8\times200\times10^9\times10^{-5}}$ $y = -\dfrac{10^3}{10^4} = 0.1\,(\text{m})$	B1	Allow -0.1
				[1]	
7	(iv)	(*A*)	W = density × volume × g = $8000\times(\dfrac{a^2}{4}\times10)\times g = 2\times10^5\times a^2$ (with g =10)	B1	
				[1]	

Question			Answer	Marks	Guidance
7	(iv)	(*B*)	$y_{\max} = \dfrac{WL^3}{8EI}$ for new beam $= \dfrac{2 \times 10^5 \times a^2 \times 10^3}{8 \times 200 \times 10^9 \times \frac{a^4}{48}}$ $1600a^2 = 48 \times 2$ $a = \sqrt{96/1600} \approx 0.244949..$ (m)	 M1 M1 A1	Guidance to anticipate most likely method: Substitute everything into y_{max} for new beam, including their expression for W from (iv) Equate to their value for y_{max} from (iii) and attempt to solve Accept any valid solution method
				[3]	
				[11]	

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