

# Wednesday 15 June 2022 – Afternoon A Level Further Mathematics B (MEI)

Y431/01 Mechanics Minor

## Time allowed: 1 hour 15 minutes



#### You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If you need extra space, use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by  $gm s^{-2}$ . When a numerical value is needed use g = 9.8 unless a different value is specified in the question.
- Do not send this Question Paper for marking. Keep it in the centre or recycle it.

### **INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has 8 pages.

### ADVICE

• Read each question carefully before you start your answer.

#### Answer **all** the questions.

[2]

[4]

- 1 Newton's gravitational constant, G, is approximately  $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ .
  - (a) Find the dimensions of G.

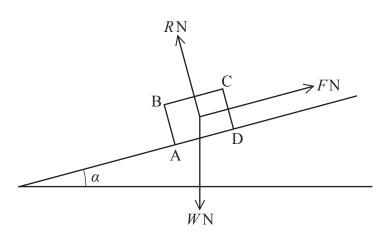
The escape velocity, v, of a body from a planet's surface, is given by the formula

 $v = k G^{\alpha} M^{\beta} r^{\gamma},$ 

where M is the planet's mass, r is the planet's radius and k is a dimensionless constant.

**(b)** Use dimensional analysis to find  $\alpha$ ,  $\beta$  and  $\gamma$ .

2 The diagram below shows the cross-section through the centre of mass of a uniform block of weight WN, resting on a slope inclined at an angle  $\alpha$  to the horizontal. The cross-section is a rectangle ABCD. The slope exerts a frictional force of magnitude FN and a normal contact force of magnitude RN.



- (a) Explain why a triangle of forces may be used to model the scenario. [2]
- (b) In the space provided in the Printed Answer Booklet, draw such a triangle, fully annotated, including the angle  $\alpha$  in the correct position. [2]

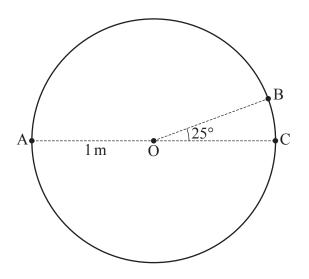
The coefficient of friction between the block and the slope is  $\mu$ .

(c) Given that the block is in limiting equilibrium, use your diagram in part (b) to show that  $\mu = \tan \alpha$ . [2]

It is given that AB = 8.9 cm and AD = 11.6 cm. The coefficient of friction between the slope and the block is 1.35. The slope is slowly tilted so that  $\alpha$  increases.

(d) Determine whether the block topples first without sliding or slides first without toppling. [2]

**3** A rough circular hoop, with centre O and radius 1 m, is fixed in a vertical plane. A, B and C are points on the hoop such that A and C are at the same horizontal level as O, and OB makes an angle of 25° above the horizontal, as shown in the diagram.



A bead P of mass 0.3 kg is threaded onto the hoop. P is projected vertically downwards from A on two separate occasions.

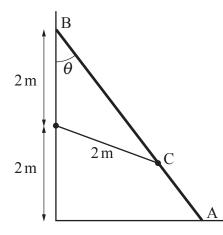
- The first time, when P is projected with a speed of  $4 \text{ m s}^{-1}$ , it first comes to rest at B.
- The second time, when P is projected with a speed of  $vm s^{-1}$ , it first comes to rest at C.

The situation is modelled by assuming that during the motion of P the magnitude of the frictional force exerted by the hoop on P is constant.

<b>(a)</b>	Determine the value of <i>v</i> .	[5]
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(b) Comment on the validity of the modelling assumption used in this question. [1]

4 A uniform beam AB of mass 6 kg and length 5 m rests with its end A on smooth horizontal ground and its end B against a smooth vertical wall. The vertical distance between the ground and B is 4 m, and the angle between the beam and the downward vertical is  $\theta$ . To prevent the beam from sliding, one end of a light taut rope of length 2 m is attached to the beam at C and the other end of the rope is attached to a point on the wall 2 m above the ground, as shown in the diagram.



(a) By considering the value of  $\cos \theta$ , determine the distance BC. [2]

An object of mass 75 kg is placed on the beam at a point which is x m from A.

It is given that the tension in the rope is T N and the magnitude of the normal contact force between the ground and the beam is R N.

- (b) By taking moments about B for the beam, show that 25R + 3675x 16T = 19110. [4]
- (c) Given that the rope can withstand a maximum tension of 720 N, determine the largest possible value of *x*.

- 5 Point A lies 20 m vertically below a point B. A particle P of mass 4m kg is projected upwards from A, at a speed of  $17.5 \text{ m s}^{-1}$ . At the same time, a particle Q of mass m kg is released from rest at point B. The particles collide directly, and it is given that the coefficient of restitution in the collision between P and Q is 0.6.
  - (a) Show that, immediately after the collision, P continues to travel upwards at 0.7 m s<sup>-1</sup> and determine, at this time, the corresponding velocity of Q.
    [8]

In another situation, a particle of mass 3m kg is released from rest and falls vertically. After it has fallen 10 m, it explodes into two fragments. Immediately after the explosion, the lower fragment, of mass 2m kg, moves vertically downwards with speed  $v_1 \text{ m s}^{-1}$ , and the upper fragment, of mass m kg, moves vertically upwards with speed  $v_2 \text{ m s}^{-1}$ .

- (b) Given that, in the explosion, the kinetic energy of the system increases by 72%, show that  $2v_1^2 + v_2^2 = 1011.36$ . [3]
- (c) By finding another equation connecting  $v_1$  and  $v_2$ , determine the speeds of the fragments immediately after the explosion. [6]

**6** Fig. 6.1 shows a light rod ABC, of length 60 cm, where B is the midpoint of AC. Particles of masses 3.5 kg, 1.4 kg and 2.1 kg are attached to A, B and C respectively.

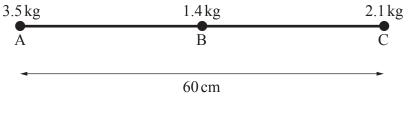


Fig. 6.1

The centre of mass is located at a point G along the rod.

(a) Determine the distance AG.

Two light inextensible strings, each of length 40 cm, are attached to the rod, one at A, the other at C. The other ends of these strings are attached to a fixed point D. The rod is allowed to hang in equilibrium.

(b) Determine the angle AD makes with the vertical.

The two strings are now replaced by a single light inextensible string of length 80 cm. One end of the string is attached to A and the other end of the string is attached to C. The string passes over a smooth peg fixed at D. The rod hangs in equilibrium, but is not vertical, as shown in **Fig. 6.2**.

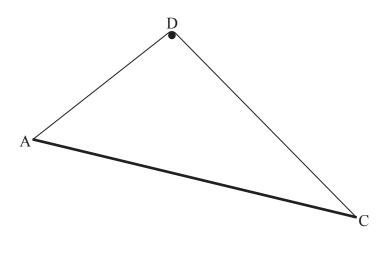


Fig. 6.2

(c)	Explain why angle ADG and angle CDG must be equal.	2]

(d) Determine the tension in the string.

[6]

[2]

[3]

### **END OF QUESTION PAPER**



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