Arithmetic series

\[ S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\} \]

Geometric series

\[ S_n = \frac{a(1 - r^n)}{1 - r} \]

\[ S_\infty = \frac{a}{1 - r} \quad \text{for} \quad |r| < 1 \]

Binomial series

\[ (a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}b^n \quad (n \in \mathbb{N}), \]

where \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \)

\[ (1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)(n-2)!}{r!}x^r + \ldots \quad (|x| < 1, \ n \in \mathbb{R}) \]

Differentiation

\[
\begin{array}{|c|c|}
\hline
f(x) & f'(x) \\
\hline
\tan kx & k \sec^2 kx \\
\sec x & \sec x \tan x \\
cot x & -\csc^2 x \\
cosec x & -\csc x \cot x \\
\hline
\end{array}
\]

Quotient Rule \( y = \frac{u}{v}, \ \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \)

Differentiation from first principles

\( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)

Integration

\[
\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + c
\]

\[
\int f'(x)\left( f(x) \right)^n \, dx = \frac{1}{n+1} \left( f(x) \right)^{n+1} + c
\]

Integration by parts \( \int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx \)

Small angle approximations

\( \sin \theta \approx \theta, \ \cos \theta \approx 1 - \frac{1}{2}\theta^2, \ \tan \theta \approx \theta \) where \( \theta \) is measured in radians
Trigonometric identities

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)
\]

Numerical methods

Trapezium rule: \[
\int_a^b y \, dx \approx \frac{1}{2}h \left\{ (y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1}) \right\}, \text{ where } h = \frac{b-a}{n}
\]
The Newton-Raphson iteration for solving \( f(x) = 0 \): \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \)

Probability

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]
\[
P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B) \quad \text{ or } \quad P(A \mid B) = \frac{P(A \cap B)}{P(B)}
\]

Sample variance

\[
s^2 = \frac{1}{n-1}S_{xx} \quad \text{where} \quad S_{xx} = \sum(x_i - \overline{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - nx^2
\]

Standard deviation, \( s = \sqrt{\text{variance}} \)

The binomial distribution

If \( X \sim B(n, p) \) then \( P(X = r) = \binom{n}{r} p^r q^{n-r} \) where \( q = 1 - p \)

Mean of \( X \) is \( np \)

Hypothesis testing for the mean of a Normal distribution

If \( X \sim N(\mu, \sigma^2) \) then \( \overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \) and \( \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \)

Percentage points of the Normal distribution

<table>
<thead>
<tr>
<th>( p )</th>
<th>10</th>
<th>5</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z )</td>
<td>1.645</td>
<td>1.960</td>
<td>2.326</td>
<td>2.576</td>
</tr>
</tbody>
</table>

Kinematics

Motion in a straight line

\[
v = u + at
\]
\[
s = ut + \frac{1}{2}at^2
\]
\[
s = \frac{1}{2}(u + v)t
\]
\[
v^2 = u^2 + 2as
\]
\[
s = vt - \frac{1}{2}at^2
\]

Motion in two dimensions

\[
v = u + at
\]
\[
s = ut + \frac{1}{2}at^2
\]
\[
s = \frac{1}{2}(u + v)t
\]
\[
v^2 = u^2 + 2as
\]
\[
s = vt - \frac{1}{2}at^2
\]
Answer **all** the questions.

**Section A (60 marks)**

1. A curve for which \( y \) is inversely proportional to \( x \) is shown below.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Find the equation of the curve. [2]

2. The function \( f(x) = \sqrt{x} \) is defined on the domain \( x \geq 0 \).

The function \( g(x) = 25 - x^2 \) is defined on the domain \( \mathbb{R} \).

(a) Write down an expression for \( f(x) \). [1]

(b) (i) Find the domain of \( f(x) \). [3]

(ii) Find the range of \( f(x) \). [2]

3. An infinite sequence \( a_1, a_2, a_3, \ldots \) is defined by \( a_n = \frac{n}{n+1} \), for all positive integers \( n \).

(a) Find the limit of the sequence. [1]

(b) Prove that this is an increasing sequence. [3]
4 In this question you must show detailed reasoning.

Determine the exact solutions of the equation $2 \cos^2 x = 3 \sin x$ for $0 \leq x \leq 2\pi$. [5]

5 A curve is defined implicitly by the equation $2x^2 + 3xy + y^2 + 2 = 0$.

(a) Show that $\frac{dy}{dx} = -\frac{4x + 3y}{3x + 2y}$. [3]

(b) In this question you must show detailed reasoning.

Find the coordinates of the stationary points of the curve. [4]

6 A hot drink is cooling. The temperature of the drink at time $t$ minutes is $T \degree C$.

The rate of decrease in temperature of the drink is proportional to $(T - 20)$.

(a) Write down a differential equation to describe the temperature of the drink as a function of time. [2]

(b) When $t = 0$, the temperature of the drink is 90 $\degree C$ and the temperature is decreasing at a rate of 4.9 $\degree C$ per minute.

Determine how long it takes for the drink to cool from 90 $\degree C$ to 40 $\degree C$. [6]
A student is trying to find the binomial expansion of $\sqrt{1-x^3}$.

She gets the first three terms as $1 - \frac{x^3}{2} + \frac{x^6}{8}$.

She draws the graphs of the curves $y = \sqrt{1-x^3}$, $y = 1 - \frac{x^3}{2}$ and $y = 1 - \frac{x^3}{2} + \frac{x^6}{8}$ using software.

(a) Explain why $1 - \frac{x^3}{2} + \frac{x^6}{8} \geq 1 - \frac{x^3}{2}$ for all values of $x$. [1]

(b) Explain why the graphs suggest that the student has made a mistake in the binomial expansion. [1]

(c) Find the first four terms in the binomial expansion of $\sqrt{1-x^3}$. [3]

(d) State the set of values of $x$ for which the binomial expansion in part (c) is valid. [1]

(e) Sketch the curve $y = 2.5 \sqrt{1-x^3}$ on the grid in the Printed Answer Booklet. [2]

(f) In this question you must show detailed reasoning.

The end of a bus shelter is modelled by the area between the curve $y = 2.5 \sqrt{1-x^3}$, the lines $x = -0.75$, $x = 0.75$ and the $x$-axis. Lengths are in metres.

Calculate, using your answer to part (c), an approximation for the area of the end of the bus shelter as given by this model. [4]
The curves \( y = h(x) \) and \( y = h^{-1}(x) \), where \( h(x) = x^3 - 8 \), are shown below.

The curve \( y = h(x) \) crosses the \( x \)-axis at \( B \) and the \( y \)-axis at \( A \).

The curve \( y = h^{-1}(x) \) crosses the \( x \)-axis at \( D \) and the \( y \)-axis at \( C \).

(a) Find an expression for \( h^{-1}(x) \). [2]

(b) Determine the coordinates of \( A \), \( B \), \( C \) and \( D \). [5]

(e) Determine the equation of the perpendicular bisector of \( AB \). Give your answer in the form \( y = mx + c \), where \( m \) and \( c \) are constants to be determined. [4]

(d) Points \( A \), \( B \), \( C \) and \( D \) lie on a circle.

Determine the equation of the circle. Give your answer in the form \( (x-a)^2 + (y-b)^2 = r^2 \), where \( a \), \( b \) and \( r^2 \) are constants to be determined. [5]
Answer all the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

9 Show that $y = x$ has the same gradient as $y = \sin x$ when $x = 0$, as stated in line 5. [2]

10 In this question you must show detailed reasoning.

Fig. C2.2 indicates that the curve $y = \frac{4x(\pi - x)}{\pi^2} - \sin x$ has a stationary point near $x = 3$.

- Verify that the $x$-coordinate of this stationary point is between 2.6 and 2.7. [5]

11 Show that, for the angle $45^\circ$, the formula $\sin \theta \approx \frac{4\theta(180 - \theta)}{40500 - \theta(180 - \theta)}$ given in line 28 gives the same approximation for the sine of the angle as the formula $\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ given in line 23. [3]

12 (a) Show that $\cos x = \sin \left(x + \frac{\pi}{2}\right)$. [2]

(b) Hence show that $\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ gives the approximation $\cos x \approx \frac{\pi^2 - 4x^2}{\pi^2 + x^2}$, as stated in line 31. [3]

END OF QUESTION PAPER