Oxford Cambridge and RSA

Thursday 19 May 2022 – Afternoon

Level 3 Certificate Core Maths A (MEI)

H868/01 Introduction to Quantitative Reasoning

Insert

Time allowed: 2 hours

INSTRUCTIONS
- Do **not** send this Insert for marking. Keep it in the centre or recycle it.

INFORMATION
- This Insert contains the pre-release material that you have already seen.
- This document has **8** pages.

A Leaves as thermometers

Plants are obviously influenced by the climate in which they grow. It is observed that warm climates contain a higher proportion of plant species with smooth-edged leaves as shown in **Fig. A.1**. Cooler climates have a higher proportion of plant species with toothed-edged leaves as in **Fig. A.2**.

Fig. A.1

Fig. A.2

The scatter diagram in **Fig. A.3** shows the percentages of species of plant with smooth-edged leaves and the mean annual temperatures for 11 regions in South America. Note the false origin. There is a strong positive correlation.

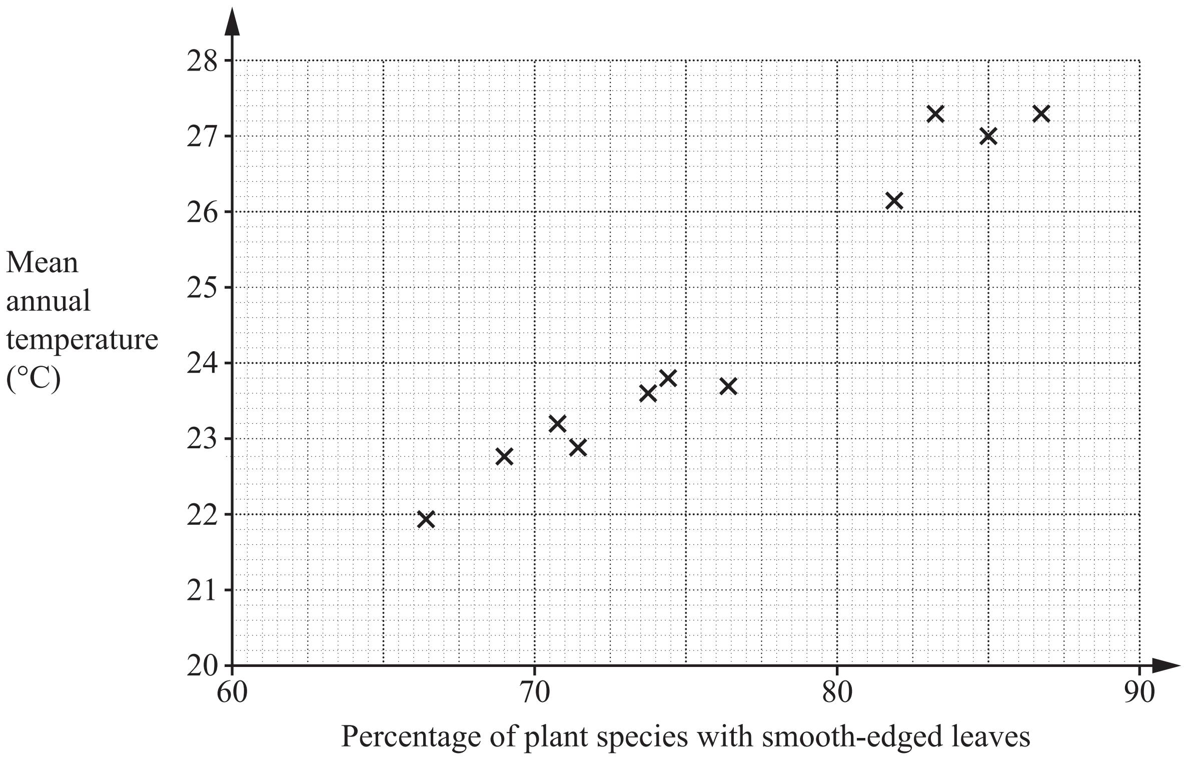


Fig. A.3

Providing there are sufficient leaf fossil specimens of a particular age it is possible to estimate the mean annual temperature at the time the leaves were living. **Fig. A.4** shows some fossilised leaves from about 56 million years ago.

Fig. A.4

B Centre pivot irrigation

Centre pivot irrigation is a method for irrigating crops. It consists of a supported water pipe that pivots and moves around a centre point. As the pipe rotates, water is pumped through nozzles. These nozzles are spaced along the pipe in such a way as to ensure that watering of the crops is even. The pipe typically takes between 12 and 36 hours to make a complete revolution.

Some photographs of centre pivot irrigation systems are shown below.
The centre pivot (the water pump) in **Fig. B.1** and the system in action in **Fig. B.2**.

Fig. B.1

Fig. B.2

Some watering pipes are a kilometre long, giving a circular irrigated region of one kilometre radius. These circles and others from shorter pipes are visible from an aeroplane as in **Fig. B.3** or even from space as illustrated in **Fig. B.4**.

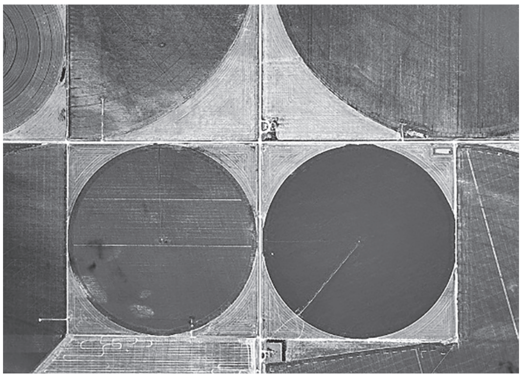


Fig. B.3

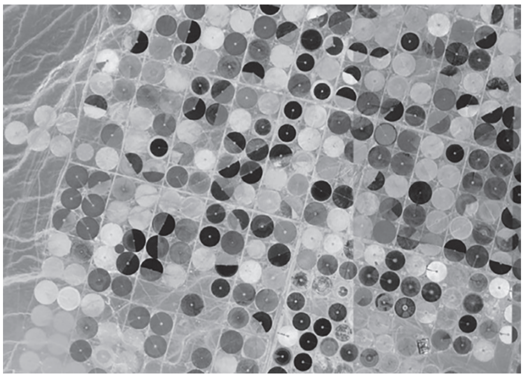


Fig. B.4

Where sufficient underground water is available, centre pivot irrigation can be used in desert regions; it is also used in the mid-west of the USA. However, there are concerns that these systems are using up too much underground water; it may not be replaced naturally for hundreds of years.

Fields tend to be square; therefore simple centre pivot irrigation cannot water their corners, as seen in **Fig. B.3**. However, in more sophisticated systems a rotating spraying system at the end of the pipe reduces the area that does not get watered.

C Land speed record

Speed can be measured using units such as mph, metres per second or $km\,h^{-1}$.
Acceleration (and deceleration) is measured in units such as miles/hour^2, $m\,s^{-2}$ or $km\,h^{-2}$.

The world land speed record is still given in miles per hour (mph). It is generally stated correct to 3 decimal places. At present (2020) it is 763.035 mph. It was set in 1997 by the car named Thrust SSC (supersonic car) shown in **Fig. C.1**.

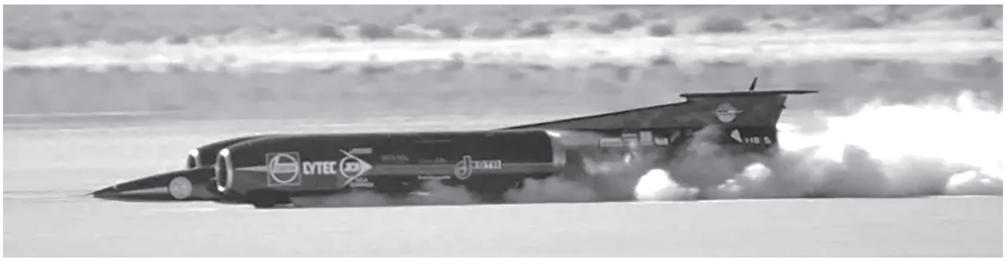


Fig. C.1

Until recently work was in progress to raise the record to over 1000 mph with the rocket assisted car Bloodhound SSC shown in **Fig. C.2**. So far it has "only" reached speeds just over 600 mph. At the time of writing the Bloodhound SSC is up for sale and so further attempts at 1000 mph are on hold.

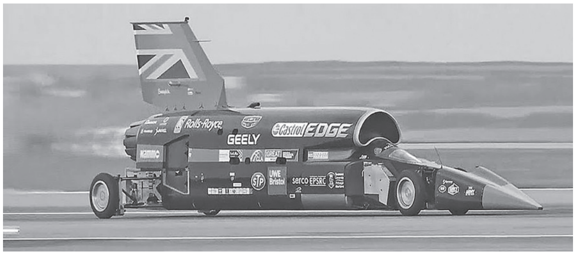


Fig. C.2

As world record speeds are given in miles per hour and times in seconds it is helpful to have an expression linking these two quantities.

If a car covers N miles in T seconds, its average speed can be written as

- $\frac{N}{T}$ miles **per second**

or

- $60 \times \frac{N}{T}$ miles **per minute**

or

- $60 \times 60 \times \frac{N}{T} = \frac{3600N}{T}$ miles **per hour**.

So, the average speed V in mph of a car taking T seconds to cover N miles is given by

$$V = \frac{3600N}{T}.$$

According to the FiA (Fédération Internationale de l'Automobile), which verifies car speed records, a car's speed is the average speed achieved covering a measured distance of 1 mile once in each direction or run – a total distance of 2 miles. It is a flying start, so the car may cross the start line at full speed.

For example, in 1935, the Bluebird took a total time of 23.91 seconds to cover both runs of the measured mile (a total distance of 2 miles). Using the formula on the previous page gives an FiA official speed of

$$V = \frac{3600 \times 2}{23.91} = 301.129\,\text{mph (3 dp)}.$$

D Near-Earth Objects

Impact craters are formed when objects such as asteroids or meteorites crash into the surfaces of planets or moons. Typical impact speeds are up to $70\,\text{km}\,\text{s}^{-1}$.

Impact craters like those shown below have been observed on most of the planets and their moons. There are impact craters on Earth but these are difficult to see because they are worn away by wind and rain.

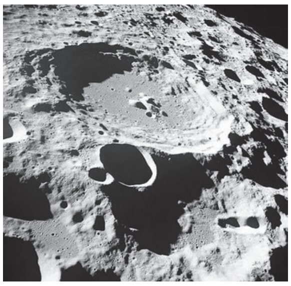


Fig. D.1 The Moon

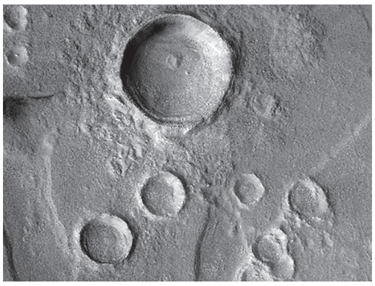


Fig. D.2 Mars

Fig. D.3 Mercury

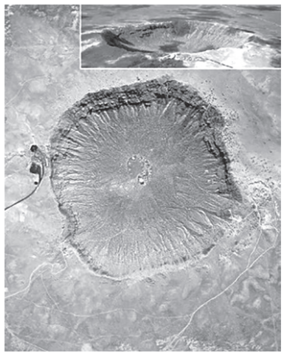


Fig. D.4 Arizona

Asteroids or meteorites that are likely to pass close to the Earth or even strike it are called Near-Earth Objects (NEOs). About 50 000 years ago an NEO hit the Earth in what is now Arizona. It made an impact crater about a mile across; this is shown in **Fig. D.4**.

Large NEOs that do impact the Earth release vast amounts of energy, together with clouds of debris that can block sunlight for many years. Over the 4 billion years of its history, Earth has been hit a number of times by NEOs that have changed the development of life on our planet.

The SI unit for energy is the joule (J). The impact energy of the NEO that caused the Arizona crater in **Fig. D.4** is estimated to have been 5×10^{16} J, an amount of energy almost impossible to imagine.

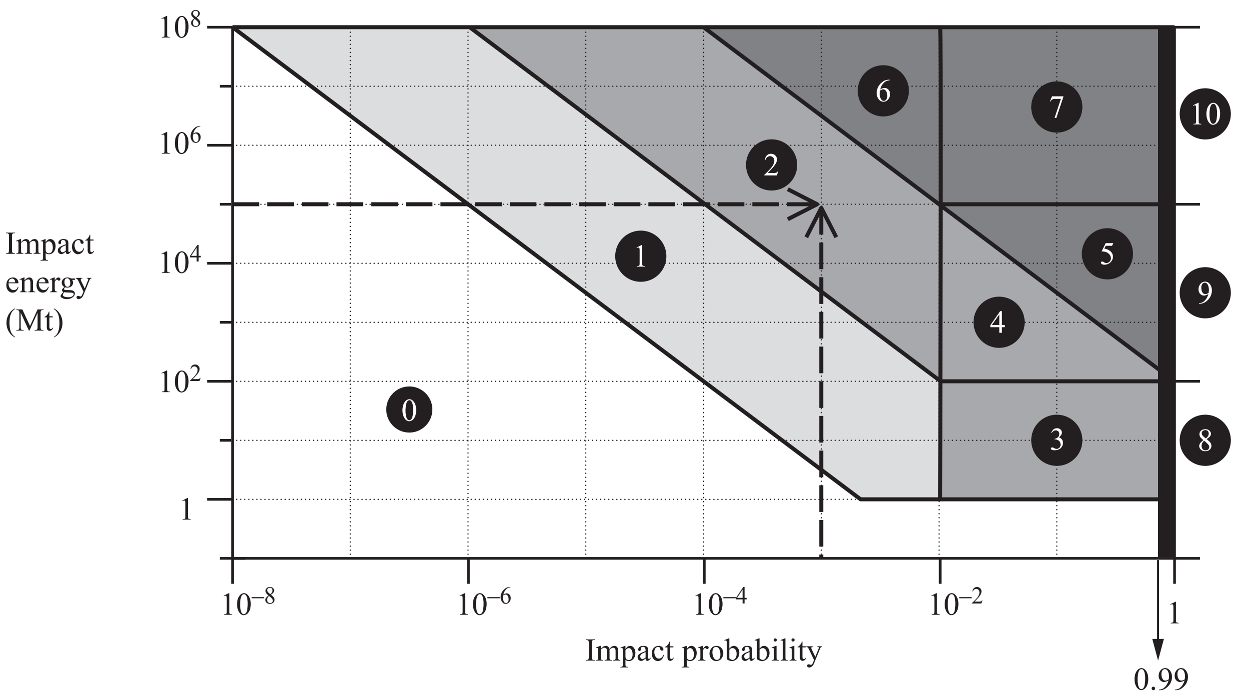
To make it easier to handle the very large numbers involved, the explosive energy of a gigantic explosion is often given as the number of millions of tons of TNT that would have the same effect. A million tons of TNT or megaton (Mt) is equivalent to 4.2×10^{15} J of energy. So, the Arizona crater involved an explosion of energy $(5 \times 10^{16}\,\text{J}) \div (4.2 \times 10^{15}\,\text{J})$ which is 12 Mt correct to 2 sf.

NEOs are tracked by NASA and ESA (European Space Agency) who model their impact energy if they should hit Earth and the probability of their doing so. Diameters are calculated from the objects' brightness using a model connecting the two. Uncertainties arising from the tracking model and from measurements are taken into account when the probability is estimated. This gives warning of any possible impact years before it might happen. Over 10 000 NEOs are on databases.

The seriousness of risk of NEO earth impact is measured on a scale 0 to 10 called the Torino scale. The value on the Torino scale for a particular NEO is found by plotting it as a point on **Fig. D.5**, treating it like a scatter diagram; the variables are Impact probability on the horizontal axis and Impact energy on the vertical. (A commonly used alternative label for the vertical axis is Kinetic energy.)

Using **Fig. D.5**, an object having impact probability of 10^{-3} and calculated impact energy of 10^5 Mt would be rated 2 (region ❷) on this scale, This is classed as no cause for public concern but needing further observation by astronomers.

The Torino scale (0 to 10)



Torino Scale	0	2	5	8	10
Effect	Not a hazard, no risk	No concern but needs more observations	Threatening, regional devastation	Total destruction local to impact	Global climate change, threat to civilisation

Fig. D.5