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Introduction

Our examiners’ reports are produced to offer constructive feedback on candidates’ performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates’ performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.
Paper 2 series overview

H640/02 is the second of three compulsory components in the A Level assessment. It contributes 36.4% of the total A Level and assesses content from pure mathematics and statistics.

Candidates are expected to have studied statistics using the large data set and to have routinely used spreadsheets, graphing and statistical software when studying this course.

To do well in this component, candidates need to be familiar with the command words detailed in the specification and to use their calculators efficiently in a variety of contexts, such as calculating binomial probabilities or using the inverse Normal function. When interpreting statistical diagrams and tables they need to relate their comments to the question which is asked, and to avoid irrelevant speculation. Their comments should be supported by numerical arguments where possible.

<table>
<thead>
<tr>
<th>Candidates who did well on this paper generally did the following:</th>
<th>Candidates who did less well on this paper generally did the following:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• set out their solutions clearly, showing full details of working in questions which required a longer response</td>
<td>• made elementary algebraic and arithmetic errors</td>
</tr>
<tr>
<td>• made efficient and appropriate use of their calculators when working with the binomial and Normal distribution</td>
<td>• described what they saw on diagrams rather than relating what they inferred to the question</td>
</tr>
<tr>
<td>• made inferences from statistical diagrams and information obtained by calculation and related their findings back to the original question</td>
<td>• did not express answers clearly and concisely when a sentence or two of explanation was required</td>
</tr>
<tr>
<td>• applied their knowledge of the pre-release material in a reasoned and directed way</td>
<td>• gave answers which suggested a very limited exposure to the pre-release material</td>
</tr>
<tr>
<td>• were able to interpret and use information presented in the software printout.</td>
<td>• resorted to using their calculator when it was clear some written reasoning was required (for example in Question 16 (a)).</td>
</tr>
</tbody>
</table>

Assessment for learning

The majority of candidates are competent calculators of statistical measures from raw data, but a significant minority were not able to calculate the variance given summary statistics.

Similarly, many candidates did not use the information given in the printout in Question 12 to conduct the hypothesis test. Instead, many recalculated the p-value, the z-score or the critical region – and often went astray in the process. They also wasted valuable time.

Many candidates were not able to identify the correct Normal distribution to test the mean, often confusing the sample mean with the hypothesised population mean.

All candidates appeared to be familiar with cumulative frequency diagrams and scatter diagrams, but a significant minority were unable to extract relevant information to answer the questions.

It seemed that a significant minority of candidates were not familiar with the pre-release material.

Consequently, it is suggested that in some cases candidates would benefit from teaching which emphasises process and interpretation and inference in equal measure, rather than teaching which may focus on the traditional approach of calculation and drawing diagrams.
Section A overview

Section A proved accessible to most candidates, with some candidates earning close to full marks. Even those who struggled with the paper as a whole could expect to earn half marks in this section.

Question 1

1 Express \( \cos \theta + \sqrt{3} \sin \theta \) in the form \( R \cos(\theta - \alpha) \), where \( R \) and \( \alpha \) are exact values to be determined. \([4]\)

Candidates who did well on this usually found \( R \) by using Pythagoras and \( \theta \) from \( \tan \theta = \sqrt{3} \), although some successfully compared \( \cos \theta + \sqrt{3} \sin \theta \) with \( R \cos \theta \cos \alpha + R \sin \theta \sin \alpha \), generally finding \( \theta \) first.

Candidates who did less well neglected to express the final answer in the requested form, made a slip in finding \( R \) (4 was sometimes seen) or decimalised their answer for \( \theta \).

Candidates who did not do well started incorrectly with \( \tan \theta = \frac{1}{\sqrt{3}} \) or were unable to make a correct initial step.

Question 3 (a)

3 (a) On the axes in the Printed Answer Booklet, sketch the curve with equation \( y = 3 \times 0.4^x \). \([3]\)

Candidates who did well in this question drew a smooth decreasing curve for the majority of the domain given on the axes, clearly marking the correct \( y \)-intercept.

Candidates who did less well identified most of the key features, but either drew a curve which was too steep in the second quadrant or not clearly asymptotic to the \( x \)-axis in the first quadrant and/or did not cut the \( y \)-axis at \((0, 3)\).

Candidates who did not do well in this question drew an increasing curve.
Question 3 (b)

(b) Given that $3 \times 0.4^x = 0.8$, determine the value of $x$ correct to 3 significant figures.

The overwhelming majority of candidates obtained full marks in this question. Candidates who did less well gave their final answer to a different precision to the 3 s.f. requested, or made a slip when calculating, for example, $\frac{\ln 0.8}{\ln 0.4}$.

Candidates who did not do well made an initial incorrect step such as $3 \times 0.4^x = 1.2^x$ or $\ln 3 \times \ln 0.4^x = \ln 0.8$.

Some resorted to the solver function on their calculator to find $x$. The use of the command word “determine” is a steer to candidates that the latter approach could only gain very limited credit at best.

Question 4 (a)

4 A survey of university students revealed that
- 31% have a part-time job but do not play competitive sport.
- 23% play competitive sport but do not have a part-time job.
- 22% do not play competitive sport and do not have a part-time job.

(a) Show this information on a Venn diagram.

The overwhelming majority of candidates obtained full marks in this question. The very few candidates who did not do well misplaced 0.22 – usually in the intersection of the two sets.

Question 4 (b)

A student is selected at random.

(b) Determine the probability that the student plays competitive sport and has a part-time job.

Most candidates earned both marks on this question. Candidates who did less well either made a slip with the arithmetic or lost the accuracy mark by leaving the answer as a percentage.
**Question 5**

5 Tom conjectures that if \( n \) is an **odd** number greater than 1, then \( 2^n - 1 \) is prime.

Find a counter example to disprove Tom’s conjecture. [3]

Candidates who did well in this question adopted a systematic approach and usually identified \( 2^9 - 1 \) as a counter-example, stating that is divisible by 7. Some candidates scored full marks by identifying a larger value of \( n \), together with the correct factor – presumably following some experimentation on their calculator which was not shown.

Candidates who did less well evaluated the formula for one or more odd values of \( n \), but did not identify a counter example.

Candidates who did not do well substituted an even value of \( n \) or \( n = 1 \), and thought they had answered the question successfully. A few candidates opted solely for very large values of \( n \) and hoped that the number expressed in standard form would suffice.

**Question 6**

6 \( X \) is a continuous random variable such that \( X \sim N(\mu, \sigma^2) \).

On the sketch of this Normal distribution in the Printed Answer Booklet, shade the area bounded by the curve, the \( x \)-axis and the lines \( x = \mu \pm \sigma \). [2]

Candidates who did well in this question identified a symmetrical region with boundaries approximately at the asymptotes on the curve.

Candidates who did less well identified a symmetrical region but did not realise that the inflections on the curve are one standard deviation away from the mean.

Candidates who did not do well did not identify a region under the curve which was symmetrical about the turning point.
Section B overview

This section proved accessible to the majority of candidates, with many earning full marks on two or more complete questions. However, some did not supply adequate supporting arguments when answering the statistics questions.

Question 8 (a)

8 Ali conducted an investigation into the distances ridden by those members of a cycling club who rode at least 120 km in a training week. She grouped all the distances into intervals of length 10 km and then constructed a cumulative frequency diagram, which is shown below.

![Cumulative Frequency Diagram]

(a) Explain whether the data Ali used is a sample or a population. [1]

Candidates who did well in this question related their answer to the information given in the question, usually referring to all the distances over 120 km.

Candidates who did not do well identified the data as a sample or stated that it's a population but restricted their reasoning to a general comment such as “because it’s all the members of the cycling club”.

Question 8 (b)

The club is taking part in a competition. Eight team members and one reserve are to be selected. The club captain decides that the team members should be those cyclists who rode the furthest during the training week, and that the reserve should be the cyclist who rode the next furthest.

(b) Use the graph to estimate the shortest distance cycled by a team member. [1]

Candidates who did well in this question identified a distance of between 161 and 163 km from the graph.

Candidates who did not do well often gave a value between 120 and 123 km, which arose from not understanding that they were being asked to estimate the shortest distance ridden by a team member, rather than the shortest distance over 120 km ridden by a member of the club.

Question 8 (c)

The captain’s best friend rode 156 km in the training week and was selected as reserve. Ali complained that this was unjustifiable.

(c) Explain whether there is sufficient evidence in the diagram to support Ali’s complaint. [1]

Candidates who did well in this question stated that there was evidence to support Ali’s complaint and then gave an appropriate argument to support this. Usually this was either based on the reserve should have ridden the ninth furthest, but the captain’s friend rode the fourteenth furthest, or that ten riders had ridden more than 160 km, but the friend had only ridden 156 km.

Candidates who did not do well sometimes gave a correct supporting argument but said this didn’t support Ali’s complaint, or they thought that the total number of riders was 60 and so went awry with the numerical argument. Some misunderstood completely and referenced the median or the interquartile range.
Question 9 (a)

At the beginning of the academic year, all the pupils in year 12 at a college take part in an assessment. Summary statistics for the marks obtained by the 2021 cohort are given below.

\[ n = 205 \quad \Sigma x = 23042 \quad \Sigma x^2 = 2591716 \]

Marks may only be whole numbers, but the Head of Mathematics believes that the distribution of marks may be modelled by a Normal distribution.

(a) Calculate

- The mean mark
- The variance of the marks

[2]

Candidates who did well in this question used their calculators efficiently to find both requested values.

Candidates who did less well found the mean successfully, but either mistook the mean squared deviation for the variance, or thought the variance was \( \sum x^2 - nx^2 \).

Question 9 (b)

(b) Use your answers to part (a) to write down a possible Normal model for the distribution of marks.

[2]

Candidates who did well in this question wrote down the Normal distribution with the correct values obtained in part (a).

A follow through M1 was available for those who had the wrong value for the variance, which was often earned by those who did less well. Some dropped a mark by dividing 8.8 by 205.

Candidates who did not do well generally didn’t make the connection between the values they had calculated in part (a) and the parameters of the Normal model they were being asked to identify.

Question 9 (c)

One candidate in the cohort scored less than 105.

(c) Determine whether the model found in part (b) is consistent with this information.

[3]

Candidates who did well in this question used their Normal model to calculate \( P(X < 104.5) \), multiplied their answer by 205 and compared their answer with 1. Some compared their probability with 1/205 and a small minority worked with the Inverse Normal function successfully.

Candidates who did less well did not spot the need for a continuity correction and lost the accuracy mark. Candidates who did not do well often applied a two standard deviation check to see if 105 was an outlier.
Question 9 (d)

(d) Use the model to calculate an estimate of the number of candidates who scored 115 marks. [2]

Candidates who did well in this question found \( P(114.5 < X < 115.5) \) and multiplied by 205.

Candidates who did less well earned the method mark having worked with the wrong parameters.

Candidates who did not do well worked with the Normalpdf function on their calculator, occasionally obtaining a correct answer fortuitously. Some misunderstood what was required and worked with \( P(X > 115) \) or \( P(X < 115) \).

Question 10 (a)

10 The parametric equations of a curve are

\[ x = 2 + 5\cos\theta \text{ and } y = 1 + 5\sin\theta, \text{ where } 0 \leq \theta \leq 2\pi. \]

(a) Determine the cartesian equation of the curve. [3]

Candidates who did well in this question usually expressed \( \cos\theta \) and \( \sin\theta \) in terms of \( x \) and \( y \) respectively, before squaring both sides and using Pythagoras to eliminate \( \theta \). Candidates who spotted the radius and centre from the parametric equations usually obtained the cartesian equation very efficiently.

Candidates who did less well made a slip such as neglecting to square 5 or made errors such as \((x - 2)^2 = x^2 - 2^2\). Candidates who squared both sides first sometimes omitted the terms in \( \cos\theta \) and \( \sin\theta \), or were unable to eliminate these terms following a correct expansion.

Question 10 (b)

(b) Hence or otherwise, find the equation of the tangent to the curve at the point (5, -3), giving your answer in the form \( ax + by + c = 0 \), where \( a \), \( b \) and \( c \) are integers to be determined. [4]

Candidates who did well in this question recognised the equation of a circle in part (a), and that the tangent is perpendicular to the radius, so found the gradient of the tangent quickly and efficiently. They then went on to obtain the equation with ease.

Candidates who did less well worked with parametric or implicit differentiation, often successfully, to find \( \frac{dy}{dx} \). However, the substitution often went astray, with \(-\frac{3}{4}\) being a common wrong answer for the gradient. In many cases only the final accuracy mark was lost. A significant minority of candidates were able to score 2 or 3 marks out of 4 in this part, even following 0 marks in part (a).
Question 11 (b)

(b) Find $p$ in terms of $k$.  

Candidates who did well in this question equated the sum of the probabilities to 1 and used this result to make $k$ the subject.

Candidates who did less well made $p$ the subject of the formula.

Question 11 (c)

(c) Determine, in terms of $k$, the expected number of times Nina rolls a 12.

Candidates who did well in this question multiplied their answer to part (b) by 120 and by $k$ to obtain the correct expression.

Candidates who did less well multiplied by 120 or $k$, but not both.

Candidates who did not do well left their answer in terms of $p$ rather than $k$.

Question 11 (d)

(d) Given that Nina rolls a 12 on 32 occasions, calculate an estimate of the value of $k$.

Candidates who did well in this question generally set 32 equal to their correct answer to part (c) to find $k$. However, some candidates who were unable to complete the previous part spotted that $11p = \frac{120 - 32}{120}$ and hence found $k$ successfully.

Candidates who did less well followed through an incorrect expression from part (c).

Candidates who did not do well generally made no meaningful progress with this part.
Question 11 (e)

Nina rolls the die a further 30 times.

(e) Use your answer to part (d) to calculate an estimate for the probability that she obtains a 12 exactly 8 times in these 30 rolls.  

Candidates who did well in this part used the binomial distribution n successfully with $\frac{4}{15}$ to obtain the correct result.

Candidates who did less well followed through with an incorrect value of $k$.

Candidates who did not do well did not recognise that use of the binomial distribution was required here.
Question 12 (a), (b) and (c)

A retailer sells bags of flour which are advertised as containing 1.5 kg of flour. A trading standards officer is investigating whether there is enough flour in each bag. He collects a random sample and uses software to carry out a hypothesis test at the 5% level. The analysis is shown in the software printout below.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Test of a Mean</td>
<td>[ \text{Null Hypothesis } \mu = 1.5 ]</td>
</tr>
<tr>
<td>Alternative Hypothesis</td>
<td>( \leq ) ( &gt; ) ( \neq )</td>
</tr>
</tbody>
</table>

Sample
- Mean: 1.44
- \( \sigma \): 0.24
- \( N \): 32

Z Test of a Mean
- Mean: 1.44
- \( \sigma \): 0.24
- Result:
  - SE: 0.0424
  - \( N \): 32
  - \( Z \): -1.4142
  - \( P \): 0.0786

(a) State the hypotheses the officer uses in the test, defining any parameters used. [2]

(b) State the distribution used in the analysis. [3]

(c) Carry out the hypothesis test, giving your conclusion in context. [3]

Candidates who did well in this question identified the correct hypotheses in part (a), but may have omitted “population” in their definition of \( \mu \). In part (b) they identified the correct distribution but may have lost an accuracy mark by identifying the variable as \( X \) rather than \( \bar{X} \) or by quoting the variance as \( \frac{0.24}{\sqrt{32}} \). In part (c) they generally used the information to conduct the hypothesis test: usually they worked with the \( p \)-value, but some compared -1.4142 with -1.645 before noting that there is insufficient evidence to reject the null hypothesis. They may have lost the final mark due to either an over-assertive statement or a statement which omitted one or more of the elements in bold in the mark scheme. Some candidates who did not use the information in the printout, but instead recalculated the \( p \)-value or the \( z \)-score, nevertheless did so correctly and achieved all 3 marks in part (c).

Candidates who did less well in this question were not able to define \( \mu \) correctly in part (a). In part (b) one or both parameters were incorrect, and in part (c) they may have made a slip in recalculating values for the hypothesis test and missed the final accuracy mark as a result.

Candidates who did not do well may have earned the first mark in part (a), but in part (b) just written “Normal distribution”. In part (c) they recalculated the \( p \)-value or \( z \)-score – often from an incorrect distribution – resulting in the loss of all the marks in part (c).
Exemplar 1

12(a) $H_0: \mu = 1.5$
$H_1: \mu < 1.5$

Where $\mu$ is the mean weight of a bag of flour.

12(b) $X \sim N(1.44, \frac{0.29^2}{8})$

12(c) $0.0786 > 0.05$

So as there is insufficient evidence to accept $H_1$, so we accept $H_0$.

The evidence suggests that the mean weight of a bag of flour is 1.5.

In part (a) the correct hypotheses are stated, but the definition of $\mu$ is incomplete – it needs to be identified as the population mean weight of a bag of flour.

In part (b) the response contains a common misunderstanding: the sample mean, 1.44, is quoted as a parameter of the distribution, instead of 1.5, the hypothesised population mean. Note that even if this had been correct, the final A1 would have been withheld because the variable was quoted as $X$, not $\bar{X}$.

In spite of the misunderstanding in part (b), this candidate was able to recover by adopting the expected approach and using the information given in the printout. Note that the final mark is lost because the final conclusion is too assertive. It is a common misconception that a failure to reject $H_0$ means $H_0$ must be true, as opposed to there simply being not enough evidence to suggest the alternative hypothesis may be true.
Question 13 (a) and (b)

13 Records from the 1950s showed that 35% of human babies were born without wisdom teeth. It is believed that as part of the evolutionary process more babies are now born without wisdom teeth. In a random sample of 140 babies, collected in 2020, a researcher found that 61 were born without wisdom teeth.

The researcher made the following statement.

“This shows that the percentage of babies born without wisdom teeth has increased from 35%.”

(a) Explain whether this statement can be fully justified. [1]

(b) Conduct a hypothesis test at the 5% level to determine whether there is any evidence to suggest that more than 35% of babies are now born without wisdom teeth. [7]

Candidates who did well in this question identified the need for a census rather than a sample in part (a), or they commented that different samples may give different results, or that a sample may not be representative. In part b they identified the hypotheses correctly, calculated $P(X \geq 61)$ correctly and compared their answer with 0.05, thus rejecting the null hypothesis. They may have lost the second B mark by not defining $p$ correctly (or at all), or perhaps the final A mark, either by giving a final statement which was too assertive, or possibly too vague – e.g., “more babies are born without wisdom teeth.” A few candidates produced fully correct work apart from defining $p$ in terms of babies with wisdom teeth and making the same mistake in the final statement. A very small number of candidates scored full marks from using a Normal approximation (with continuity corrections).

Candidates who did less well commented that the sample was too small in part (a). In part (b) they may not have defined $p$ correctly and went on to find $P(X \geq 60)$ or $P(X \geq 62)$, thereby losing at least 2 A marks.

Candidates who did not do well may have set up the hypotheses correctly, but went on to calculate $P(X= 61)$. Some attempted to work with the Normal distribution but were unable to identify the correct parameters.
Question 14 (a)

14 **Fig. 14.1** shows the curve with equation \( y = \frac{1}{1 + x^2} \), together with 5 rectangles of equal width.

![Graph showing the curve and rectangles](image)

**Fig. 14.1**

**Fig. 14.2** shows the coordinates of the points A, B, C, D, E and F.

<table>
<thead>
<tr>
<th>Point</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>( y )</td>
<td>1</td>
<td>0.96154</td>
<td>0.86207</td>
<td>0.73529</td>
<td>0.60976</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**Fig. 14.2**

(a) Use the 5 rectangles shown in **Fig. 14.1** and the information in **Fig. 14.2** to show that a lower bound for \( \int_{0}^{1} \frac{1}{1 + x^2} \, dx \) is 0.7337, correct to 4 decimal places. \[2\]

Candidates who did well in this question showed the appropriate calculation using the values provided and displayed the calculated value to 5 or 6 decimal places before stating the given answer.

Candidates who did less well showed the calculation but didn’t give the value to a higher precision before stating the given value.

Candidates who did not do well used the trapezium rule formula.
Question 14 (b)

(b) Use the 5 rectangles shown in Fig. 14.1 and the information in Fig. 14.2 to calculate an upper bound for \( \int_0^1 \frac{1}{1 + x^2} \, dx \) correct to 4 decimal places. \([2]\)

Candidates who did well in this question showed the appropriate calculation using the values provided or used their answer to part (a) and added 0.2\times0.5 to obtain the correct answer.

Candidates who did not do well used the trapezium rule.

Question 14 (c)

(c) Hence find the length of the interval in which your answers to parts (a) and (b) indicate the value of \( \int_0^1 \frac{1}{1 + x^2} \, dx \) lies. \([1]\)

Candidates who did not do well in this question simply quoted the interval as 0.7337 to 0.8337.

Question 14 (d)

Amit uses \( n \) rectangles, each of width \( \frac{1}{n} \), to calculate upper and lower bounds for \( \int_0^1 \frac{1}{1 + x^2} \, dx \), using different values of \( n \). His results are shown in Fig. 14.3.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>upper bound</td>
<td>0.80998</td>
<td>0.79779</td>
<td>0.79162</td>
</tr>
<tr>
<td>lower bound</td>
<td>0.75998</td>
<td>0.77279</td>
<td>0.77912</td>
</tr>
</tbody>
</table>

Fig. 14.3

(d) Find the length of the smallest interval in which Amit now knows \( \int_0^1 \frac{1}{1 + x^2} \, dx \) lies. \([2]\)

Candidates who did well in this question evaluated 0.79162 – 0.77912

Candidates who did less well made a slip in the arithmetic or gave the interval as 0.77912 to 0.79162.

A few spoiled their answer by finding the midpoint of the interval.
Question 15 (b)

According to the model, the life expectancy at birth in Liberia in 2025 is estimated to be 65.83 years.

(b) Explain whether each of these two estimates is likely to be reliable. [2]

Candidates who did well in this question identified that the first estimate comes from interpolation and the second one from extrapolation and commented on reliability accordingly.

Candidates who did less well may have commented that the first estimate is reliable because it’s interpolation, and then lost the second mark by commenting that the second estimate is less reliable (or less likely to be reliable) because it’s extrapolation. Alternatively, they may have earned the second mark but lost the first mark due to an equivalent comparative comment.

Candidates who did not do well in this question presented arguments based on other factors.

Question 15 (c)

(c) Use your knowledge of the pre-release material to explain whether this model could be used to obtain a reliable estimate of the life expectancy at birth in other countries in 1995. [1]

Candidates who did well in this question identified that life expectancy at birth varies across different countries.

Candidates who did not do well may not have mentioned life expectancy at birth in their answer, instead commenting on other variables. Alternatively, they based their argument on the diagram given in the question, rather than their knowledge of the pre-release material.
Question 15 (d)

Fig. 15.2 shows the life expectancy at birth between 1960 and 2010 for Italy and South Africa.

![Graph showing life expectancy at birth for Italy and South Africa]

**Fig. 15.2**

(d) Use your knowledge of the pre-release material to

- Explain whether series 1 or series 2 represents the data for Italy.
- Explain how the data for South Africa differs from the data for most developed countries.

Candidates who did well in this question identified the series for Italy correctly and commented that life expectancy at birth is generally higher in Europe. They noted that life expectancy at birth started to decrease in South Africa from around 1990, which is unlike most developed countries.

Candidates who did less well may have identified series 1 as Italy, or simply said series 2 is Italy because it’s a developed country. They earned the second mark for a correct comment about the data for South Africa.

Candidates who did not do well may have thought that South Africa referred to the continent rather than the country, or they limited their comments to variables other than life expectancy at birth.
Question 15 (f)

Sundip states that as GDP per capita increases, life expectancy at birth increases.

(f) Explain to what extent the information in Fig. 15.3 supports Sundip’s statement. [2]

Candidates who did well in this question identified positive association between GDP per capita and life expectancy at birth for a suitable range of values of GDP per capita and commented that this supported Sundip’s statement. They went on to note that the association did not hold for the remainder of the values of values of GDP per capita and commented that this did not support Sundip’s statement.

Candidates who did less well identified only one of the above criteria.

Candidates who did not do well may have identified the key features in the diagram and described them well but did not relate their observations to Sundip’s statement. Many candidates were too vague in their response, commenting that Sundip’s statement was supported to some extent because there was some positive association. A significant minority of candidates went off at a tangent and wrote a number of sentences which contained nothing relevant.
Question 16 (a)

16 The equation of a curve is

\[ y = 6x^4 + 8x^3 - 21x^2 + 12x - 6. \]

(a) In this question you must show detailed reasoning.

Determine

- The coordinates of the stationary points on the curve.
- The nature of the stationary points on the curve.
- The x-coordinate of the non-stationary point of inflection on the curve. [12]

Candidates who did well in this question solved \( \frac{dy}{dx} = 0 \) successfully using the Factor theorem and used the second derivative to identify a local minimum at \( x = -2 \). They may have incorrectly concluded that because the second derivative is zero at \( x = \frac{1}{2} \) this must be a point of inflection, but many went on to test the gradient either side and reached the conclusion correctly. Some neglected to find the \( y \)-values at the stationary points, and some slipped up in the calculation of the \( y \)-values or the values of \( \frac{dy}{dx} \) when determining the nature of the turning point at \( x = \frac{1}{2} \). They then went on to correctly identify the x-value at the non-stationary inflection.

Candidates who did less well found the required x-values correctly but did not understand that the second derivative test can be indecisive. They didn’t find the \( y \)-values at the stationary points correctly (or at all) and were not able to use the second derivative to determine the nature of the stationary point at \( x = -2 \).

Candidates who did not do well may have found \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) correctly but made no further progress. A few candidates solved \( y = 0 \) or simply reached for their calculators and didn’t score.

Question 16 (b)

(b) On the axes in the Printed Answer Booklet, sketch the curve whose equation is

\[ y = 6x^4 + 8x^3 - 21x^2 + 12x - 6. \] [3]

Candidates who did well in this question sketched a curve of the correct shape in all four quadrants. They may have lost an accuracy mark if one of the x-intercepts was out of tolerance or neglected to identify the y-intercept.

Candidates who did less well sketched a curve of the correct shape, but did not identify the y-intercept, and one or both of the x-intercepts were out of tolerance.

Candidates who did not do well often had an extra stationary inflection in the third quadrant, or a non-stationary inflection in the fourth quadrant.
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