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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y434/01 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper Y434/01 series overview

This optional paper counts for 16⅔ % of the qualification OCR A Level Further Mathematics B (MEI) (H645). There is one examination paper which lasts 1 hour 15 minutes. Candidates are expected to know the content of A Level Mathematics and the Core pure mandatory paper (Y420).

Candidates should know how to use the iterative capability of a calculator in the examination. No credit is given for writing down solutions generated by equation solvers.

It is expected that candidates will have routinely used a spreadsheet throughout the course. In the examination candidates will be given output from spreadsheets and may be asked: what certain cells represent; to explain or give formulae for certain cells; to give solutions and justify their accuracy; to comment on errors, convergence or order.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• made efficient use of the iterative capability of their calculator when implementing algorithms • demonstrated an understanding of spreadsheet output and articulated this clearly and concisely • articulated their understanding of how different algorithms succeed or fail in the given context • used error analysis effectively to generate an improved solution, and interpreted their answers correctly • explained how computers and calculators may work with values that are stored to a different precision than they are displayed.	• were only partly successful in using the iterative capability of their calculator • demonstrated only limited understanding of spreadsheet output • did not seem to have a clear understanding of how different methods may succeed or fail, or were unable to articulate their understanding clearly • did not recognise situations in which extrapolation was appropriate • did not appreciate that stored values and displayed values in a spreadsheet and/or calculator are not usually the same.

Assessment for learning

Candidates should note that the values stored in spreadsheets are stored to a greater precision than the values displayed. But the stored values are not (usually) exact – they are simply rounded to a greater precision.

Misconception

Many candidates thought that the criterion for convergence of a fixed point iteration is whether $|g'(x_0)| < 1$, as opposed to whether $|g'(\alpha)| < 1$, where α is the root.

Question 1 (a), (b) and (c)

1 $C = 3.7622$ and $S = 3.6269$ are used to approximate $\cosh 2$ and $\sinh 2$ respectively.

(a) Determine whether these approximations are the result of chopping or rounding the values of $\cosh 2$ and $\sinh 2$. **[1]**

(b) Calculate the relative error when $C^2 - S^2$ is used to approximate $\cosh^2 2 - \sinh^2 2$, giving your answer correct to **3** significant figures. **[2]**

(c) **Without** doing any further calculations, explain whether the same value for the relative error is obtained when $(C-S)^2$ is used to approximate $(\cosh 2 - \sinh 2)^2$. **[1]**

Candidates who did well in this question showed the values of cosh2 and sinh2 in part (a) to 5 decimal places or more, and reached the correct conclusion. In part (b) they showed the correct calculation for relative error and nearly always gave the answer to the requested precision. In part (c) a few candidates realised that although the arithmetical operations are the same, i.e. squaring and subtracting, they are in reverse order, which results in a different answer for the relative error.

Candidates who did less well usually obtained the answer to part (a) correctly, but lost the accuracy mark in part (b) either by making a slip, or more usually by giving the answer to a different precision. In part (c) they often stated that the relative errors would be the same, or explained why the two expressions are different rather than why the two relative errors are different.

Question 2 (a), (b) and (c)

2 The table shows some values of x and the associated values of $y = \mathrm{f}(x)$.

x	2.75	3	3.25
$\mathrm{f}(x)$	0.920799	1	1.072858

(a) Calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$ using the forward difference method, giving your answer correct to **5** decimal places. **[2]**

(b) Calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$ using the central difference method, giving your answer correct to **5** decimal places. **[2]**

(c) Explain why your answer to part **(b)** is likely to be closer than your answer to part **(a)** to the true value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$. **[1]**

Candidates who did well in this question answered parts (a) and (b) correctly, having shown all necessary working. In part (c) their explanation usually centred on the order of the two methods, although a few used a geometrical argument successfully.

Candidates who did less well gave one or both of their answers to parts (a) and (b) to a different precision to 5 decimal places, or made a slip in the calculation. In part (c) they only referred to the central difference method, and made no comparison with the forward difference method.

Question 2 (d)

When $x = 5$ it is given that $y = 1.4645$ and $\frac{dy}{dx} = 0.1820$, correct to **4** decimal places.

(d) Determine an estimate of the error when $f(5)$ is used to estimate $f(5.024)$. **[2]**

Candidates who did well in this question generally went straight to 0.024×0.182 and obtained the correct answer. Some candidates found the approximation to f(5.024) and subtracted 1.4645.

Candidates who did less well earned M1 by showing 0.024×0.182 embedded in the calculation to approximate f(5.024), often stating the approximation to f(5.024) as the final answer.

Question 3 (a), (b), (c) and (d)

3 The equation $f(x) = \sin^{-1}(x) - x + 0.1 = 0$ has a root α such that $-1 < \alpha < 0$.

Alex uses an iterative method to find a sequence of approximations to α. Some of the associated spreadsheet output is shown in the table.

	C	D	E
4	r	x_r	$f(x_r)$
5	0	−1	−0.4707963
6	1	−0.8	−0.0272952
7	2	−0.787691	−0.0193610
8	3	−0.7576546	−0.0020574
9	4	−0.7540834	−0.0001740
10	5		
11	6		

The formula in cell D7 is

=(D5*E6 − D6*E5)/(E6 − E5)

and equivalent formulae are in cells D8 and D9.

(a) State the method being used. **[1]**

(b) Use the values in the spreadsheet to calculate x_5 and x_6, giving your answers correct to **7** decimal places. **[3]**

(c) State the value of α as accurately as you can, justifying the precision quoted. **[1]**

Alex uses a calculator to check the value in cell D9, his result is -0.7540832686.

(d) Explain why this is different to the value displayed in cell D9. **[1]**

Candidates who did well in this question recognised that the secant method was being used, applied it successfully in part (b) and usually went on to quote the answer correct to 5 d.p. by correctly comparing their calculated values. In part (d) they may have understood that the calculator is working with less precise values, but may have spoiled their answer by making a comment such as "whereas the spreadsheet uses the stored values, which are exact." Some candidates simply said that the spreadsheet stores values to a higher precision than it stores them, which was insufficient for the mark.

Candidates who did less well may have quoted the method as false position or fixed point iteration in part (a). Nevertheless, in part (b) they usually went on to use the formula successfully to compute x_5 and x_6, although they may have lost one or both accuracy marks by giving their answers to a different precision to the one requested. In part (c) some candidates quoted one of the expected answers, but neglected to justify the precision quoted – or gave a spurious reason. In part (d) they were not able to explain why the numbers generated are different; a common misunderstanding was "the calculator is more accurate".

Question 4 (a)

4 **Fig. 4.1** shows part of the graph of $y = e^x - x^2 - x - 1.1$.

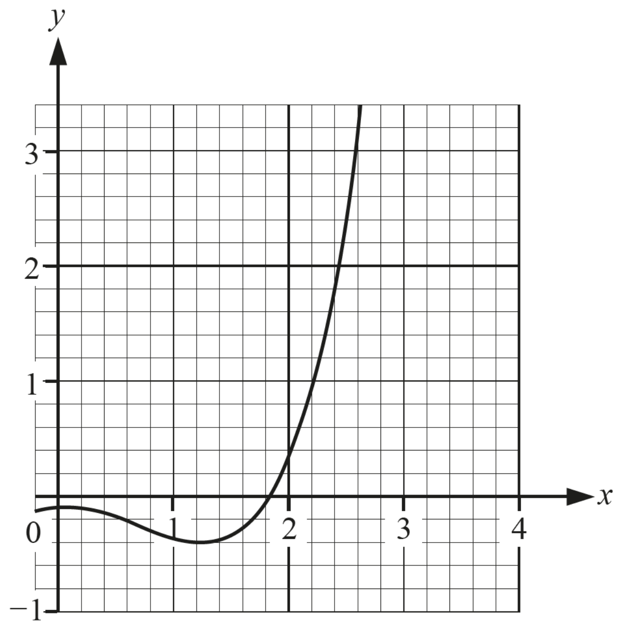

Fig. 4.1

The equation $e^x - x^2 - x - 1.1 = 0$ has a root α such that $1 < \alpha < 2$.

Ali is considering using the Newton-Raphson method to find α. Ali could use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$.

(a) **Without** doing any calculations, explain whether Ali should use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$, or whether using either starting value would work equally well. **[2]**

Candidates who did well in this question commented that drawing a tangent to the curve at $x = 2$ would lead to a new approximation which was much closer to the root, whereas drawing a tangent to the curve at $x = 1$ would lead to a new approximation which was further away from the root. A variety of ways of articulating these ideas were seen; not all of them were concise. They went on to conclude that Ali should start with $x = 2$

Candidates who did less well usually explained why starting with $x = 1$ may lead to divergence, or at best slow convergence, concluding that it would be better to start at $x = 2$ without explaining that doing this would in fact be successful.

Candidates who did not do well often discussed the gradient of the curve at $x = 2$ and $x = 1$, concluding that starting at $x = 2$ would not work because the gradient is too steep.

Question 4 (b)

Ali is also considering using the method of fixed point iteration to find α. Ali could use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$.

Fig. 4.2 shows parts of the graphs of $y = x$ and $y = \ln(x^2 + x + 1.1)$.

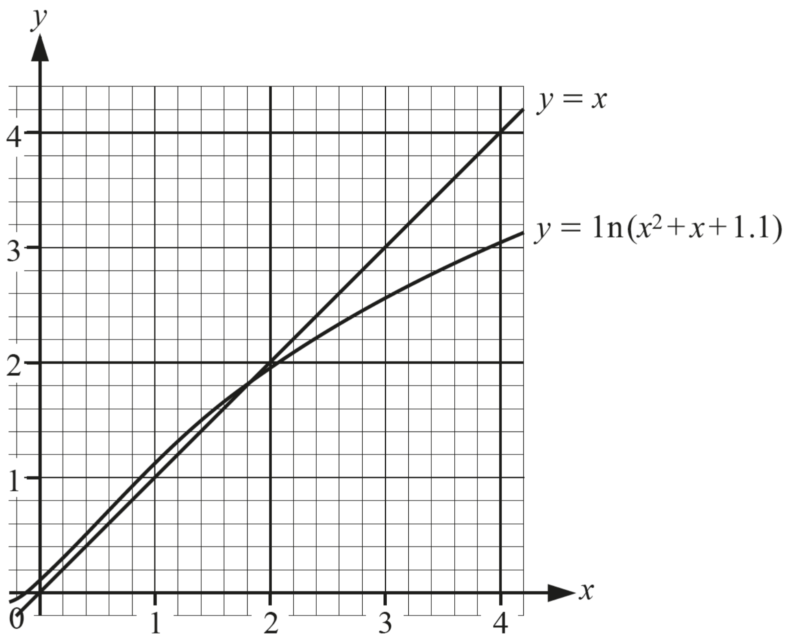

Fig. 4.2

(b) **Without** doing any calculations, explain whether Ali should use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$ or whether either starting value would work equally well. **[2]**

Candidates who did well in this question stated that starting with either value would work equally well and justified their observation by referring to the magnitude of the gradient of $y = \ln(x^2 + x + 1.1)$ at $x = \alpha$. They may have gone on to observe that starting with $x = 2$ might be slightly better as it's closer to the root.

Candidates who did less well made the correct statement, but based their justification on the gradient of $y = \ln(x^2 + x + 1.1)$ at $x = 1$ and at $x = 2$.

Candidates who did not do well stated that one of the values (usually $x = 2$) would lead to divergence.

Question 4 (d)

(d) Explain whether the values in column O suggest that Ali used the Newton-Raphson method or the iterative formula $x_{n+1} = \ln\left(x_n^2 + x_n + 1.1\right)$ to find this sequence of approximations to α. **[2]**

Candidates who did well in this question did so by two different routes. Either they commented that the ratio of differences is decreasing, which suggests faster than first order convergence, which means Ali probably used the Newton-Raphson method. Alternatively, they commented that the ratio of differences is not constant, which suggests convergence is not first order and so Ali probably didn't use fixed point iteration as this is usually a first order method – so they probably used Newton-Raphson.

Candidates who did less well made a correct observation about the ratio of differences, but followed this up with faulty reasoning. For example, "The ratio of differences is not constant, so convergence is second order, so Ali used Newton-Raphson."

Question 5 (a), (b) and (c)

5 Kai uses the midpoint rule, trapezium rule and Simpson's rule to find approximations to $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$, where a and b are constants. The associated spreadsheet output is shown in the table. Some of the values are missing.

	F	G	H	I
3	n	M_n	T_n	S_{2n}
4	1	0.2436699	0.1479020	
5	2	0.2306967		

(a) Write down a suitable spreadsheet formula for cell H5. **[2]**

(b) Complete the copy of the table in the Printed Answer Booklet, giving the values correct to **7** decimal places. **[4]**

(c) Use your answers to part **(b)** to determine the value of $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$ as accurately as you can, justifying the precision quoted. **[3]**

Candidates who did well in this question wrote down a correct spreadsheet formula in part (a) and calculated the correct values in part (b), giving final answers to the correct precision, A minority of candidates went on to use Richardson's extrapolation successfully, and a few made a sensible inference from their result to earn full marks in this part.

Candidates who did less well made a slip in part (a), such as omitting "=" or writing = 0.5×(G4 + H4) instead of =0.5*(G4+H4). In part (b) they may have made a slip in one of the calculations, or given one or more answer to a different precision. A few candidates calculated S_2 but not S_4, or vice versa. In part (c) they did not understand that extrapolation was required, and simply compared their approximations from part (b).

Question 6 (a)

6 Charlie uses fixed point iteration to find a sequence of approximations to the root of the equation

$\sin^{-1}(x) - x^2 + 1 = 0$.

Charlie uses the iterative formula $x_{n+1} = \mathrm{g}(x_n)$, where $\mathrm{g}(x_n) = \sin\left(x_n^2 - 1\right)$.

Two sections of the associated spreadsheet output, showing x_0 to x_6 and x_{102} to x_{108}, are shown in **Fig. 6.1**.

r	x_r	difference	ratio
0	0		
1	−0.841471	−0.84147	
2	−0.287798	0.553673	−0.65798
3	−0.793885	−0.50609	−0.91405
4	−0.361379	0.432507	−0.85461
5	−0.763945	−0.40257	−0.93078
6	−0.404459	0.359486	−0.89299

102	−0.596302	0.004626	−0.95886
103	−0.600738	−0.00444	−0.95911
104	−0.596484	0.004254	−0.95887
105	−0.600564	−0.00408	−0.95910
106	−0.596652	0.003912	−0.95888
107	−0.600404	−0.00375	−0.95909
108	−0.596806	0.003598	−0.95889

Fig. 6.1

(a) Use the information in **Fig. 6.1** to find the value of the root as accurately as you can, justifying the precision quoted. **[4]**

Candidates who did well in this question extrapolated to infinity using the last iterate, the last difference and a suitable version of –0.95889 for the ratio. Many went on to infer an appropriate precision for their final answer, which was explained either with reference to x_{108} or in terms of the improvement in accuracy which is likely from this method.

Candidates who did less well made a sign error in the extrapolation – usually with the difference but occasionally with the ratio, or did a partial extrapolation.

Exemplar 1

6(a)

~~ratio of differences = $\frac{-0.9504 + -0.95899}{2}$~~

~~$= -0.95899$~~

~~$-0.596806 - 0.0035908 \times \frac{0.95899}{1 - 0.95899}$~~

$\therefore$ ratio of differences $= \frac{-0.9504 + -0.95899}{2}$

= oscillating, average absolute value =

$= -0.95899$

$-0.596806 + 0.0035908 \times \frac{-0.95899}{1 - -0.95899}$

$= -0.59871909$ extrapolation to ∞

agrees with -0.59696809

to $-0.59(9)$

$\Rightarrow$ root $=$ ~~...~~ -0.60,

as accurate as possible, since it agrees with most long iterations, and the extrapolated value.

-0.459856965

The candidate extrapolates to infinity with viable values of *x, d* and *r*. Unfortunately the calculation goes astray, resulting in a value outside tolerance. Consequently the second accuracy mark is not earned. This automatically rules out the award of the final accuracy mark – the first 3 marks must have been earned for the award of this mark. It should be noted that some justification of the precision quoted in the final answer is needed for the award of the final accuracy mark.

Question 6 (b)

The relaxed iteration $x_{n+1} = (1-\lambda)x_n + \lambda g(x_n)$, with $\lambda = 0.51$ and $x_0 = 0$, is to be used to find the root of the equation $\sin^{-1}(x) - x^2 + 1 = 0$.

(b) Complete the copy of **Fig. 6.2** in the Printed Answer Booklet, giving the values of x_r correct to **7** decimal places and the values in the difference column and ratio column correct to **3** significant figures.

r	x_r	difference	ratio
0	0		
1			
2			
3			
4		-0.000192	
5		-1.99×10^{-7}	0.00103
6		-1.82×10^{-10}	0.000914

Fig. 6.2

[4]

Candidates who did well in this question used the iterative capability of their calculator to generate the 5 requested iterates, and went on to successfully find the differences and the ratios. They may have lost an accuracy mark by making a slip – usually with one of the ratios – or by giving all the values to a different precision.

Candidates who did less well successfully found the iterates, but went astray calculating the differences and the ratios – usually by adding an extra difference or omitting one of the differences.

Question 6 (d)

(d) Explain why extrapolation could not be used in this case to find an improved approximation using this sequence of iterates. **[1]**

Candidates who did well in this question referred to the ratio of differences not converging to a constant.

Candidates who did not do well thought that the approximation was sufficiently accurate so no extrapolation was needed.

Question 7 (a)

7 Sam decided to go on a high-protein diet. Sam's mass in kg, M, after t days of following the diet is recorded in **Fig. 7.1**.

t	0	10	20	30
M	88.3	80.05	78.7	78.85

Fig. 7.1

A difference table for the data is shown in **Fig. 7.2**.

t	M	ΔM	$\Delta^2 M$	$\Delta^3 M$
0	88.3			
10	80.05			
20	78.7			
30	78.85			

Fig. 7.2

(a) Complete the copy of the difference table in the Printed Answer Booklet. **[1]**

Question 7 (b)

Sam's doctor uses these data to construct a cubic interpolating polynomial to model Sam's mass at time t days after starting the diet.

(b) Find the model in the form $M = at^3 + bt^2 + ct + d$, where a, b, c and d are constants to be determined. **[4]**

Exemplar 2

7(b)

$$M = \cancel{0}\ 88\cdot3 + \frac{(t-0)(-8\cdot25)}{10} + \frac{(t-0)(t-10)(6\cdot9)}{2! \times 10^2} \qquad h=10$$

$$+ \frac{(t-0)(t-10)(t-20)(-5\cdot4)}{3! \times 10^3}$$

$$= 88\cdot3 - 0\cdot825t + 0\cdot0345(t^2 - 10t)$$

$$- 0\cdot0009(t^2 - 10)(t - 20)$$

$$= 88\cdot3 + 0\cdot0345t^2 - 1\cdot17t - 0\cdot0009(t^3 - 20t^2 - 10t + 200)$$

$$= -0\cdot0009t^3 + 0\cdot0525t^2 - 1\cdot161t + 88\cdot12$$

$$\text{Ans: } M = -0\cdot0009t^3 + 0\cdot0525t^2 \cancel{-1\cdot17t} - 1\cdot161t + 88\cdot12$$

The candidate makes the correct substitution in the formula, earning M1. Unfortunately there are then three errors in expanding $t(t-10)(t-20)$, resulting in three out of four coefficients in the final answer being incorrect, so none of the 3 available accuracy marks were earned. Note that the candidate expressed the final answer in the requested form with the correct variables – a significant minority left their answer in terms of *x* or omitted *M* from the final answer, losing an accuracy mark for otherwise fully correct work.

Question 7 (c)

Subsequently it is found that when $t = 40$, $M = 78.7$ and when $t = 50$, $M = 80.05$.

(c) Determine whether the model is a good fit for these data. **[2]**

Candidates who did well in parts (a), (b) and (c) completed the difference table correctly and worked competently with Lagrange's formula in part (b) to produce a cubic formula. They may have made a slip with one of the coefficients or lost an accuracy mark by leaving their answer in terms of a different variable (usually *x*). In part (c) they evaluated *M* correctly but may have lost the final mark by deciding that the model is a good fit, when it clearly isn't.

Candidates who did less well may have made a slip in part (a), and then accessed only the available FT marks in parts (b) and (c).

Candidates who did not do well did not know how to use Lagrange's formula in part (b) and consequently couldn't access the marks in part (c).

Question 7 (d), (e) and (f)

(d) By completing the extended copy of **Fig. 7.2** in the Printed Answer Booklet, explain why a quartic model may be more appropriate for the data. **[2]**

(e) Refine the doctor's model to include a quartic term. **[3]**

(f) Explain whether the new model for Sam's mass is likely to be appropriate over a longer period of time. **[2]**

Candidates who did well in parts (d), (e) and (f) completed the difference table correctly in part (d) and commented appropriately. In part (e) they earned both method marks, usually by starting from scratch with Lagrange's formula, and a few candidates went on to obtain the correct quartic polynomial. In part (c) they gave their conclusion in context, and explained why the model could not be viable in the long run.

Candidates who did less well competed the difference table successfully, but may have only earned 1 or 2 method marks in part (e). In part f they may have stated that the model is viable because it has used more data, or they may not have related their conclusion to the context.

Candidates who did not do well may have used their solver functions to obtain the correct quartic polynomial in part (e), often after an incorrect answer in part (b). A significant minority of candidates either omitted parts (e) and (f) or made no significant progress.

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