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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y421/01 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

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Paper Y421/01 series overview

This is one of the two major examination components for the A Level examination for GCE Further Mathematics B. The paper consists of two Sections, A and B.

The report that follows will concentrate on aspects of the candidates' performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this paper in this reformed A Level qualification produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on all the associated mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The specification includes some guidance about the level of written evidence required in assessment question; this was provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed.

The word 'determine' in a question does not simply imply that candidates should find the answer but, to quote the specification, 'this command word indicates that justification should be given for any results found, including working where appropriate.' This command word featured in Questions 3 (a), 4 (a), 5 (b), 6 (b), 7 (b) (iii), 8 (d), 9 (b), 10 (b), 11 (b), 12 (a) and 13 (d). The phrase 'Show that' generally indicates that the answer has been given, and that candidates should provide an explanation that has enough detail to cover every step of their working. This command phrase features in Questions 6 (a), 7 (a), 8 (a), 8 (b), 9 (a), 10 (a), 11 (a), 12 (b), 13 (a) and 13 (b). While there is no specific level of working needed to justify responses to questions which use the command word 'find …', method marks may still be available for valid attempts that do not result in a correct response. Standard advice (included in the specification) is that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation). Regardless of the final required accuracy, candidates should be careful of not rounding prematurely, but also take care to avoid over specifying rounded answers where the context does not support that level of accuracy.

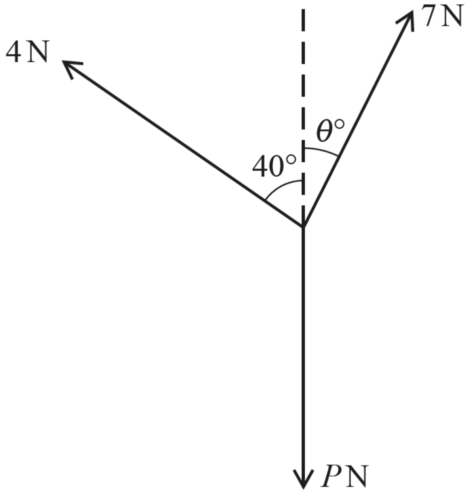
One general point with regards to the answering of certain mechanics questions should be made in this overview. This is, that unless told otherwise, the value that candidates should use for the acceleration due to gravity, g, is 9.8 and not 9.81 or 10 (and this value is stated explicitly on the front cover of the examination paper).

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• used formal language and notation • defined the letters used for speed, time, etc. in unstructured questions • understood the level of response required for the command words used in the questions • made efficient use of their calculator • read questions carefully and provided the responses that were requested.	• made careless mistake in algebraic manipulation • did not give sufficient evidence on 'Show that' and 'Determine' questions • used imprecise notation or language • did not read the question carefully and gave responses not to the required degree of accuracy (or did not use all the information given in the question).

Section A

Question 1 (a)

1



Three forces of magnitudes 4 N, 7 N and P N act at a point in the directions shown in the diagram.

The forces are in equilibrium.

(a) Draw a closed figure to represent the three forces. **[1]**

This question was answered extremely well, however, occasionally candidates did not either label all three sides of their triangle or did not include both given angles. Candidates would be advised to include the angles inside rather than outside of their triangle.

Question 1 (b) (i)

(b) Hence, or otherwise, find the following.

(i) The value of θ. **[2]**

Candidates were equally split between using the sine rule on their closed figure or resolving horizontally (from the given diagram). Regardless of the method used nearly all candidates were successful in finding the required angle.

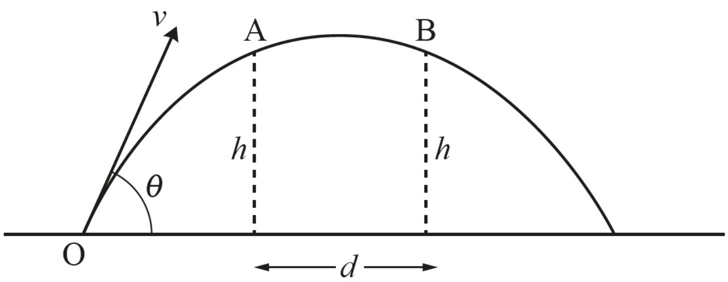
Question 1 (b) (ii)

(ii) The value of P. **[2]**

Again, candidates were equally split between either resolving vertically or using the cosine rule to find P. Only a few candidates applied the sine rule (which in most cases was much easier to apply than the cosine rule).

Question 2

2



A particle is projected with speed v from a point O on horizontal ground. The angle of projection is θ above the horizontal. The particle passes, in succession, through two points A and B, each at a height h above the ground and a distance d apart, as shown in the diagram.

You are given that $d^2 = \dfrac{v^\alpha \sin^2 2\theta}{g^\beta} - \dfrac{8hv^2\cos^2\theta}{g}$.

Use dimensional analysis to find α and β. **[4]**

It was clear that several candidates had a fundamental misunderstanding of dimensional analysis. Most candidates started off by writing $\mathrm{L}^2 = \dfrac{(\mathrm{LT}^{-1})^\alpha}{(\mathrm{LT}^{-2})^\beta} - \dfrac{\mathrm{L}(\mathrm{LT}^{-1})}{\mathrm{LT}^{-2}}$ and while this is not strictly incorrect many did not realise that as each term of the equation had to have the same dimensions that it was therefore the case that either $\mathrm{L}^2 = \dfrac{(\mathrm{LT}^{-1})^\alpha}{(\mathrm{LT}^{-2})^\beta}$ or $\dfrac{(\mathrm{LT}^{-1})^\alpha}{(\mathrm{LT}^{-2})^\beta} = \dfrac{\mathrm{L}(\mathrm{LT}^{-1})}{\mathrm{LT}^{-2}}$. A number of candidates who started with the statement that $\mathrm{L}^2 = \dfrac{(\mathrm{LT}^{-1})^\alpha}{(\mathrm{LT}^{-2})^\beta} - \dfrac{\mathrm{L}(\mathrm{LT}^{-1})}{\mathrm{LT}^{-2}}$ did not make any significant progress (or incorrectly treated it as an equation and therefore started to combine like terms). The most successful candidates were those that realised that $\mathrm{L}^2 = \dfrac{(\mathrm{LT}^{-1})^\alpha}{(\mathrm{LT}^{-2})^\beta}$ and then formed and solved two equations in α and β.

Misconception

There were two common misconceptions in this question. The first was that the dimensions of *g* contained M (possibly due to the incorrect assumption that *g* is a force) and that angles are not dimensionless; many candidates assumed that the angles given in this question had the dimensions of length.

Question 3 (a)

3 A particle, of mass 2 kg, is placed at a point A on a rough horizontal surface. There is a straight vertical wall on the surface and the point on the wall nearest to A is B. The distance AB is 5 m. The particle is projected with speed $4.2\,\mathrm{m\,s^{-1}}$ along the surface from A towards B. The particle hits the wall directly and rebounds. The coefficient of friction between the particle and the surface is 0.1.

(a) Determine the speed of the particle immediately before impact with the wall. **[4]**

It was interesting to note that in this question most candidates attempted the first part by using the work-energy principle rather than the expected use of $v^2 = u^2 + 2as$. Many candidates initially stated the acceleration of the particle incorrectly as 0.98 but then correctly used –0.98 to determine the speed of the particle immediately before impact.

Question 3 (b)

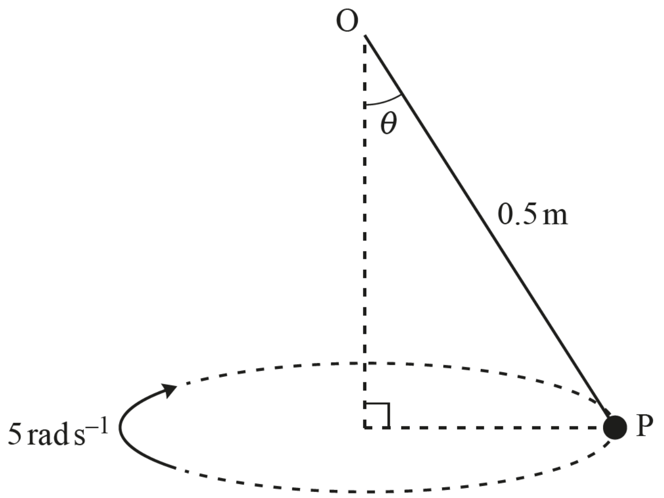
The magnitude of the impulse that the wall exerts on the particle is 9.8 N s.

(b) Find the speed of the particle immediately after impact with the wall. **[2]**

Apart from the standard sign errors when applying the result that impulse is equal to the change in momentum, most candidates correctly found the speed of the particle immediately after impact with the wall.

Question 4 (a)

4



The diagram shows a particle P, of mass 0.1 kg, which is attached by a light inextensible string of length 0.5 m to a fixed point O.

P moves with constant angular speed $5\,\text{rad}\,\text{s}^{-1}$ in a horizontal circle with centre vertically below O. The string is inclined at an angle θ to the vertical.

(a) Determine the tension in the string. **[3]**

This was the best answered question on the entire paper with most candidates scoring full marks in all three parts. A few candidates in the first part assumed that the radius of the horizontal circle was 0.5 rather than $0.5\sin\theta$. However, this was rare with the vast majority correctly applying Newton's Second Law horizontally and resolving the tension in the string as $T\sin\theta$.

Question 4 (b)

(b) Find the value of θ. **[2]**

Nearly all candidates correctly resolved vertically to obtain the correct value of θ.

Question 4 (c)

(c) Find the kinetic energy of P. **[2]**

Again, this question was answered extremely well with almost all candidates applying $\frac{1}{2}mv^2$ with both a correct mass and with $v = 5r$.

Question 5 (a)

5 At time t seconds, where $t \geqslant 0$, a particle P of mass 2 kg is moving on a smooth horizontal surface. The particle moves under the action of a constant horizontal force of $(-2\mathbf{i}+6\mathbf{j})$ N and a variable horizontal force of $(2\cos 2t\,\mathbf{i}+4\sin t\,\mathbf{j})$ N. The acceleration of P at time t seconds is $\mathbf{a}\,\mathrm{m\,s^{-2}}$.

(a) Find $\mathbf{a}$ in terms of t. **[2]**

Almost all candidates correctly added the two given forces together and divided by the given mass to obtain a correct expression for **a**. Candidates are reminded that the use of brackets in questions involving vectors is vital so that it is clear to examiners which terms are linked to which components.

Question 5 (b)

The particle P is at rest when $t = 0$.

(b) Determine the speed of P at the instant when $t = 2$. **[5]**

While most candidates correctly integrated their expression from (a) to obtain the corresponding velocity vector some either ignored the need for a vector constant of integration or assumed that it would be a zero. Some candidates who did correctly obtain the velocity at t = 2 as $\mathbf{v} = \left(\frac{1}{2}\sin 4 - 2\right)\mathbf{i} + \left(8 - 2\cos 2\right)\mathbf{j}$ either did not find the magnitude (even though the question asked for the speed of P at this time) or gave a response of 6.31 (indicating that their calculator was in degrees rather than radians).

Assessment for learning

When a question involves the integration or differentiation of trigonometric functions candidates are reminded that the angle will be measured in radians and not degrees (and this will often not be explicitly mentioned in the question).

Section B

Question 6 (a)

6 **In this question the box should be modelled as a particle.**

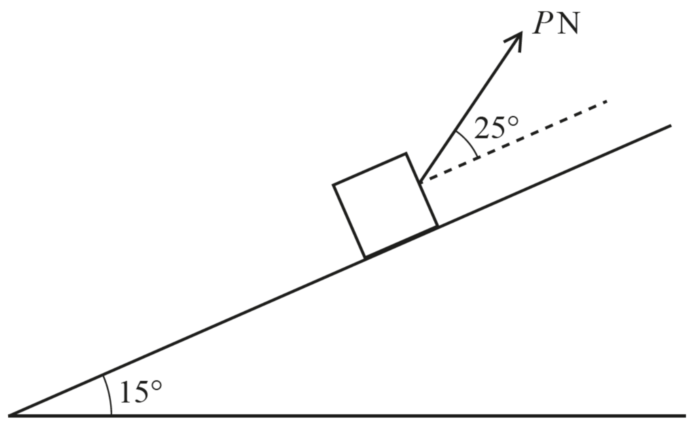
A box of mass m kg is placed on a rough slope which makes an angle of α with the horizontal.

(a) Show that the box is on the point of slipping if $\mu = \tan\alpha$, where μ is the coefficient of friction between the box and the slope. **[2]**

This piece of book work was a relatively straightforward introduction to Section B with almost all candidates correctly resolving perpendicular and parallel to the inclined plane to obtain the given result. Only occasionally did a candidate believe that the information provided for part (b) had any relevance with this first part.

Question 6 (b)

A box of mass 5 kg is pulled up a rough slope which makes an angle of 15° with the horizontal. The box is subject to a constant frictional force of magnitude 3 N. The speed of the box increases from $2\,\mathrm{m\,s^{-1}}$ at a point A on the slope to $5\,\mathrm{m\,s^{-1}}$ at a point B on the slope with B higher up the slope than A. The distance AB is 10 m.



The pulling force has constant magnitude P N and acts at a constant angle of 25° above the slope, as shown in the diagram.

(b) Use the work–energy principle to determine the value of P. **[5]**

Some candidates did not read the question carefully and instead gave a solution using Newton's Second Law and constant acceleration formulae (therefore scoring no marks). The most common errors when applying the work-energy principle were to either include the gravitational potential energy term twice or failing to resolve the pulling force parallel to the plane (and so had an equation containing $10P$ rather than the correct $10P\cos 25$).

Question 7 (a)

7 Two small uniform smooth spheres A and B, of masses 2 kg and 3 kg respectively, are moving in opposite directions along the same straight line towards each other on a smooth horizontal surface. Sphere A has speed $2\,\text{m s}^{-1}$ and B has speed $1\,\text{m s}^{-1}$ before they collide. The coefficient of restitution between A and B is e.

(a) Show that the velocity of B after the collision, in the original direction of motion of A, is $\frac{1}{5}(1+6e)\,\text{m s}^{-1}$ and find a similar expression for the velocity of A after the collision. **[5]**

This question was answered extremely well with most candidates correctly applying the conservation of linear momentum and Newton's experimental law consistently to derive the given response for the velocity of B after collision. Some candidates however, did not read the question carefully and did not find a similar expression for the velocity of A after impact even though the question explicitly said that this expression should be given in 'the original direction of motion of A'. Therefore, the only acceptable response was $\frac{1}{5}(1-9e)$ and not $\frac{1}{5}(9e-1)$ (which was from assuming that the direction of motion of A was reversed in the collision).

Assessment for learning

Unless a question explicitly mentions the direction of motion of the bodies immediately after direct impact, candidates are advised to assume that both bodies move in whichever direction is being taken as the positive direction of motion. Therefore, candidates are better off assuming that both bodies are moving in the same direction of motion after impact, this then eliminates the majority of sign errors which are all too common in this type of question.

Question 7 (b) (i)

(b) The following three parts are independent of each other, and each considers a different scenario regarding the collision between A and B.

(i) In the collision between A and B the spheres coalesce to form a combined body C. State the speed of C after the collision. **[1]**

Nearly all candidates correctly stated the speed of C correctly (as 0.2).

Question 7 (b) (ii)

(ii) In the collision between A and B the direction of motion of A is reversed.
Find the range of possible values of e. **[2]**

Although most candidates scored at least one mark for realising that $\frac{1}{5}(1-9e)<0$ only the most able included an upper bound on the value of *e* and hence it was rare for examiners to see the fully correct response of $\frac{1}{9}<e\leqslant 1$.

Question 7 (b) (iii)

(iii) The total loss in kinetic energy due to the collision is 3 J.
Determine the value of e. **[4]**

Although most candidates had the correct method for finding the value of *e*, many made careless algebraic (or sign) slips when solving the equivalence of the equation $\frac{11}{2}-\left(\frac{1-9e}{5}\right)^2-\frac{3}{2}\left(\frac{1+6e}{5}\right)^2=3$.

Question 8 (a)

8 A particle P is projected from a fixed point O with initial velocity $u\mathbf{i}+ku\mathbf{j}$, where k is a positive constant. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the horizontal and vertically upward directions respectively. P moves with constant gravitational acceleration of magnitude g.
At time $t \geqslant 0$, particle P has position vector $\mathbf{r}$ relative to O.

(a) Starting from an expression for $\ddot{\mathbf{r}}$, use integration to derive the formula

$$\mathbf{r} = ut\mathbf{i}+\left(kut-\tfrac{1}{2}gt^2\right)\mathbf{j}.$$ **[4]**

This question was answered extremely well although several candidates did not do as requested and instead used constant acceleration formulae to derive the given result for the position vector of P. Of those that did start from a correct acceleration vector and used integration, some did not then include appropriate constants of integration or give specific detail when deriving the given result.

Question 8 (b)

The position vector $\mathbf{r}$ of P at time $t \geqslant 0$ can be expressed as $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, where the axes O$x$ and Oy are horizontally and vertically upwards through O respectively. The axis Ox lies on horizontal ground.

(b) Show that the path of P has cartesian equation

$$gx^2 - 2ku^2x + 2u^2y = 0.$$ **[3]**

Almost all candidates were successful in removing the vectors from part (a) and eliminating *t* to obtain the given cartesian equation.

Question 8 (c)

(c) Hence find, in terms of g, k and u, the maximum height of P above the ground during its motion. **[3]**

Although some candidates did not use the response to part (b) to tackle part (c) (even though the command, 'Hence' was used in (c)) most used differentiation (either implicitly or by first rearranging to make *y* the subject) rather than the discriminant of the quadratic equation in *x* to find the maximum height of P above the ground during its motion.

Question 8 (d)

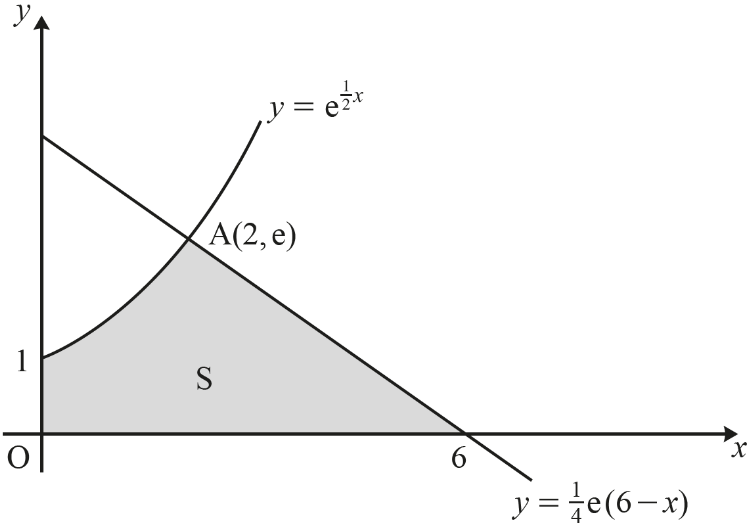
The maximum height P reaches above the ground is equal to the distance OA, where A is the point where P first hits the ground.

(d) Determine the value of k. **[3]**

Most candidates answered this question extremely well with most correctly setting their response to part (c) equal to the expression found from substituting *y* = 0 into the given response from part (b). Occasional algebraic errors were seen in solving the corresponding quadratic equation for *k*.

Question 9 (a)

9 [In this question you may use the facts that for a uniform solid right circular cone of height h and base radius r the volume is $\frac{1}{3}\pi r^2 h$ and the centre of mass is $\frac{1}{4}h$ above the base on the line from the centre of the base to the vertex.]



The diagram shows the shaded region S bounded by the curve $y = e^{\frac{1}{2}x}$ for $0 \leqslant x \leqslant 2$, the x-axis, the y-axis, and the line $y = \frac{1}{4}e(6-x)$.

The line $y = \frac{1}{4}e(6-x)$ meets the curve $y = e^{\frac{1}{2}x}$ at the point A with coordinates $(2, e)$.

The region S is rotated through 2π radians about the x-axis to form a uniform solid of revolution T.

(a) Show that the x-coordinate of the centre of mass of T is $\dfrac{3(5e^2+1)}{7e^2-3}$. **[8]**

This question was answered extremely well by most candidates with only a few not reading the question correctly and instead calculating the *x*-coordinate of the centre of mass of a corresponding uniform lamina instead of the required solid of revolution. Nearly all candidates who correctly stated that $V_1\bar{x} = \pi\int_0^2 x\left(e^{\frac{1}{2}x}\right)^2 \, dx$ completed the required integration by parts correctly (and then went on to derive the given response). Those candidates who did not read the question carefully spent time calculating the volume and centre of mass of the cone formed by rotating the straight line about the *x*-axis even though the question explicitly stated that results about the volume and centre of mass of uniform solid cone could be assumed.

Question 9 (b)

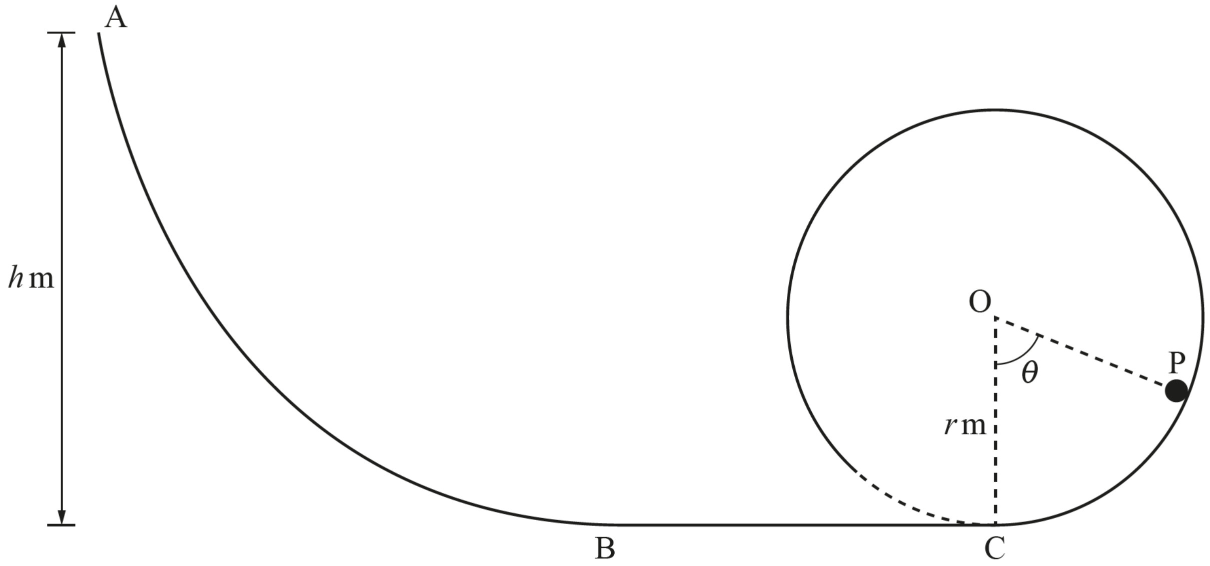
Solid T is freely suspended from A and hangs in equilibrium.

(b) Determine the angle between AO, where O is the origin, and the vertical. **[3]**

Candidates found this question one of the trickiest ones of the entire paper with only the most able realising that the required angle was given by $\arctan\left(\dfrac{\dfrac{3\left(5e^2+1\right)}{7e^2-3}-2}{e}\right)+\arctan\left(\dfrac{2}{e}\right)$. While a few candidates attempted this question using either the cosine rule or the scalar product, very few were successful.

Question 10 (a)

10



A small toy car runs along a track in a vertical plane.

The track consists of three sections: a curved section AB, a horizontal section BC which rests on the floor, and a circular section that starts at C with centre O and radius r m.

The section BC is tangential to the curved section at B and tangential to the circular section at C, as shown in the diagram.

The car, of mass m kg, is placed on the track at A, at a height h m above the floor, and released from rest. The car runs along the track from A to C and enters the circular section at C. It can be assumed that the track does not obstruct the car moving on to the circular section at C.

The track is modelled as being smooth, and the car is modelled as a particle P.

(a) Show that, while P remains in contact with the circular section of the track, the magnitude of the normal contact force between P and the circular section is

$$mg\left(3\cos\theta - 2 + \frac{2h}{r}\right)\text{N},$$

where θ is the angle between OC and OP. **[7]**

The responses to this question were mixed and while some candidates set their working out in a logical manner, examiners did report that at times it was difficult to follow some candidates' working.

Assessment for learning

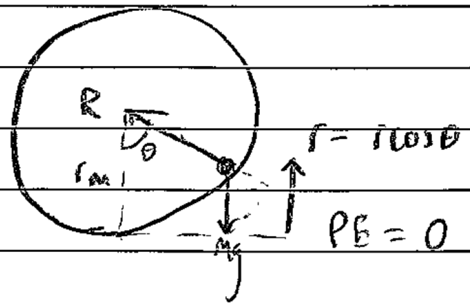
When a question involves multiple stages of working that include the need to introduce a certain number of variables that have not been defined in the question it is the responsibility of the candidate to define the variables they use. In this question many candidates did not make it explicitly clear which letters were being used for the speed of the car during the different parts of its motion.

Exemplar 1

10(a)

$mgh = \frac{1}{2} m v_B^2$

$V_B = \sqrt{2gh}$



at general posn speed = V

~~ca~~ conservation of ME

$\frac{1}{2} m \times 2gh = \frac{1}{2} mv^2 + mgr(1 - \cos\theta)$

$\frac{1}{2} v^2 = \cancel{m}gh - gr(1 - \cos\theta)$

$v^2 = 2gh - 2gr + 2gr\cos\theta$

Radially $F = ma$

$\cancel{m}\ R - mg\cos\theta = \frac{mv^2}{r}$

$R = mg\cos\theta + \frac{m}{r}(2gh - 2gr + 2gr\cos\theta)$

$= mg\left(\cos\theta + \frac{2h}{r} - 2 + 2\cos\theta\right)$

$= mg\left(3\cos\theta + \frac{2h}{r} - 2\right)$

This response scored full marks and showed all the correct detail required to follow the candidate's method and working towards the correct result.

Question 10 (b)

(b) Hence determine, in terms of r, the least possible value of h so that P can complete a vertical circle. **[2]**

This question was answered extremely well with most candidates realising that the least value of h was found by considering the normal contact force (from (a)) equal to zero when P was at the highest point of the vertical circle (so when $\theta = 180^\circ$).

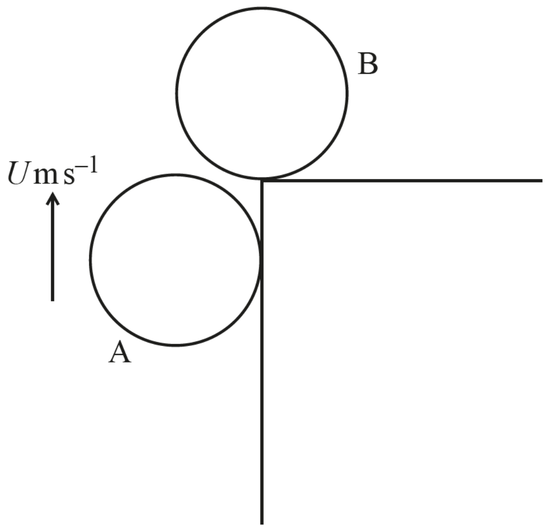
Question 10 (c)

(c) Apart from not modelling the car as a particle, state one refinement that would make the model more realistic. **[1]**

Many candidates did not correctly state one refinement that would make the model more realistic (for example, include air resistance in the model or modelling the track as rough) but instead gave a modelling assumption that already existed for the problem. Many candidates did not make it clear what the refinement was, for example, just stating 'air resistance' does not make it clear what form the refinement is going to take.

Question 11 (a)

11



The diagram shows two small identical uniform smooth spheres, A and B, just before A collides with B. Sphere B is at rest on a horizontal table with its centre vertically above the edge of the table. Sphere A is projected vertically upwards so that, just before it collides with B, the speed of A is $U\,\mathrm{m\,s^{-1}}$ and it is in contact with the vertical side of the table. The point of contact of A with the vertical side of the table and the centres of the spheres are in the same vertical plane.

(a) Show that on impact the line of centres makes an angle of 30° with the vertical. **[1]**

Many candidates answered this question correctly and were able to consider the fact that $\sin\theta = \frac{r}{2r}$, where *r* was the common radius of the spheres and θ was the angle between the line of centres at impact and the vertical. However, many candidates left this question blank.

Question 11 (b)

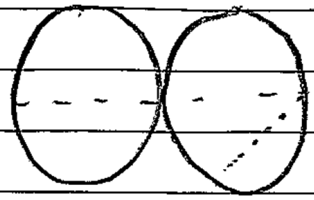
The coefficient of restitution between A and B is $\frac{1}{2}$. After the impact B moves freely under gravity.

(b) Determine, in terms of U and g, the time taken for B to first return to the table. **[7]**

Many candidates struggled with the unfamiliar nature of this question on oblique impact. Although some candidates correctly applied the conservation of linear momentum and Newton's experimental law parallel to the line of centres many included unknown angles for the velocity components after impact. Candidates are strongly advised, in such problems, to only ever use angles given in the question and then only consider the components of the velocity parallel and perpendicular to the line of centres (as required). So, in this problem those that correctly stated these equations as $v_A + v_B = U\cos 30$ and $v_A - v_B = -\frac{1}{2}U\cos 30$ were the most successful at finding the speed of B after impact. Of those that attempted to find the time taken for B to first return to the table only the most successful realised the need to consider the vertical component of B's velocity with the corresponding equation(s) of constant acceleration.

Exemplar 2

11(b)



$(U\sin 60)\frac{1}{2} = Bv\cancel{4} + BA$

$\cancel{m}U\sin 60 = \cancel{m}Bv\cancel{4} - \cancel{m}BA$

$\cancel{2}B = \frac{3}{\cancel{2}4}U\sin 60$

$s = x$
$u = \frac{3}{4}U\sin 60$ $\quad v = u + at$
$v = 0$
$a = -9.8$
$t = ?$

$0 = \frac{3}{4}U\sin 60 - 9.8t$

$9.8t = \frac{3}{4}U\sin 60$ but 2x time so

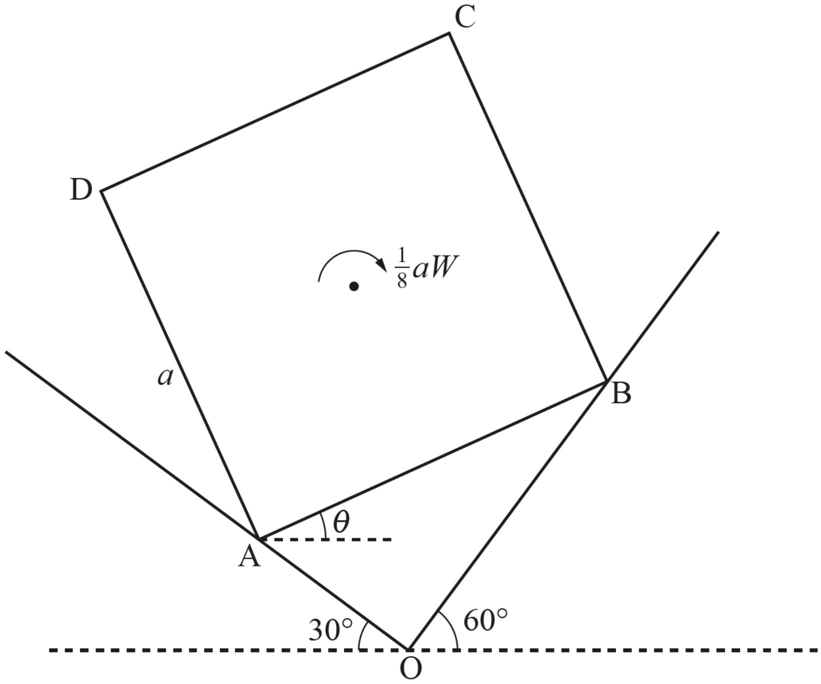
$t = \cancel{0.1326U}\ \frac{\frac{3}{4}\sin 60 U}{g}$

$t = \frac{3}{4g}U\sin 60 = \frac{3\sqrt{3}}{8g}U$

This response gained 5 out of the 7 marks. The candidate's response is easy to follow with suitable detail at each stage indicating the methods (and notation) being employed. The candidate correctly derived the speed of B after impact by applying both the conservation of linear momentum and Newton's experimental law correctly. Their error is a conceptual one of assuming that the speed of B after impact (a scalar quantity) can be substituted directly into the equation *v* = *u* + *at* (a vector equation) without realising that *u* in this case would need to be $\left(\frac{3}{4}U\sin 60\right)\cos 30$ as the components needs to be considered vertically to find the time to maximum height before being doubled to find the time to return to the table. This error meant the loss of the final 2 marks in this part.

Question 12 (a)

12



The diagram shows a uniform square lamina ABCD, of weight W and side-length a. The lamina is in equilibrium in a vertical plane that also contains the point O. The vertex A rests on a smooth plane inclined at an angle of 30° to the horizontal. The vertex B rests on a smooth plane inclined at an angle of 60° to the horizontal.

OA is a line of greatest slope of the plane inclined at 30° to the horizontal and OB is a line of greatest slope of the plane inclined at 60° to the horizontal.

The side AB is inclined at an angle θ to the horizontal and the lamina is kept in equilibrium in this position by a clockwise couple of magnitude $\frac{1}{8}aW$.

(a) By resolving horizontally and vertically, determine, in terms of W, the magnitude of the normal contact force between the plane and the lamina at B. **[6]**

Candidates found this the most demanding question on the paper even though in both parts (a) and (b) candidates were told the method to use in order to make the question more accessible. The main issues in this part were in determining the required angle when resolving vertically and horizontally (with only the most able realising that only the angle of 30 degrees was required). Those that did resolve correctly in the two given directions usually went on to solve the simultaneous equations correctly.

Question 12 (b)

(b) By taking moments about A, show that θ satisfies the equation

$$2(\sqrt{3}+2)\sin\theta - 2\cos\theta = 1.$$ **[5]**

As with part (a), candidates found this question very demanding with only the most able being able to take moments correctly about A. Although most could deal with the addition of a couple to maintain equilibrium very few found the moment of the weight or the moment of the contact force at B correctly.

Question 12 (c)

(c) Verify that $\theta = 22.4°$, correct to **1** decimal place. **[2]**

Many candidates did not realise how to verify that the angle was 22.4 in this question. Most candidates either tried to solve the given equation, or simply substituted the given value into the equation and commented that the value was 'close to' 1. Of those candidates who did realise that a 'sign change' test was the best way to verify the result, most opted for using 22.35 and 22.45 and compared their results with 1 (rather than setting up an expression equal to zero and showing a sign change from negative to positive (or vice-versa)).

Question 13 (a)

13 In this question take $g = 10$.

A particle P of mass 0.15 kg is attached to one end of a light elastic string of modulus of elasticity 13.5 N and natural length 0.45 m. The other end of the string is attached to a fixed point O. The particle P rests in equilibrium at a point A with the string vertical.

(a) Show that the distance OA is 0.5 m. **[3]**

This question was answered extremely well with most candidates correctly showing the distance OA was 0.5 (by resolving vertically and using Hooke's law). However, a few candidates did not use g = 10 and so lost the final accuracy mark (in this and the next question).

Question 13 (b)

At time $t = 0$, P is projected vertically downwards from A with a speed of $1.25\,\mathrm{m\,s^{-1}}$. Throughout the subsequent motion, P experiences a variable resistance R newtons which is of magnitude 0.6 times its speed (in $\mathrm{m\,s^{-1}}$).

(b) Given that the downward displacement of P from A at time t seconds is x metres, show that, while the string remains taut, x satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 200x = 0.$$ **[3]**

The responses to this question were mixed. While some candidates correctly set up the equation of motion as $0.15\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 0.15g - \frac{13.5}{0.45}(0.05 + x) - 0.6\frac{\mathrm{d}x}{\mathrm{d}t}$ and were then successful in deriving the given differential equation, many started with the extension as x rather than $0.05 + x$. This meant that they could only obtain the given equation by eliminating the weight term from the equation which was not mathematically correct (even though it led to the 'correct' response).

Question 13 (c)

(c) Verify that $x = \frac{5}{56}e^{-2t}\sin(14t)$. **[6]**

While most candidates did indeed verify this solution by substituting it into the differential equation from part (b) and checking that when $t = 0$, $\frac{dx}{dt} = 1.20$ and $x = 0$, several candidates instead solved the given differential equation.

Misconception

The word 'verify' does not have the same mathematical meaning as 'show that' – many candidates in this question solved the second order differential equation and used the given conditions to calculate the two arbitrary constants. While this approach could score all 6 marks it was a time-consuming way of tackling the problem. Furthermore, the solution to differential equations (apart from those found via SHM) are not required for this unit and it may not always be possible to solve the differential equation that candidates are being asked to verify in this unit.

Question 13 (d)

(d) Determine whether the string becomes slack during the motion. **[5]**

Most candidates who attempted this final question incorrectly considered the expression for x from part (c) equal to 0.05. The correct method was to first find the second time when the particle would be at rest and then find the corresponding value of x at this time (and then compare the magnitude of this value with 0.05). Of those that did consider the displacement of the particle when it was at rest, many considered only the first time when it would be at rest which, due to the motion of P, would be a position of maximum extension therefore guaranteeing that the string would be taut.

Exemplar 3

13(d)	$\frac{dx}{dt} = -\frac{5}{28} e^{-2t} \sin(14t) + \frac{5}{4} e^{-2t} \cos(14t)$
	$\frac{dx}{dt} = 0$
	$\frac{5}{28} \sin(14t) = \frac{5}{4} \cos(14t)$
	$\tan(14t) = 7$
	$14t = 1.428899 , 4.57$
	$t = 0.326$ s
	$x = -0.046$m
	The maximum displacement above the equilibrium
	point is 0.046 m
	as 0.046 < 0.05 m, the string does not go slack.

This response was fully correct. Each line of working was extremely clear and the level of detail in the algebraic working to show that string does not go slack was impressive.

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