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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y544/01 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper Y544/01 series overview

This is an option paper for Further Mathematics. It assesses Discrete Mathematics through graphs and networks, algorithms, linear programming and the application of these to problems including critical path analysis and game theory.

Candidates who did well on this paper generally did the following:	**Candidates who did less well on this paper generally did the following:**
• attempted all questions and gave responses that were appropriate for the number of marks available • worked neatly and explained their working where appropriate • answered written responses precisely and unambiguously.	• did not give written responses to questions asking for an explanation • worked in a muddled way and misread their own letters or numerical values • did not read the questions carefully enough.

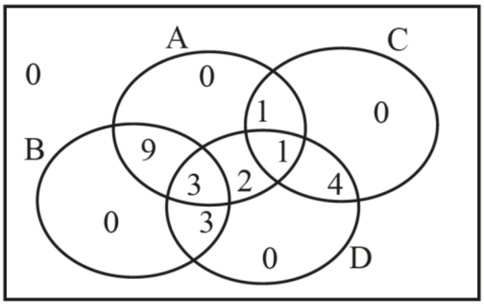
Assessment for learning

Centres should advise candidates that erased work can sometimes still show through on the scanned copy and may lead to ambiguities especially when it has been overwritten or in diagrams.

Question 1 (a)

1 Four children, A, B, C and D, discuss how many of the 23 birthday parties held by their classmates they had gone to. Each party was attended by at least one of the four children.

The results are shown in the Venn diagram below.



(a) Construct a complete graph K_4, with vertices representing the four children and arcs weighted to show the number of parties each **pair** of children went to. **[3]**

Usually done well although a few candidates omitted the arc with zero weight and some weighted the arcs to show the number of parties attended by 'this pair and neither of the others'

Question 1 (b)

(b) State a piece of information about the number of parties the children went to that is shown in the Venn diagram but is not shown in the graph. **[1]**

Most candidates were able to give a valid answer

Question 1 (c)

A fifth child, E, also went to some of the 23 parties shown in the Venn diagram.
Every party that E went to was also attended by at least one of A, B, C and D.

- A was at 8 of these parties, B at 7, C at 5 and D at 8.
- These include 5 parties attended by both A and B, 2 by both A and C, 3 by both A and D, 3 by both B and D and 4 by both C and D.
- These include 1 party attended by A, B and D and 1 party attended by A, C and D.

(c) Use the inclusion-exclusion principle to determine the number of parties that E went to. **[2]**

Several correct solutions and some who mixed the methods to get 23 – 13 = 10. Most candidates showed their working clearly and most did show enough evidence that they had tried to use inclusion-exclusion appropriately for more than two sets, even if there were a small number of errors or omissions.

Question 2 (a)

2 The table below shows the activities involved in a project together with the immediate predecessors and the duration of each activity.

Activity	Immediate predecessors	Duration (minutes)
A	-	4
B	-	1
C	A	2
D	A, B	5
E	D	1
F	B, C	2
G	D, F	5
H	E, F	4

(a) Model the project using an activity network. **[3]**

Many correct networks although some drew scheduling diagrams and a few tried to use activity-on-node. In this specification activity networks are drawn using activity-on-arc.

Some candidates omitted the directions of the arcs and some had several dummy activities to accommodate the precedences, when it could be done using only 4 dummy activities.

OCR support

The use of activity-on-arc is covered in the OCR Delivery guide for Discrete Mathematics section 7.05

Question 2 (b)

(b) Determine the minimum project completion time. **[3]**

Usually done well, some candidates did not show evidence of using a forward pass through their network and some had attempted a forward pass but had at least one arc where the early start time was greater than the early finish time.

Question 2 (c)

(c) Calculate the total float for each non-critical activity. **[3]**

Many correct answers although a few candidates forgot the independent float on activity B.

Question 3 (a)

3 A para relay team of 4 swimmers needs to be chosen from a group of 7 swimmers.

(a) How many ways are there to choose 4 swimmers from 7? **[1]**

Almost always correct

Question 3 (b)

There are no restrictions on how many men and how many women are in the team.
The group of 7 swimmers consists of 5 men and 2 women.

(b) How many ways are there to choose a team with more men than women? **[2]**

Many correct answers, but a few who calculated the number of ways of choosing 3 men and 2 women but then added these and a few who left out the women completely.

Question 3 (c)

The physical impairment of each swimmer is given a score.
The scores for the swimmers are
3 4 4 6 7 8 9

The total score for the team must be 20 or less.

(c) How many different valid teams are possible? **[2]**

Most candidates could find some valid teams but often they forgot that the two scores of 4 corresponded to different swimmers.

Exemplar 1

3(c)	3, 4, 4, 6 = 17	3, 4_1, 6, 7 = 20
	3, 4, 4, 7 = 18	3, 4_2, 6, 7 = 20
	3, 4, 4, 8 = 19	So 6 teams possible
	3, 4, 4, 9 = 20	
	~~4, 4, 6, 7 = 21 – not valid~~	
	~~4 So there are 4 valid teams.~~	

This candidate has distinguished between the two people with a score of 4, when only one of them is involved, by calling them 4_1 and 4_2.

Question 3 (d)

The order of the swimmers in the team is now taken into consideration.

(d) In total, how many different arrangements are there of valid teams? **[1]**

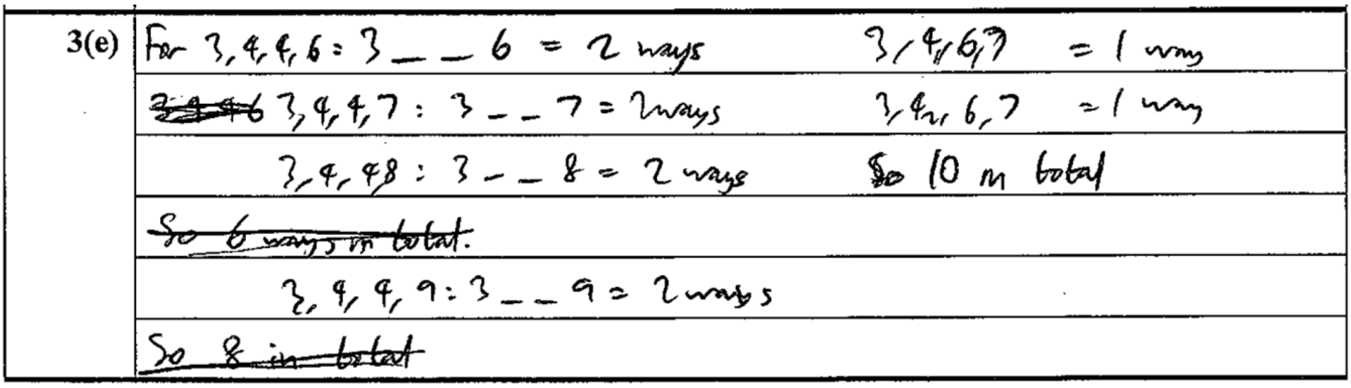
Many candidates realised that each team could be rearranged in 4! ways, some tried to treat the two scores of 4 as being the same swimmer.

Question 3 (e)

(e) In how many of these valid teams are the scores of the swimmers in increasing order?
For example, 3, 4, 4, 8 but not 4, 3, 4, 8. **[2]**

Most candidates recognised here that the two swimmers with a score of 4 could be swapped, although they did not always deal with the 3, 4, 6, 7 cases appropriately.

Exemplar 2

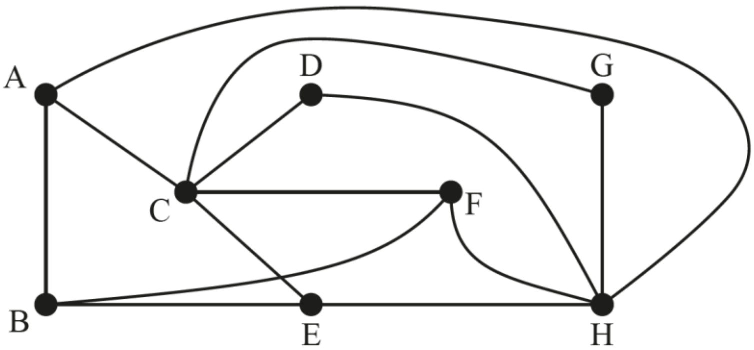


3(e)	For 3, 4, 4, 6 : 3 _ _ 6 = 2 ways	3, 4, 6, 7 = 1 way
	~~3 4 4 6~~ 3, 4, 4, 7 : 3 _ _ 7 = 2 ways	3, 4, 6, 7 = 1 way
	3, 4, 4, 8 : 3 _ _ 8 = 2 ways	So 10 in total
	~~So 6 ways in total.~~	
	3, 4, 4, 9 : 3 _ _ 9 = 2 ways	
	~~So 8 in total~~	

Here the candidate has indicated that the four selections on the left each start with 3 and finish with another score with two gaps between them with 2 ways to fill the gaps with the 4's, and that there is one order for each of the other selections, 3 4_1 6 7 and 3 4_2 6 7.

Question 4 (a)

4 A connected graph is shown below.



(a) Write down a path through exactly 7 of the vertices. **[1]**

A valid path through exactly 7 vertices was usually found. A very small number of candidates repeated a vertex.

Assessment for learning

The meanings of the terms walk, trail, path, cycle and route are given in the specification (item 7.02c).

Question 4 (b)

(b) Write down a cycle through exactly 6 of the vertices. **[1]**

A few candidates did not close their cycle or had repeated vertices within the cycle.

Question 4 (c)

(c) Explain why Ore's theorem cannot be used to decide whether or not this graph is Hamiltonian. **[2]**

Some candidates did not know what Ore's theorem says, and several confused 'Hamiltonian' with 'Eulerian'. Some candidates stated that Ore's theorem is necessary but not sufficient to show that a graph is Hamiltonian but did not demonstrate that for this particular graph the conditions were not satisfied (and hence Ore's theorem was inconclusive).

Some candidates thought that the requirement in the statement of Ore's theorem that $n > 3$ referred to the vertex degrees rather than the number of vertices, and some thought that they needed a planar graph (although this may have been due to confusion with Euler's formula).

Assessment for learning

Candidates need to know the definitions and results used with graphs.

A Hamiltonian graph contains a cycle that passes through every node exactly once with no repeats.

Ore's theorem is given in the specification (item 7.02i).

Question 4 (d)

(d) Prove that the graph is not Hamiltonian. **[2]**

Many correct answers, apart from the candidates who thought that Hamiltonian meant either 'Eulerian' or 'planar'.

Assessment for learning

When a question asks candidates to prove a result they must give a reasoned argument, a specific case is not enough.

Question 4 (e)

The following colouring algorithm can be used to determine whether a connected graph is bipartite or not. The algorithm colours each vertex of a graph in one of two colours, ① and ②.

STEP 1 Choose a vertex and assign it colour ①.
STEP 2 If any vertex is adjacent to another vertex of the same colour, stop.
Otherwise assign colour ② to each vertex that is adjacent to a vertex with colour ①.
STEP 3 If any vertex is adjacent to another vertex of the same colour, stop.
Otherwise assign colour ① to each vertex that is adjacent to a vertex with colour ②.
STEP 4 Repeat STEP 2 and STEP 3 until all vertices are coloured.
STEP 5 If there are no adjacent vertices of the same colour then the graph is bipartite, output the word "bipartite".
Otherwise the graph is not bipartite, output the words "not bipartite".

(e) Use this algorithm, starting at vertex A, to determine whether the graph is bipartite, or not. **[2]**

The 'colours' ① and ② were correctly assigned by most candidates. Those who used other notation needed to give a key to explain what represented ① and what represented ②. The use of actual colours does not show up on the scanned scripts and should be discouraged.

Assessment for learning

Candidates need to know that the use of coloured pens or pencil does not show up properly on the scanned scripts.

Question 4 (f)

(f) Explain what Kuratowski's theorem tells you about the graph. **[3]**

Most candidates knew what Kuratowski's theorem says. Many were able to identify that this graph contains $K_{3,3}$ as a subgraph (although sometimes the terms subgraph, subdivision, contraction were misused, such as claiming that the graph in the question was a subgraph of $K_{3,3}$). Fewer candidates were able to unambiguously give the two sets {A, E, F}, {B, C, H} that defined the subgraph (with no contractions of vertices required).

Some candidates said that the given graph was $K_{5,\,3}$. Although it was a bipartite graph with 5 vertices in one set and 3 in the other, it was not the complete bipartite graph on these sets.

Question 4 (g)

(g) Show that the graph has thickness 2. **[2]**

Most candidates were able to draw the graph using two layers, fewer remembered to say that the graph is non-planar so one layer is not sufficient. A small number of candidates described the graph as being $K_{3,3}$ with extra arcs to attach D and G, but then either claimed that $K_{3,3}$ as one layer or quoted that $K_{3,3}$ has thickness 2 without any explanation of why. To show that a graph has thickness 2 candidates need to show that the thickness is both greater than 1 and less than or equal to 2.

Question 5 (a)

5 In each turn of a game between two players they simultaneously each choose a strategy and then calculate the points won using the table below. They are each trying to maximise the number of points that they win.

In each cell the first value is the number of points won by player 1 and the second value is the number of points won by player 2.

		Player 2		
		X	Y	Z
Player 1	A	(6, 0)	(1, 7)	(5, 6)
	B	(9, 4)	(2, 6)	(8, 1)
	C	(6, 8)	(1, 3)	(7, 2)

(a) Find the play-safe strategy for each player. **[3]**

Some candidates confused which players the values referred to and some tried to find column maxima. Some working needed to be seen, for example the minimum pay-off for player 1 in each row and the minimum pay-off for player 2 in each column.

Assessment for learning

When a question asks candidates to 'find' a result (rather than just 'write down' the result) they must show some working.

Question 5 (b)

(b) Explain why player 2 would never choose strategy Z. **[2]**

Most candidates identified that strategy Y dominated (or strictly dominated) strategy Z although the explanations were sometimes rather minimal. Ideally candidates would explicitly show the three inequalities 7 > 6, 6 > 1 and 3 > 2 (and no others).

Exemplar 3

5(b)	7 > 6, 6 > 1, 3 > 2 ∴ strategy Y dominates strategy Z for player 2, so Z should never be played.

This candidate has shown the three relevant inequalities to demonstrate that Y is dominant over Z and concluded that Z should never be played.

Question 5 (c)

(c) Find the Nash equilibrium solution(s) or show that there is no Nash equilibrium solution. **[2]**

Several good solutions, although some candidates used repeated dominance to reduce to the cell (B, Y), missing the fact that if player 1 chose strategy C then player 2 would do best by playing strategy X.

The easiest way to demonstrate the comparisons for finding Nash equilibrium solutions is to write down the best strategy for player 2 for each of player 1's choices (= Y, Y and X for rows A, B and C, respectively) and the best strategy for player 1 for each of player 2's choices (= B for each column).

Question 5 (d) (i)

Player 2 chooses strategy X with probability p and strategy Y with probability $1-p$.

You are given that when player 1 chooses strategy A, the expected number of points won by each player is the same.

(d) **(i)** Calculate the value of p. **[3]**

Some candidates used the differences between the gains for the two players, which often led to muddled reasoning. Many candidates used the expected number of points for player 1 using each strategy (A, B and C) rather than the expected number of points for player 1 and for player 2 when player 1 chooses A.

Question 5 (d) (ii)

(ii) Determine which player expects to win the greater number of points when player 1 chooses strategy B. **[2]**

Some candidates used their value of *p* appropriately although some only substituted into the expression for player 1 (ignoring player 2) and some substituted into the expressions for row A or row C. There were also far too many candidates who made numerical errors in their calculations.

Question 6 (a) (i)

6 A linear programming problem is

$$\begin{aligned} &\text{Maximise } P = 2x - y \\ &\text{subject to} \quad 3x + y - 4z \leqslant 24 \\ &\qquad\qquad\quad 5x - 3z \leqslant 60 \\ &\qquad\qquad\quad -x + 2y + 3z \leqslant 12 \\ &\text{and } x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0 \end{aligned}$$

(a) **(i)** Represent this problem as an initial simplex tableau. **[2]**

Usually correct although some candidates did not deal with the signs in the objective row and a few lost signs elsewhere. A small minority put the -3 from the second constraint in the *y* column.

Question 6 (a) (ii)

(ii) Carry out **one** iteration of the simplex algorithm. **[3]**

Many correct answers. Some candidates had numerical slips in one or more of the cells and a few had lost the structure of the table, in particular the basis columns.

Assessment for learning

When performing an iteration of the simplex algorithm it is usually easier to use fractions rather than decimals, unless the decimals are exact to, say, 3 s.f.

Rounded decimals can generate errors in subsequent iterations.

Question 6 (b) (i)

After two iterations the resulting tableau is

P	x	y	z	s	t	u	RHS
1	0	$\frac{5}{11}$	0	$-\frac{6}{11}$	$\frac{8}{11}$	0	$30\frac{6}{11}$
0	1	$-\frac{3}{11}$	0	$-\frac{3}{11}$	$\frac{4}{11}$	0	$15\frac{3}{11}$
0	0	$-\frac{5}{11}$	1	$-\frac{5}{11}$	$\frac{3}{11}$	0	$5\frac{5}{11}$
0	0	$\frac{34}{11}$	0	$\frac{12}{11}$	$-\frac{5}{11}$	1	$10\frac{10}{11}$

(b) (i) Write down the basic variables after two iterations. **[1]**

Several candidates left this part out, those who did attempt it sometimes listed the values of all the variables (or of *P*, *x*, *y* and *z*) and those who realised that *P*, *x* and *z* were basic (but not *y*) sometimes did not write *u* as well.

Question 6 (b) (ii)

(ii) Write down the exact values of the basic feasible solution for x, y and z after two iterations. **[1]**

Often correct, although some candidates claimed a non-zero value for *y* and some claimed that *y* could not be found.

Structure of a simplex tableau

Each tableau should be a shorthand for an identity matrix (the basis columns) and another matrix (the non-basis columns) together with a vector of non-negative values (the RHS column).

Question 6 (b) (iii)

(iii) State what you can deduce about the optimal value of the objective for the original problem. **[2]**

From the given tableau the current value of *P* could be seen (although some candidates used their values of *x* and *y* to find *P*), however fewer realised that further iterations could be carried out (since there is still a negative value in the objective row) so this was not necessarily the optimum value (and in fact, since there are no zero values in the RHS column it was not the optimum).

Some candidates said that the optimum could not be (or was not) integer-valued. Although the question did not require further iterations, some candidates appeared to have calculated the optimum value for *P* (which was 36 when $x = 18$, $y = 0$, $z = 10$, $s = 10$, $t = 0$, $u = 0$).

Question 6 (c)

You are now given that, in addition to the constraints above, $x+y+z=9$.

(c) Use the additional constraint to rewrite the original constraints in terms of x and y but not z. **[4]**

Most candidates correctly substituted for *z* in the first three constraints, very few also substituted for *z* in the constraint $z \geq 0$.

Question 6 (d)

(d) Explain why the simplex algorithm cannot be applied to this new problem without some modification. **[2]**

The issue here was the third constraint which should now either have looked like $-4x - y \leq -15$, which is not in the standard form because $-15 < 0$ (and hence the origin is not in the feasible region), or have looked like $4x + y \geq 15$, which would require modification before a slack variable could be used and then would have become $-4x - y \leq -15$ giving the same issue as before regarding -15 being negative.

Some candidates did not identify which constraint was causing the problem and some worried about the number of constraints or introduced the non-negativity of *z* at this point.

Question 7 (a)

7 A building has 7 CCTV cameras, A to G, at the junctions of some of the corridors.

The cameras at the junctions and the lengths of the 11 corridors between them, in metres, are shown in the table below.

	A	B	C	D	E	F	G
A		64	60	111			
B	64			72	103		
C	60			66		58	
D	111	72	66		32	127	
E		103		32			82
F			58	127			75
G					82	75	

(a) Model this information as a network. **[1]**

Most candidates were able to construct the network without errors.

Question 7 (b)

(b) Use an appropriate algorithm to calculate the minimum distance from A to each of the other vertices. **[3]**

Several candidates filled in the table of minimum distances but gave no evidence of the use of Dijkstra's algorithm. The specification says that candidates should be able to choose an appropriate algorithm to solve a practical problem (item 7.04f) so we need to see the use of the algorithm in the working. Dijkstra's algorithm should be carried out as described in the formula booklet.

Question 7 (c)

The run-time of an algorithm for finding this minimum distance is proportional to the total number of **comparisons** used. For a network with n vertices, the worst case is when the algorithm is applied to a network based on the complete graph K_n.

In each pass

- A vertex is made permanent and the temporary label at all adjacent vertices that are not yet permanent are updated. This uses 1 comparison for every such vertex (adjacent to the permanent label) that previously already had a temporary label.
- The best temporary labels at all vertices that do not yet have permanent labels are then compared to determine the next vertex to become permanent. If there are k such vertices this involves $k-1$ comparisons.

(c) By considering the number of comparisons of each type in each iteration, show that the algorithm uses a total of 6 comparisons when it is applied to a network based on the complete graph K_4. **[2]**

Most candidates achieved the value 6 but not always correctly. Initially there is one permanent label but no temporary labels so in the first pass there are no comparisons to find temporary labels, but then the temporary labels are filled in and compared giving 2 comparisons to find the next permanent label. In the second pass there are 2 comparisons to update the temporary labels and 1 comparison to find the next permanent label. In the third, and final, pass there is 1 comparison to update the last temporary label and 0 comparisons to decide that this must become permanent.

Many candidates essentially counted the number of 'calculations' at each vertex in the style of applying Prim's algorithm to K_4, without recognising that the initial temporary labels involved no calculations. Some candidates quoted the formula given in the stem to part (d) instead of considering the number of comparisons of each type in each iteration.

Question 7 (d)

You are given that the total number of comparisons used when the algorithm is applied to a network based on K_n is $(n-1)(n-2)$.

A computer takes 0.03 seconds to apply this algorithm on a network based on K_7.

(d) Calculate, to **1** decimal place, how many seconds it will take the computer to apply the algorithm to a network based on K_{70}. **[2]**

Several candidates calculated the time of 4.7 seconds, although some calculated (70-1)(70-2)/(7-1)(7-2) and then squared it before scaling the time.

Unusually, the question asked for the actual time taken to apply the algorithm rather than an estimation of the approximate run-time. Some candidates assumed that an estimate was required and used the fact that the algorithm has quadratic order to calculate a time of 3 seconds.

Assessment for learning

The answer space included the word 'seconds' to try to avoid candidates writing 4.7s, which can easily look like 4.75. In general, if the units are not given in the answer space candidates should be encouraged to use 'sec' or 'seconds' rather than 's'.

Question 7 (e) (i)

The manager wants to construct a tour (a closed route) that passes each camera.

(e) **(i)** Find a lower bound for the length of this tour by initially deleting D. **[2]**

Most candidates deleted vertex D but then some found a tour for the reduced network (a closed route through every vertex but which may repeat vertices) instead of finding a lower bound by constructing a minimum spanning tree for the reduced network and adding on the two (different) least weight arcs to join D to the tree.

Question 7 (e) (ii)

(ii) Find an upper bound for the length of this tour by using the nearest neighbour algorithm starting from D. **[1]**

To show the use of the nearest neighbour algorithm candidates should list the vertices in the order in which they were visited, the length of the resulting tour then gives an upper bound.

Question 7 (e) (iii)

(iii) Deduce the length of the shortest possible tour. Briefly explain your reasoning. **[1]**

The lower bound was not a tour, to include D in a tour two of the arcs connected to D must be used. Replacing DC by BD in the lower bound gave the least increase and gave the tour that had already been found as the upper bound. Using any other arcs could not give a shorter tour, so the upper bound gave the shortest tour which had length 443 metres.

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