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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y542/01 Summer 2022 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper Y542/01 series overview

The general standard of work on this paper was pleasingly high and many good marks were seen. Most candidates seemed well prepared, and many lessons from previous papers have been learnt. There are still issues with stating hypotheses and conclusions, and the Central Limit Theorem remains widely misunderstood, but this was the best-answered Y542 paper since the new specification was introduced.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• stated hypotheses in terms of the underlying population • in dealing with combinations of normally distributed random variables, they explained their notation and showed clearly the mean and variance of each distribution they considered • stated conclusions to hypotheses tests appropriately, particularly when not rejecting the null hypotheses, for example "there is insufficient evidence that the model is not true" • showed clearly that they realised that the Central Limit Theorem refers to two different distributions, the underlying ("parent") distribution and the distribution of the sample mean.	• gave vague statements of hypotheses which could have referred to either the population or the sample • did not make their notation clear. They indicated some vague combination of variables (often writing 3X instead of $X_1 + X_2 + X_3$) and did not show clearly how they obtained the means and variances • stated conclusions to hypothesis tests too assertively, especially when not rejecting the null hypothesis, for example, "there is significant evidence that the model is correct" • demonstrated that their understanding of the Central Limit Theorem was unclear, often implying that the underlying distribution could be changed provided you took a large sample.

Misconception

?

Hypotheses describe the actual state of the underlying population, and not the sample. Thus hypotheses involving a correlation coefficient shoud be stated in the form 'H_0: $\rho = 0$, H_1: $\rho > 0$, where ρ is the population correlation coefficient between temperature and the number of computer failures', or equivalent. Use of the symbol ρ should help candidates to distinguish between the population parameter ρ and the sample coefficient r.

It is wrong to say 'H_0: there is evidence that ...'. The hypotheses concern what is actually the case, not whether we have evidence that it is the case. Words such as "evidence" belong in the conclusion, not in the hypotheses.

Hypotheses for goodness-of-fit tests should be stated in a form such as 'H_0: the population of absences is in the ratio 3:2:2:2:3', or equivalent. It is wrong to say 'H_0: the sample is in the ratio 3:2:2:2:3', etc., as we are trying to make inferences about the population (and in any case the sample obviously is not in this ratio but in the ratio 28:16:24:16:36). It is also wrong to say 'H_0: the data are consistent with the ratio 3:2:2:2:3'; the hypotheses do not concern the sample but the population, and such words as "consistent" should be used only in the conclusions.

Question 1 (a) (ii)

(ii) Find $P(50 < R \leqslant 150)$. **[2]**

There were the predictable confusions between 49, 50 and 51, or 149, 150 and 151, but otherwise this was usually correct. A few candidates tried to use 0.01 as well as 0.99.

Question 1 (b)

The researcher incorrectly believes that the variance of a random variable X with **any** discrete probability distribution is given by the formula $[\mathrm{E}(X)]^2 - \mathrm{E}(X)$.

(b) Show that, for the type of distribution stated in part **(a)**, they will obtain the correct value of the variance, regardless of the value(s) of the parameter(s). **[2]**

The question explicitly said "regardless of the value(s) of the parameter(s)", so it was surprising that a substantial number of candidates answered the question purely numerically, substituting 0.01 into the two variance formulae. Most who used the algebraic formulae were successful.

Question 2 (a)

2 The directors of a large company believe that there are more computer failures in the Head Office when temperatures are higher. They obtain data for the Head Office for the maximum temperature, T°C, and the number of computer failures, X, on each of 12 randomly chosen days.

(a) State which of the following words can be applied to T.

Dependent Independent Controlled Response **[1]**

A majority, but not a large one, knew that the only correct word was "independent".

Question 2 (c) (i)

(c) The directors wish to investigate their belief using a significance test at the 1% level.

(i) Explain why a 1-tail test is appropriate in this situation. **[1]**

Successful answers were written in the context of the question, for example "the directors are concerned whether computer failures increase with higher temperature, not just with higher or lower temperatures".

Question 2 (c) (ii)

(ii) Carry out the test. **[6]**

A large number of candidates stated their hypotheses in terms of the population parameter ρ, but, as has been made clear on specimen papers, they should state that this was the population parameter (as opposed to the sample value), as well as mentioning the context. Successful responses included, for example, "where ρ is the population correpation coefficient between temperature and number of failures." Those who used only verbal statements could not score full marks. Comparison and conclusions were usually correct, although some did not give their conclusion in context.

Question 3

3 **In this question you must show detailed reasoning.**

A discrete random variable V has the following probability distribution, where p and q are constants.

v	0	1	2	3
$\mathrm{P}(V = v)$	p	q	0.12	0.2

It is given that $\mathrm{E}(V) = \mathrm{Var}(V)$.

Determine the value of p and the value of q. **[8]**

This question was often very well done, and was a good source of marks for many. Only a few used $\mathrm{Var}(V) = \mathrm{E}(V^2)$ (which leads to a contradiction) or $\mathrm{Var}(V) = \mathrm{E}(V^2) - \mathrm{E}(V)$. Those who tried to use $\mathrm{Var}(V) = \mathrm{E}[V - \mathrm{E}(V)]^2$ found the algebra very much harder.

Most candidates who obtained two solutions to a quadratic equation such as $q^2 + 1.68q - 0.7344 = 0$ correctly gave a reason for rejecting the solution –2.04, such as "q must be positive".

A few candidates assumed that "expectation = variance" implies that the distribution must be Poisson, but were naturally unable to make any progress. Obviously the implication is the other way round, so it cannot be used here.

Question 4 (a)

4 The manager of a car breakdown service uses the distribution Po(2.7) to model the number of punctures, R, in a 24-hour period in a given rural area. The manager knows that, for this model to be valid, punctures must occur randomly and independently of one another.

(a) State a further assumption needed for the Poisson model to be valid. **[1]**

As always, the second modelling assumption for the Poisson distribution was poorly stated. The insistence on 'constant <u>average</u> rate' might appear pedantic, but there are always candidates whose answers show that they think that events have to occur to an exactly regular pattern. For example, one candidate wrote, "The number of punctures that occur in a 24-hour period must be constant".

It has been mentioned in many previous reports that "singly" is usually not an appropriate modelling assumption. Candidates need to show discrimination in applying standard lists of modelling assumptions, and here "singly" is either meaningless or is part of the "independence" condition. What would it mean for punctures *not* to occur singly? If a car ran over a set of nails and several tyres were punctured, all that would mean is that they were not independent.

Question 4 (c)

(c) Use the model to calculate the probability that, in a randomly chosen period of 168 hours, at least 22 punctures occur. **[3]**

Almost all correctly used Po(18.9) and most obtained the correct answer, though there were naturally many instances of "P(≥ 22) = 1 – P(≤ 22)".

Question 4 (d)

The manager uses the distribution Po(0.8) to model the number of flat batteries in a 24-hour period in the same rural area, and he assumes that instances of flat batteries are independent of punctures. A day begins and ends at midnight, and a "bad" day is a day on which there are more than 6 instances, in total, of punctures and flat batteries.

(d) Assume first that both the manager's models are correct.

Calculate the probability that a randomly chosen day is a "bad" day. **[2]**

Most correctly obtained 0.0653, though a few thought that 0.065 was an answer correct to 3 significant figures.

Question 4 (e)

(e) It is found that 12 of the next 100 days are "bad" days.

Comment on whether this casts doubt on the validity of the manager's models. **[2]**

In assessing outcomes such as this, candidates should first make a clear comparison. The natural comparisons were 0.12 with 0.0653, or 12 with 6.53. Quite a lot of candidates approached the question as if it were a hypothesis test, by calculating P(≥ 12) from B(100, 0.0653). (Those who calculated P(= 12) did not gain full credit.) Those who used 0.142 from the previous part could score full marks, though the corresponding "hypothesis test" calculation would have to be P(≤ 12) as 12 is now less than the expected value. Treating the question as if it were a hypothesis test is not wrong, but candidates should be encouraged to use quick informal comparisons where appropriate, in real life as well as in exams.

Candidates had to say whether doubt was cast on the models used, and this should be done without being too definite. "It casts doubt" is fine, but "the model is not valid" did not score the final mark.

Question 5 (a)

5 A company uses two drivers for deliveries.

Driver A charges a fixed rate of £80 per day plus £2 per mile travelled on that day.

Driver B charges a fixed rate of £120 per day plus £1.50 per mile travelled on that day.

On each working day the total distance, in miles, travelled by each driver is a random variable with the distribution N(83, 360).

(a) Find the probability that driver A charges the company less than £235.00 for a randomly chosen day's deliveries. **[4]**

Many found this easy, often by working out that if A charges less than £235, then $X < 77.5$.

Question 5 (b)

(b) Find the probability that the total charge to the company of three randomly chosen days' deliveries by driver A is at least £300 more than the total charge of two randomly chosen days' deliveries by driver B. **[6]**

It was often very hard to see what candidates were doing in this question, through a combination of insufficient working and unexplained notation. To give themselves the maximum chance of obtaining marks if anything went wrong, it would have been wise to say things like

"Using charges A_1, A_2, A_3, B_1, B_2,

$A_1 = 80 + 2X_1 \sim N(246, 1440)$ and $B_1 = 120 + 1.5X_4 \sim N(244.5, 810)$", or

"Using distances X_1, X_2, X_3, X_4, X_5,

$X_1 + X_2 + X_3 \sim N(249, 1080)$ and $X_4 + X_5 \sim N(166, 720)$".

Some candidates were unable to take account of the two issues of adding different days and multiplying by constant factors 2 and 1.5. Naturally many treated $X_1 + X_2 + X_3$ as $3X$ and thereby obtained the wrong variances. A few assumed that the distances travelled by each driver were the same on each day, which is not only implausible but ignores the statement that the days are randomly chosen.

Exemplar 1

5(b)	$\cancel{3\times 83 = 249}$ $\cancel{2\times 83 = 186}$
	$3\times 83 - 2\times 83 = 83$
	$9\times 360 + 4\times 360 = 4680$
	$\cancel{X\sim N(}$
	$\cancel{80+}$
	$A: \cancel{X\sim} N(\cancel{160+160}, 3240)$
	$B: \cancel{X\sim N(}$
	$\cancel{A: X\sim N}$ $A\sim N(738, 3240)$
	$B\sim N(489, 1440)$
	$(A-B)\sim N(249, 4680)$
	$P(A-B \geq 300) = 0.228$
	$\underline{\underline{0.228}}$

Although this candidate has indicated intermediate means and variances, it is not clear what A and B stand for, nor how the factors of 2 and 1.5 have been used. All that the examiner can do is to mark on the basis of what has been written down, and anything that is incorrect can get no credit if there is insufficient explanation.

Question 6 (a)

6 The random variable X was assumed to have a normal distribution with mean μ. Using a random sample of size 128, a significance test was carried out using the following hypotheses.

H_0: $\mu = 30$

H_1: $\mu > 30$

It was found that $\sum x = 3929.6$ and $\sum x^2 = 123\,483.52$. The conclusion of the test was to reject the null hypothesis.

(a) Determine the range of possible values of the significance level of the test. **[5]**

This was a variation on the usual hypothesis test question and more candidates than usual made the usual mistakes: not multiplying the sample variance by 128/127, not dividing the variance by 128 to get the variance of the sample mean, or, sometimes, obtaining the sample variance by subtracting the square of the population mean, rather than the square of the sample mean.

The final answer had to be given as an inequality. Most candidates knew that the significance level had to be greater than their calculated result.

Question 6 (b)

(b) It was subsequently found that X was not normally distributed.

Explain whether this invalidates the conclusion of the test. **[2]**

This question revealed widespread misunderstanding of the Central Limit Theorem. A good answer was "No, because *n* is large, so we can use the central limit theorem to say that the sample mean is approximately normally distributed, even if the population is not normally distributed."

Misconception

The Central Limit Theorem (CLT) is widely misunderstood. Candidates should understand that the CLT relates two different distributions: the distribution of the population as a whole (the 'parent distribution', typically the distribution of X), and the distribution of the sample mean $\bar{X}$. It therefore relates two different random variables, X and $\bar{X}$, which may well have differently shaped distributions.

A correct statement is: the sample mean $\bar{X}$ is approximately normally distributed if the sample is sufficiently large (typically $n > 25$), regardless of the distribution of X. Candidates should make it clear in their answers that it is the sample mean that can be assumed to have a normal distribution. Here are some common misconceptions:

- It is **not** a statement about parameter values. The mean and variance of $\bar{X}$ are μ and $\frac{\sigma^2}{n}$ for any distribution, and this has nothing to do with the CLT.
- It does **not** say that the parent distribution can be assumed to be normal for a large sample. The parent distribution is fixed; it cannot change into a normal distribution just because a lot of observations are obtained from it. If you take 500 observations from a uniform distribution and plot the results in a bar chart, the heights of the bars will be roughly the same, and certainly not bell-shaped. Perhaps candidates confuse the situation with the next bullet point.
- It does **not** say that, if the parent distribution is normally distributed, then the sample mean is normally distributed. That is a different theorem (some teachers call it the "not-the-Central-Limit-Theorem theorem"!). It is therefore incorrect to say that the CLT allows you to assume that the parent population is normally distributed.

Here are some quotations from candidates' scripts that exemplify these misconceptions.

"You can apply the central limit theorem as the variance will be divided by 128."

"As n is large, the parent population is normally distributed."

"All distributions are approximately normal."

"For such a large data set it can be assumed that the data is normally distributed." ("Data" sounds like the 128 results, not the sample mean; this candidate has made no distinction between X and $\bar{X}$.)

"If it isn't normal, we can't use the central limit theorem."

"The central limit theorem only works with normal distributions."

Exemplar 2

6(b)	It does not invalidate the result as the central limit theorem states that a sample mean of any distribution is normally distributed

This candidate has not referred to the size of the sample, which is all-important.

Question 7

7 The continuous random variable X has probability density function

$$f(x) = \begin{cases} kx^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant and n is a parameter whose value is positive.

It is given that the median of X is 0.8816 correct to 4 decimal places.

Ten independent observations of X are obtained.

Find the expected number of observations that are less than 0.8. **[8]**

An unstructured question. Some candidates did not realise that they had to use the fact that the total probability was 1 in order to obtain an equation connecting n and k. Some who correctly integrated the probability density function (PDF) then substituted 0.8816 and 0.5 the wrong way round, but many candidates correctly obtained $n = 4.5$, and it was pleasing to see that most used 4.5 and not a number such as 4.5004. (0.8816 is almost certainly rounded, so more than 4 sf for n makes no sense.)

Many candidates could not resist calculating E(X) by multiplying the PDF by x and integrating. Such candidates were obviously pre-programmed on standard questions that require E(X) and E(X^2). Those who used their n to integrate their PDF between 0 and 0.8 almost always obtained the right answer.

Question 8 (a)

8 The critical region for an r% two-tailed Wilcoxon signed-rank test, based on a large sample of size n, is $\{W_+ \leqslant 113\} \cup \{W_+ \geqslant 415\}$.

(a) Show that $n = 32$. **[3]**

It was expected that most candidates would use symmetry to calculate the mean of the distribution, and then equate it to $\frac{1}{4}n(n+1)$. In fact most added 113 and 415 to obtain the total rank and equated it to $\frac{1}{2}n(n+1)$, no doubt inspired by a previous year's question which involved the total sum of the ranks. To obtain full marks in this "show that" question, it was necessary to obtain the equation $n^2 + n - 1056 = 0$, or equivalent, and then to solve this. Merely substituting $n = 32$ does not establish that n is 32 <u>and nothing else</u>.

Question 8 (b)

(b) Using a suitable approximation, determine the value of r. **[4]**

This was generally well done, although omission of the continuity correction was common, and many candidates did not multiply P(< 113.5) by 2 (or add P(< 113.5) to P(> 414.5), which is of course the same). Candidates should be aware that significance levels need not be restricted to the usual ones of 10%, 5%, 1%, 0.5%, etc.

Question 9 (a)

9 The head teacher of a school believes that, on average, pupil absences on the days Monday, Tuesday, Wednesday, Thursday and Friday are in the ratio 3 : 2 : 2 : 2 : 3. The head teacher takes a random sample of 120 pupil absences. The results are as follows.

Day of week	Monday	Tuesday	Wednesday	Thursday	Friday
Number of absences	28	16	24	16	36

(a) Test at the 5% significance level whether these results are consistent with the head teacher's belief. **[7]**

This was a relatively routine question and was largely well done. The calculation was usually correct. There were three main errors. Hypotheses were often stated poorly or even wrongly (see the Misconceptions box), although on this occasion some latitude was given. Some used the wrong number of degrees of freedom, or the critical value from the 0.05 column instead of the 0.95 column. And the final conclusion was often wrongly stated.

Not rejecting H_0 indicates merely that the proposed model for the underlying distribution cannot be rejected; no amount of evidence from a hypothesis test can show that the null hypothesis is correct. A double negative is needed, even though it sounds clumsy: "there is insufficient evidence to show that the absences are not distributed in the ratio 3:2:2:2:3", as written by many candidates.

Exemplar 3

9(a)

Ho: The distribution follows the ratio 3:2:2:2:3

H_1: The distribution does not follow the ratio 3:2:2:2:3

$120 \div 12 = 10$

Expected Values: 30 20 20 20 30

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(28-30)^2}{30} + \frac{(16-20)^2}{30} + \ldots$$

$$\chi^2 = \frac{44}{15} \qquad df = 5-1 = 4$$

critical value = 9.488

as $\cancel{\chi^2}$ $\frac{44}{15} < 9.488$

There is insufficient evidence to suggest to reject Ho and hence the distribution does follow a 3:2:2:2:3 ratio.

This candidate has stated the hypotheses acceptably, making it clear that the ratio applies to the underlying distribution and not to the sample. However, it would have been better to refer to the "population of absences".

On the other hand, this candidate has wrongly stated the conclusion of the test. One cannot infer that the population distribution is correct; all one can say is that "there is no significant evidence that the distribution does not follow the ratio 3:2:2:2:3".

Question 9 (b)

A significance test at the 5% level is also carried out on a second, independent, random sample of n pupil absences. All the numbers of absences are integers. The ratio of the numbers of absences for each day in this sample is identical to the ratio of the numbers of absences for each day in the original sample of size 120.

(b) Determine the smallest value of n for which the conclusion of this significance test is that the data are not consistent with the head teacher's belief. **[3]**

A testing final question. The easiest way to answer it was to know, or realise, that the test statistic is proportional to *n*. All that was then needed was the inequality $3.733 \times (n/120) > 9.488$, which leads at once to $n > 304.97$. Most candidates recalculated the test statistic using expressions such as $\frac{\left(\frac{7n}{30}-\frac{3n}{12}\right)^2}{\frac{3n}{12}} + 2\times\frac{\left(\frac{4n}{30}-\frac{2n}{12}\right)^2}{\frac{2n}{12}} + \frac{\left(\frac{6n}{30}-\frac{2n}{12}\right)^2}{\frac{2n}{12}} + \frac{\left(\frac{9n}{30}-\frac{3n}{12}\right)^2}{\frac{3n}{12}} > 9.488$, which led to the same inequality after a lot more work. A few tried to use trial and improvement but were usually unsuccessful. To gain credit such candidates needed to show the value of the test statistic for each value of *n* that they used.

Only a few candidates appreciated that *n* had to be a multiple of 30, otherwise the observed frequencies could not be integers. The smallest possible value of *n* is therefore 330.

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