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AS LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H635

For first teaching in 2017

Y410/01 Summer 2022 series

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance plus where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper Y410/01 series overview

Y410/01 assesses the pure mathematics content of AS Level Further Mathematics B (MEI). Content includes complex numbers, matrices, solution of polynomial equations, proof (including proof by induction) and series, all at AS Level standard.

There were many excellent scripts seen this summer, with well over half of candidates scoring over 40 marks out of 60 and only a small minority scoring below 20. The mean score for the paper was 41 marks.

Most of the questions proved to be accessible to the majority of candidates, although Question 8(b) was challenging, even for otherwise high-scoring students.

There was little evidence that students lacked time to complete the paper, although a few scripts did not offer responses to Question 8.

The standard of presentation was generally high, with most scripts showing an appreciation of how to present mathematical arguments.

Candidates who did well on this paper generally did the following:	**Candidates who did less well on this paper generally did the following:**
• Understood complex number theory and its application to polynomial equations. • Were able to apply matrices to solve simultaneous equations and to transformations. • Used algebra accurately. • Used the language of proof correctly.	• Did not have a clear understanding of the key concepts of complex numbers. • Showed weaknesses in algebra. • Did not fully understand the application of matrices. • Were unable to present mathematical arguments fluently.

Question 1 (a) (i)

1 **(a)** **(i)** Write the following simultaneous equations as a matrix equation.

$$\begin{aligned} x+\ y+2z &= 7 \\ 2x-4y-3z &= -5 \\ -5x+3y+5z &= 13 \end{aligned}$$

[1]

This was an easy starter question with almost all candidates receiving the mark. A few just wrote the matrix of coefficients.

Question 1 (a) (ii)

(ii) Hence solve the equations. **[2]**

Most candidates used their calculator in matrix mode to gain these two marks, with once again almost all candidates picking up full marks.

Question 1 (b)

(b) Determine the set of values of the constant k for which the matrix equation

$$\begin{pmatrix} k+1 & 1 \\ 2 & k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 23 \\ -17 \end{pmatrix}$$

has a unique solution. **[3]**

Half the candidates gained full marks here by calculating the determinant, finding its roots and correctly stating there is a unique solution except for these values. Many others identified 1 and -2 but lost the final mark for not stating $k \neq 1$ and $k \neq -2$.

Question 2 (a)

2 **(a)** Show that the vector $\mathbf{i}+4\mathbf{j}+2\mathbf{k}$ is parallel to the plane $2x+y-3z=10$. **[3]**

Many candidates gained the first two marks for showing the scalar product of the vector and normal is zero. However, some lost the final mark for failing to explain adequately why this showed the vector is parallel to the plane.

Question 2 (b)

(b) Determine the acute angle between the planes $2x+y-3z=10$ and $x-y-3z=3$. **[4]**

This question was extremely well answered, with almost all candidates picking up the marks.

Question 3

3 The complex number z satisfies the equation $5(z-\mathrm{i})=(-1+2\mathrm{i})z^*$.

Determine z, giving your answer in the form $a+b\mathrm{i}$, where a and b are real. **[5]**

Most candidates gained a mark for identifying the complex conjugate as *a* - *b*i (or equivalent), expanding the brackets correctly and equating real and imaginary parts. Occasionally the final mark was lost through algebraic slips. A few candidates rearranged the equation as shown in the alternative solution given in the mark scheme. However, the crucial step is to substitute $z = a + b\mathrm{i}$ in order to separate out real and imaginary parts.

Question 4

4 **In this question you must show detailed reasoning.**

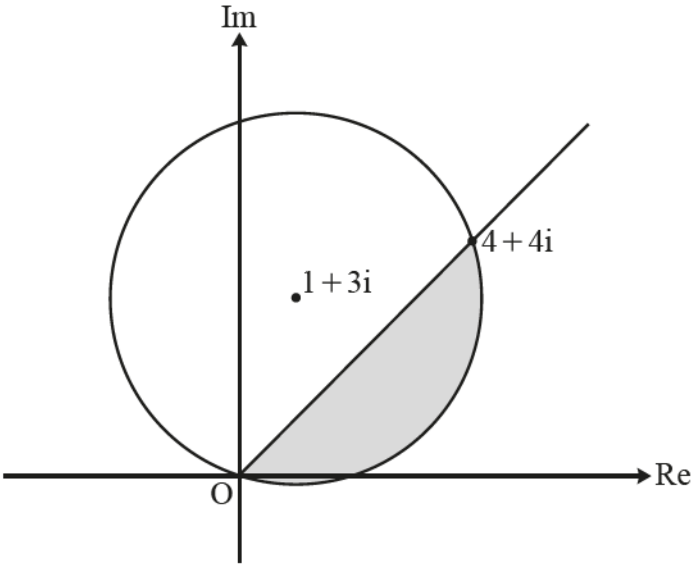
The equation $z^3+2z^2+kz+3=0$, where k is a constant, has roots α, $\frac{1}{\alpha}$ and β.

Determine the roots in exact form. **[6]**

This question was well answered, with well over half the candidates gaining all six marks. The most-common method was that shown in the mark scheme, i.e., using the product of the roots to find the –1 root and the sum to find and solve the quadratic in the other. While we did not insist on showing the roots of the quadratic are α and $1/\alpha$, we did require a clear statement that the roots are indeed -3, (1 + i√3)/2 and (1 – i√3)/2. Less successful responses sometimes did not recognise that α x $1/\alpha \times \beta = \beta$. Some other approaches were also seen, for example using the factor theorem to find *k*, then factorising the cubic.

Question 5

5 An Argand diagram is shown below. The circle has centre at the point representing $1+3\mathrm{i}$, and the half line intersects the circle at the origin and at the point representing $4+4\mathrm{i}$.



State the **two** conditions that define the set of complex numbers represented by points in the shaded segment, including its boundaries. **[5]**

Almost half of the candidates scored all five marks. The circle locus was better answered than the half line. Quite a few scripts scored 3 marks due to errors in the inequality signs.

Question 6 (a)

6 **(a)** Using standard summation formulae, show that $\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$. **[4]**

The majority of candidates scored full marks here, although some made hard work for themselves by expanding before factorising their expressions to final answer. The method of splitting the series into $\sum r^2 + 2\sum r$ was universally known.

Question 6 (b)

(b) Use induction to prove the result in part **(a)**. **[6]**

Virtually all candidates showed the result holds for $n = 1$. The inductive step of adding $(k + 1)(k + 3)$ to $k(k + 1)(2k + 7)/6$ was also often seen, together with a target for $n = k + 1$. However, again some candidates made things challenging for themselves when factorising convincingly to show this target is met. A few candidates expanded to a cubic and then stated that this factorised to $(k + 1)(k + 2)(2k + 9)$ without any working, losing a mark here. The final mark was given for a clear statement of the induction principle (shown for $n = 1$ and **if** true for $n = k$, **then** true for $n = k + 1$, so true for all n).

Exemplar 1

6(b)

Assume true for $n=k$

$\sum_{r=1}^{k} \cancel{k}r(\cancel{k}r+2) = \frac{1}{6}k(k+1)(2k+7)$

Therefore must be true for $n=k+1$

$\sum_{r=1}^{k+1} \cancel{k+1(k+3)}\ r(r+2) = \frac{1}{6}k(k+1)(2k+7) + (k+1)(k+3)$

$\frac{1}{6}(k+1)[k(2k+7) + 6(k+3)]$

$\frac{1}{6}(k+1)[2k^2+7k+6k+18]$

$\frac{1}{6}(k+1)[2k^2+13k+18]$

$\frac{1}{6}(k+1)(2k+9)(k+2)$

Assume true for $n=1$

$\sum_{r=1}^{1} \cancel{k}\ 1(1+2) = \frac{1}{6}(1)(1+1)(2(1)+7)$

$1(3) = \frac{1}{6}(2)(9)$

$3 = \frac{1}{6}(18)$

$3 = 3$

Statement is true for $n=k$, $n=k+1$ and $n=1$ so therefore statement must be true for all ~~positive~~ intergers.

This script loses one mark for not establishing that $(k + 1)(2k + 9)(k + 2)$ is the result when n = $k + 1$ (either by referring to a target or writing it as $(k + 1)[2(k + 1) + 7](k + 1 + 1)$. The final mark is lost for an insufficiently precise statement of the principle of induction.

Question 7 (a)

7 On an Argand diagram, the point A represents the complex number z with modulus 2 and argument $\frac{1}{3}\pi$. The point B represents $\frac{1}{z}$.

(a) Sketch an Argand diagram showing the origin O and the points A and B. **[2]**

Quite a few candidates converted *z* and 1/*z* to *a* + *b*i form here and used this information to sketch the points (albeit often somewhat inaccurately). If the arguments $\pi/3$ and $-\pi/3$ were used, these often did not resemble 60 degrees; however, some discretion was applied here.

Question 7 (b)

(b) The point C is such that OACB is a parallelogram. C represents the complex number w.

Determine each of the following.

- The modulus of w, giving your answer in exact form.
- The argument of w, giving your answer correct to **3** significant figures. **[7]**

The marks for converting *z* and 1/*z* to *a* + *b*i form were given here if seen in 7(a). Multiplying top and bottom by the complex conjugate was usually well done. From here, solutions were less successful. Quite a few thought that C was represented by *z* – 1/*z* rather than *z* + 1/*z*. Some vector approaches were successfully applied. A few candidates tried to use a cartesian approach by finding the equations of AC and BC and solving these simultaneously, although not many followed this approach through correctly.

Assessment for learning

It appears that the geometric interpretation of the sum of two complex numbers was not as well-known as other aspects of complex arithmetic.

Question 8 (a)

8 A transformation T of the plane has matrix **M**, where $\mathbf{M} = \begin{pmatrix} \cos\theta & 2\cos\theta - \sin\theta \\ \sin\theta & 2\sin\theta + \cos\theta \end{pmatrix}$.

(a) Show that T leaves areas unchanged for all values of θ. **[2]**

The relationship between determinant and scale factor was generally well known. However, just showing the determinant here equals 1 without any concluding statement (even just 'so the area is unchanged') loses a mark here.

Assessment for learning

Not concluding a 'Show…' question with a confirming remark quite often loses marks for candidates. Here, just showing that the determinant equals 1 without a comment on the significance of this result lost a mark.

Exemplar 2

8(a)

$DET = 1$

$\cos\theta(2\sin\theta + \cos\theta) - \sin\theta(2\cos\theta - \sin\theta)$

$2\sin\theta\cos\theta + \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta$

$= \cos^2\theta + \sin^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\therefore$ Det $= 1$ for all values of θ

Here, the candidate has correctly calculated the determinant as 1, but has not explicitly stated the significance of this for the area scale factor. One could possibly infer that they know this from the 'DET = 1' written in the first line, but candidates should be encouraged to offer clear explanations of their thinking.

Question 8 (b)

(b) Find the value of θ, where $0 < \theta < \frac{1}{2}\pi$, for which the y-axis is an invariant line of T. **[4]**

The matrix **N** is $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

This was the least well answered question in the paper. While many scripts suggested a knowledge of invariant lines, the fact that this invariant line was the y-axis meant that using a '$y = mx + c$' approach failed as the y-axis is not expressible in this form.

Misconception

An important distinction in teaching this topic is between invariant lines and invariant points. It is possible to get the correct answer here using $(0, y) \rightarrow (0, y)$ rather than $(0, y')$ and ignoring the second equation in θ. However, this is of course incorrect and lost 2 marks.

Question 8 (c) (i)

(c) (i) Find $\mathbf{MN}^{-1}$. **[2]**

The majority of scripts gained the marks here in this straightforward application of inverse matrices and matrix multiplication.

Question 8 (c) (ii)

(ii) Hence describe fully a sequence of two transformations of the plane that is equivalent to T. **[4]**

A couple of relatively easy marks here were given for 'rotation' and 'shear'. For the 'A' marks, the rotation needed to mention 'anticlockwise' and the shear needs to state the *x*-axis is fixed and the correct image of a point (for example '(0, 1) maps to (2, 1)'). Not many scripts gained full marks here, sometimes because the order of the correctly specified transformations was incorrect.

OCR support

Some candidates used the term 'scale factor 2' for the shear. This is specifically ruled out by the specification – see the note after statement m4 in the specification.

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