OCR
RECOGNISING ACHIEVEMENT

SAMPLE

OCR Level 3 Certificate in Mathematics for Engineering

UNIT H860

Component 1

Sample Paper

Time: 2 hours

Candidates answer on the question paper.

Candidate Name	

Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Write your answers in black ink.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60
- The use of calculators **is** allowed for this paper
- Include **all** working out where used
- **You are reminded of the need for clear presentation in your answers.**

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.

1. An engineer starts working on a computer model of a football stadium which is in the shape of a perfect circle.

a) In her program, she entered the radius as 5 and the co-ordinates of the centre as (6 , 3). Show that the equation of the circle she obtains is

$$x^2 + y^2 - 12x - 6y + 20 = 0 .$$

[2]

b) A straight cable needs to be installed between two poles P and Q on the circumference of the stadium. If the cable is represented by the straight line $y = x + 4$ in the computer model, show that the points P and Q will satisfy the equation

$x^2 - 5x + 6 = 0 .$

[2]

c) Factorise this equation in x and hence find the positions of the two poles P and Q.

[2]

2. An engineer monitors the voltage across the output terminals of an electrical circuit for three seconds and at one second intervals. The readings are recorded in the following table.

Time t (s)	0	1	2	3
Voltage V	0	5	0	–3

The output voltage is to be modeled using a cubic equation of this form.

$$V = at^3 + bt^2 + ct + d$$

a) Use the data in the table to determine the values of the constants *a, b, c* and *d*.

[3]

b) Use the model to determine the maximum voltage during the first two seconds.

[3]

c) Use the model to draw a graph of V against t for $t = 0$ to 4, identifying all salient points accurately.

[2]

3. A small engineering firm produces bolts. Each bolt has a probability of 0.16 of having a fault. Alice picks 3 bolts at random from the production line.

a) Find the probability that

i) All 3 bolts are faulty.

[1]

ii) Exactly 1 of the 3 bolts is faulty

[2]

b) The probability that a particular product is found faulty after manufacture is 0.001. A random sample of 500 of these products is selected and tested. Use the Poisson distribution to determine the following probabilities.

i) Exactly one item is found to be faulty.

[1]

ii) Three or more of the 500 products are found to be faulty.

[2]

4. The displacement, x mm, of an oscillating body, in an engine, at time t seconds is given by the formula:

$$x = 10e^{-50t}\sin(200\pi t)$$

a) i) What is the frequency of oscillation?

[1]

ii) What is the period of the oscillation?

[1]

iii) Sketch a graph of x against t.

[2]

b) What is the maximum amplitude?

[4]

5. A manufacturing company produces two products *X* and *Y*. Each product involves the use of three machines, A, B and C for certain amounts of time. The table below shows the time, in minutes, required on each machine for each product.

	Product *X*	Product *Y*
Machine A	5	6
Machine B	2	8
Machine C	6	2

Each machine can be operated for a maximum number of hours per week as follows: Machine A: 30 hours; Machine B: 32 hours; Machine C: 30 hours.

The unit profit obtained by selling product *X* is £20 and the unit profit obtained by selling product *Y* is £10. The company needs to determine the numbers of each product to manufacture each week to maximize its profit.

a) Taking *x* to be the number of the product *X* produced each week, and *y* to be the number of the product *Y* produced each week, formulate this problem as a linear program.

[2]

b) Draw a suitable graph showing the constraints imposed on the number of each product produced each week.

[3]

c) Clearly indicate the feasible area on the graph for all possible values of *x* and *y*.

[2]

d) Find the number of products to produce each week in order to maximize the profit. Express your answer using integer numbers only.

[2]

6. An engineer, designs a new energy efficient motor. The motor's cam is the area, bounded by $y = 2 + x^2$, the *x* axis, $x = 1$ and $x = 2$. The coordinates of the centroid of the cam are $\bar{x}$ and $\bar{y}$ are given by:

$$\bar{x} = \frac{\int_a^b xydx}{\int_a^b ydx} \quad \text{and} \quad \bar{y} = \frac{\frac{1}{2}\int_a^b y^2 dx}{\int_a^b ydx}$$

a) Sketch a graph of the bounded area.

[1]

b) Determine the values of $\bar{x}$ and $\bar{y}$.

[7]

7. When an engineer monitors an electronic circuit to ensure that it is functioning properly, he finds that the rate of change of voltage, V, with respect to time, t seconds, across the output terminals of the circuit involving a capacitor of C farads and a resister of R ohms is described by the differential equation:

$$\frac{dV}{dt} = \frac{1}{RC}(K - V)$$, where K is a constant voltage.

a) Show that $V = K(1 - e^{-t/RC})$ is a solution to this differential equation.

[2]

b) Rearrange the formula $V = K(1 - e^{-t/RC})$ to make t the subject.

[2]

c) If K= 50 volts, R = 50 000 ohms and $C = 10^{-6}$ farads, determine the time needed for the output voltage to reach 25 volts.

[2]

8. As part of safety testing toughened glass used in the construction of large office buildings, a body is projected from the top of a 100m high tower. It is projected upwards at an angle of 30° to the horizontal and with a velocity of u m/s. There are no forces acting on the body other than gravity. The body travels until it reaches the sheet of glass which is taken to be horizontal and level with the base of the tower.

a) The vertical acceleration of the body is described by the equation:

$$\frac{d^2y}{dt^2} = -g$$, where g is the acceleration due to gravity.

Prove, from first principles, that the height the body, y, above the pane of glass at time t is given by:

$$y = \frac{-gt^2}{2} + u\sin(30^{\circ})t + 100$$

[3]

b) If u = 20m/s show that the time at which the body reaches the glass is 5.65 s.

[3]

c) Show that the height y with respect to the horizontal distance traveled, x, is given by:

$$y = \frac{-g}{2}\left(\frac{x}{u\cos 30^{\circ}}\right)^2 + x\tan 30^{\circ} + 100$$

[3]

Paper Total [60]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

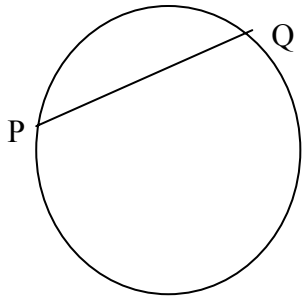
Level 3 Certificate in Mathematics for Engineering

Component 1

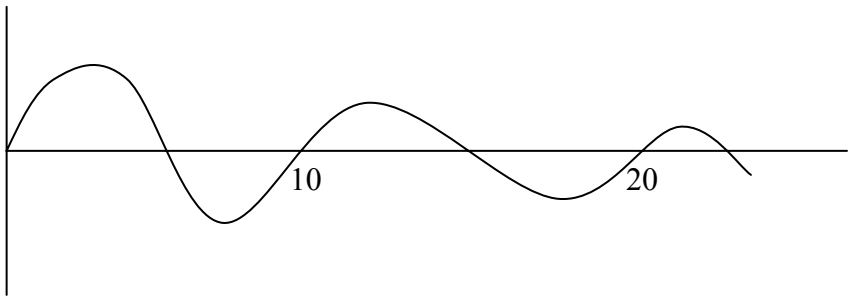
Unit H860

Sample Mark Scheme

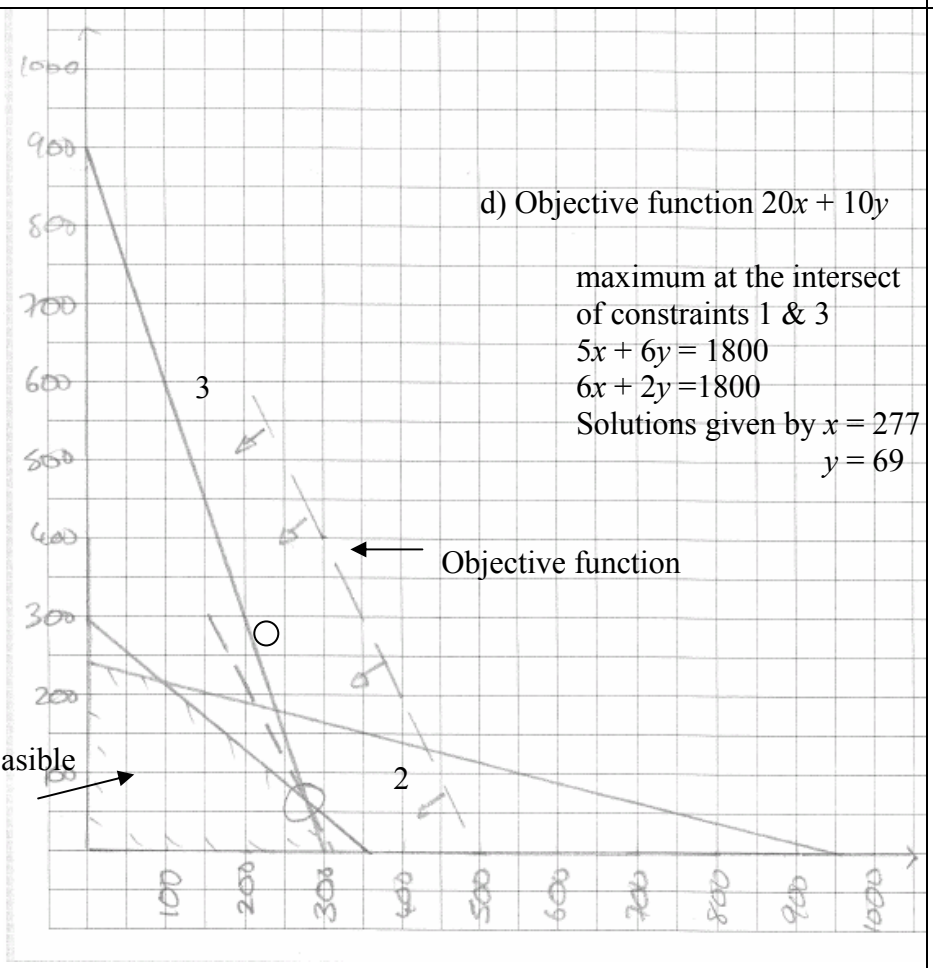
The maximum mark for this paper is 60.

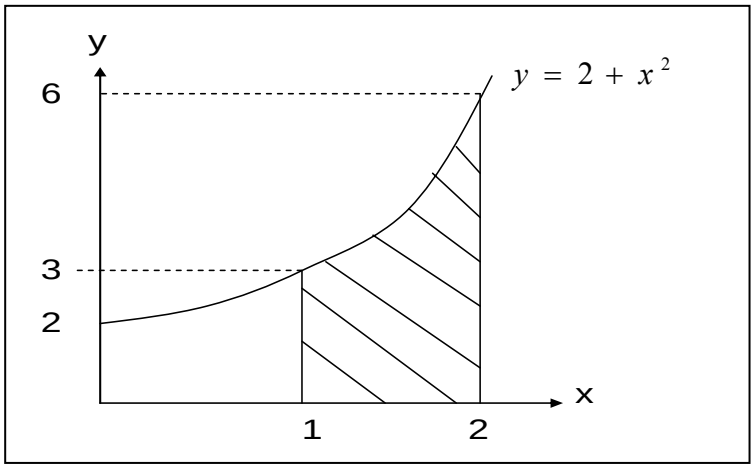
Question		Mark
1a	Radius = 5 Centre = (6 , 3) The general equation of the circle is given by: $(x-a)^2 + (y-b)^2 = r^2$ Where r is the radius and (a, b) is the centre Therefore: $(x-6)^2 + (y-4)^2 = 5^2$ $x^2 + 36 - 12x + y^2 + 9 - 6y = 25$ $x^2 + y^2 - 12x - 6y + 45 - 25 = 0$ $x^2 + y^2 - 12x - 6y + 20 = 0$	 M1 A1 2
1b	 $PQ: y = x + 4$ Substituting $y = x + 4$ in the circle equation we get: $x^2 + (x + 4)^2 - 12x - 6(x + 4) + 20 = 0$ $x^2 + x^2 + 16 + 8x - 12x - 6x - 24 + 20$ $2x^2 - 10x + 12 = 0$ *Dividing by* 2 *throughout we get:* $x^2 - 5x + 6 = 0$	 M1 A1 2

Question		Mark
1c	Equation: $x^2 - 5x + 6 = 0$ $x^2 - 3x - 2x + 6 = 0$ $x(x - 3) - 2(x - 3) = 0$ $(x - 3)(x - 2) = 0$ $x = 3$ or $x = 2$ therefore $y = x + 4$ Therefore the co-ordinates for P and Q are (3 , 7) and (2 , 6)	M1 A1 2
	Question 1 AC5.1/AC1.1 (4), AC4.2/AC10.1 (2) (6 marks)	
2a	Since $V =$ when $t = 0$, $d = 0$ Since $V = 0$ at $t = 0$ and $t = 2$, $V=t(t-2)(At+B)$ *where A and B are constant* Or forming 3 simultaneous equations and finding one coefficient $V = 5$ *when* $t=1$: $(At + B) = -1$ $V = -3$ *when* $t = 3$: $(3t + b) = -3$ *Solving gives* $A = 2$ *and* $B = -7$ *Expanding* $t(t-2)(2t-7) = V = 2t^3 - 11t^2 + 14t$: $\underline{a = 2,\ b = -11,\ c = 14,\ d = 0}$	B1 M1 A1 3
2b	$\frac{dV}{dt} = 6t^2 - 22t + 14$ and use of $\frac{dV}{dt} = 0$ $\frac{dV}{dt} = 0$ when $t = \frac{22 \pm \sqrt{484 - 4 \times 6 \times 14}}{2 \times 6} = 0.81954$ and 2.847127 Maximum occurs at 0.81954 giving $V = 5.186$ or 5.19	M1 M1 A1 3
2c	Suitable graph passing through points given in the table plus other salient points when $t = 3.5$, $t = 4$ and at the turning points. Shape correct All salient points indicated	M1 A1 2
	Question 2 AC4.2 (3), AC6.1(3), AC4.1(2) (8 marks)	
3a i	(a) (0.16)cubed = 0.004096 (accept 0.004 or better)	B1

Question		Mark
3a ii	0.16 x (0.84) squared x 6 =0.677 If 0 scored, allow sc 1 for answer omitting '6' - 0.113	M1 A1 2
3b i	Use Poisson distribution $p = (x = r) = \frac{e^{-u}\mu^{r}}{r!}$ $p = 0.001, \quad n = 500, \quad \mu = np = 0.5$ $P(x=1) = \underline{e^{-0.5}(0.5)} = 0.303(265)$	1
3b ii	$P(x=2) = \frac{e^{-0.5}(0.5)^2}{2} = 0.075816$ $P(x \geq 3) = 1 - (P(x=0) + P(x=1) + P(x=2)$ $= 1 - 0.985612$ $= 0.0143(88)$	M1 2
	Question 3 AC8.3(6) (6 marks)	
4 a i	$x = 10\, e^{-50t} \sin(200\, \pi\, t)$ $f = 100$ Hz	1
4 a ii	$T = \frac{1}{100} = 10$ ms	1
4 a iii	Damped oscillating curve Approx correct indication of scale on x axis.  	M1 A1 2

Question		Mark
4 b	$x = 10\ e^{-50t} \sin(200\ \pi\ t)$	
	$\frac{dx}{dt} = 10\ e^{-50t}\ 200\ \pi \cos(200\ \pi\ t)$	*M1
	$-500\ e^{-50t} \sin(200\ \pi\ t)$	A1
	$= 10\ e^{-50t}\ (200\ \pi \cos(200\ \pi\ t) - 50 \sin(200\ \pi\ t))$	
	$= 0$ when $200\ \pi \cos(200\ \pi\ t) = 50 \sin(200\ \pi\ t)$	M1dep*
	$200\ \pi = 50 \tan(200\ \pi\ t)$	
	$4\ \pi = \tan(200\ \pi\ t)$	
	$t = \frac{\tan^{-1} 4\pi}{200\pi} = 2.37\text{ms}$	A1
		4
	Question 4 AC2.2(4), AC3.2(4) (8 marks)	

5a	Maximise 20x + 10y, subject to: $5x + 6y \leq 1800$ $2x + 8y \leq 1920$ $6x + 2y \leq 1800$ One correct inequality Other two correct ($x \geq 0, y \geq 0$)	 B1 B1 2
5b	For constraint 1: ($x = 0$, $y = 300$); ($x = 360$, $y = 0$) For constraint 2: ($x = 0$, $y = 240$); ($x = 960$, $y = 0$) For constraint 3: ($x = 0$, $y = 900$); ($x = 300$, $y = 0$)	B1 B1 B1 3
	 	
5c 5d	At least 2 constraints correctly shaded Other constraints correctly shaded Correct point identified for maximizing profit (indicated or clear from working) Correct values for x and y.	B1 B1 2 M1 A1 2
	Question 5 AC7.4(9) (9 marks)	

6a		1
6b	$\int y\,dx = \int 2 + x^2\,dx = 2x + \frac{x^3}{3} + c$	B1
	$\int y^2 = \int 4 + 4x^2 + x^4\,dx = 4x + \frac{4x^3}{3} + \frac{x^5}{5} + c$	B1
	$\int xy\,dx = \int 2x + x^3\,dx = x^2 + \frac{x^4}{4} + c$	B1
	Condone missing '*c*' for these B1 marks.	
	$\int_1^2 y\,dx = \left[2x + \frac{x^3}{3}\right]_1^2 = \left(4 + \frac{8}{3}\right) - \left(2 + \frac{1}{3}\right) = \frac{13}{3}$	
	$= \frac{13}{3}$	
	$\int_1^2 y^2\,dx = \left[4x + \frac{4x^3}{3} + \frac{x^5}{5}\right]_1^2 = \left(8 + \frac{32}{3} + \frac{32}{5}\right) - \left(4 + \frac{4}{3} + \frac{1}{5}\right)$	
	$= \frac{293}{15}$	
	$\int_1^2 xy\,dx = \left[x^2 + \frac{x^4}{4}\right]_1^2 = (4 + 4) - \left(1 + \frac{1}{4}\right) = \frac{27}{4}$	
	For correct use of limits in any one of these three integrals For correct evaluation of one of these integrals	B1 B1
	$\bar{x} = \frac{27}{4}\frac{3}{13} = 1.55769 \qquad \bar{y} = \frac{1}{2}\frac{293}{15}\frac{3}{13} = 2.53846$	A1,A1 CAO
	Accept 1.55 or 1.56 and 2.53 or 2.54 if rounded or truncated	7
	Question 6 AC6.3 (1), AC6.4(7) (8 marks)	

7a	$v = k(1 - e^{-\frac{t}{RC}})$ ------ (1) $\frac{dv}{dt} = \frac{1}{RC}(K - v)$ -------- (2) substitute (1) in (2) $\frac{dv}{dt} = \frac{1}{RC}(K - K(1 - e^{-\frac{t}{RC}}))$ $= \frac{K}{RC} e^{-\frac{t}{RC}}$ ……(3) Convincing conclusion to given result Differentiate (1) $\frac{dv}{dt} = \frac{K}{RC} e^{-\frac{t}{RC}}$ = result (3)	B1 B1 2
7b	$V = K(1 - e^{-\frac{t}{RC}})$ $\frac{V}{K} = (1 - e^{-\frac{t}{RC}})$ $e^{-\frac{t}{RC}} = 1 - \frac{V}{K}$ $-\frac{t}{RC} = \ln(1 - \frac{V}{K})$ $t = -RC\ln(1 - \frac{V}{K})$	M1 A1 2
7c	$t = -5 \times 10^4 \times 10^{-6} \ln\left(1 - \frac{25}{50}\right)$ = 0.05 ln ½ = 34.6(47)	M1 A1 2
	Question 7 AC2.3/AC6.1 (2), AC 2.2/9.2 (2), AC 9.2 (2) (6 marks)	

8a	$\frac{d^2y}{dt^2} = -g$ $\frac{dy}{dt} = -gt + c$ $\frac{dy}{dt} = u\sin30^o$ when $t = 0$, therefore $c = u\sin30^o$ $\frac{dy}{dt} = -gt + u\sin30^o$ $y = \frac{-gt^2}{2} + u\sin(30^o)t + k$ y = 100 when $t = 0$, therefore k = 100 $y = \frac{-gt^2}{2} + u\sin(30^o)t + 100$	*M1 A1 M1dep* 3
8b	$\frac{-gt^2}{2} + 10t + 100 = 0$ $t = \frac{-10 \pm \sqrt{100 + 200g}}{-g}$ let $g = 9.8$ $\frac{-10 \pm \sqrt{100 + 1960}}{-9.8} = \frac{-10 \pm 45.387}{-9.8}$ answer must be – ve root $\frac{-10 - 45.387}{-9.8} = 5.65$	M1 A1 A1 3
8c	$x = u\cos30° t$ $t = \frac{-x}{u\cos30^o}$ substitute in 1a $y = \frac{-g}{2}\left(\frac{x}{u\cos30^o}\right)^2 + u\left(\frac{x}{u\cos30^o}\right)\sin30^o + 100$ For convincing algebra to achieve given result $y = \frac{-g}{2}\left(\frac{x}{u\cos30^o}\right)^2 + x\tan30^o + 100$	M1 M1 A1 3
	Question 8 AC1.2 (3), AC2.3(3), AC3.1/AC9.2(3) (9 marks)	

	LO1		LO2			LO3		Lo4		LO5			LO6				LO7					LO8			LO9			LO10	
AC →	**1.1**	**1.2**	**2.1**	**2.2**	**2.3**	**3.1**	**3.2**	**4.1**	**4.2**	**5.1**	**5.2**	**5.3**	**6.1**	**6.2**	**6.3**	**6.4**	**7.1**	**7.2**	**7.3**	**7.4**	**7.5**	**8.1**	**8.2**	**8.3**	**9.1**	**9.2**	**9.3**	**10.1**	**Total marks for question**
Question																													
1 (a)										X																			**2**
1 (b)										X																			**2**
1 (c)									X																				**2**
2 (a)									X																				**3**
2 (b)													X																**3**
2 (c)								X																					**2**
3 (a)																								X					**3**
3 (b)																								X					**3**
4 (a)							X																						**4**
4 (b)				X																									**4**
5 (a)																				X									**2**
5 (b)																				X									**3**
5 (c)																				X									**2**
5 (d)																				X									**2**
6 (a)															X														**1**
6 (b)																X													**7**
7 (a)					X																								**2**
7 (b)																										X			**2**
7 (c)																										X			**2**
8 (a)		X																											**3**
8 (b)					X																								**3**
8 (c)						X																							**3**
Total marks for AC	0	3	0	4	5	3	4	2	5	4	0	0	3	0	1	7	0	0	0	9	0	0	0	6	0	4	0	0	**60**