OCR
Oxford Cambridge and RSA

Tuesday 17 January 2023 – Afternoon

Level 3 Cambridge Technical in Engineering

05823/05824/05825/05873 Unit 23: Applied mathematics for engineering

Time allowed: 2 hours
C305/2301

You must have:
- the Formula Booklet for Level 3 Cambridge Technical in Engineering (inside this document)
- a ruler (cm/mm)
- a scientific calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________________

Last name ______________________________

Date of birth | D | D | M | M | Y | Y | Y | Y |

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 In this question use 1 litre $= 10^6\,\text{mm}^3$.

Fig. 1a shows a small deep fryer.

Fig. 1b is a plan view of the tank that holds the oil for frying. The plan shows a square with rounded corners; the straight edges are of length 80 mm and the rounded corners are each part of a circle of radius 40 mm.

Fig. 1a

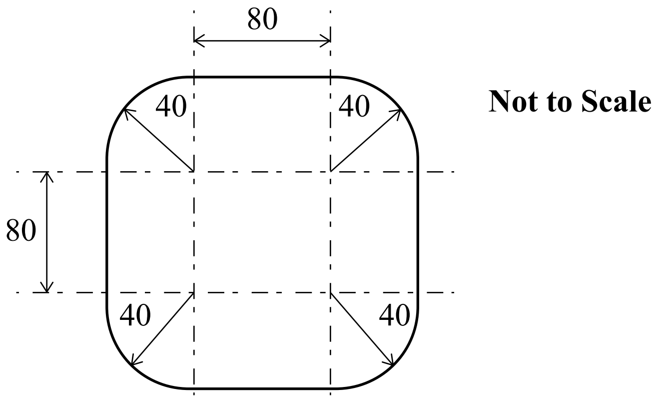

Fig. 1b

(i) Calculate the volume of oil in the tank when it is filled to a depth of 60 mm.

Give your answer in litres.

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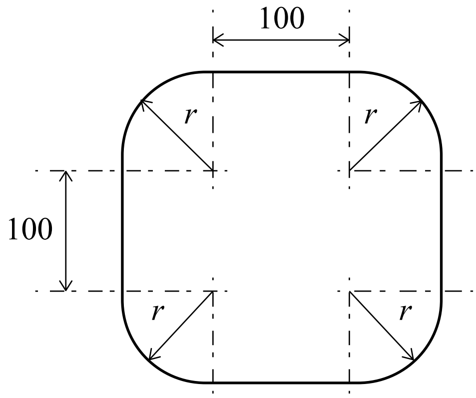
The manufacturer has designed a new deep fryer, also in the shape of a square with rounded corners. The straight edges of the new oil tank are of length 100 mm and the rounded corners are each part of a circle of radius r mm, as shown in **Fig. 2**.



Not to Scale

Fig. 2

(ii) The depth of oil in the tank is 50 mm when it contains 2 litres of oil.

Calculate the value of r.

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2 A small object O, of mass 10 kg, is suspended by two light inextensible cables, as shown in **Fig. 3**.

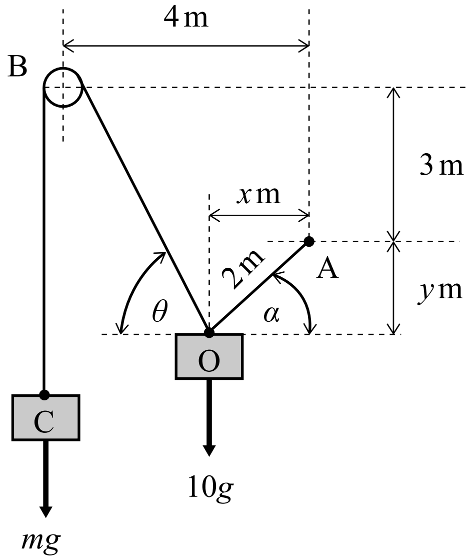

Fig. 3

The cable OA is 2 m long and is attached to a fixed point A. The other cable passes over a small freely rotating fixed pulley B and is then attached to a small object C, of mass m kg, which hangs freely.

The pulley B is 4 m horizontally and 3 m vertically from A; the object O is x m horizontally and y m vertically from A. The cable OA makes an angle α with the horizontal; the other cable makes an angle θ with the horizontal.

When $m = 0$ the mass O is supported by cable OA, which is now vertical. When $m > 0$, O is supported by both cables, and $y < 2$.

(i) Calculate α when $y = 0.5$.

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(ii) Calculate x when $y = 0.5$.

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(iii) Show that $\tan\theta = \dfrac{3 + 2\sin\alpha}{4 - 2\cos\alpha}$.

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(iv) Calculate the value of m in the case when $y = 0$.

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3 The heat transfer, $Q\,\mathrm{W\,m^{-2}}$, through the surface of a wall with a thickness of s m made from a material which has a thermal conductivity of $K\,\mathrm{W\,(m\,°C)^{-1}}$ is given by

$$Q = \frac{K}{s}D,$$

where D °C is the difference between the temperature T_1 °C, on one side of the wall and T_2 °C, on the other side, with $T_1 \geqslant T_2$. This is shown in **Fig. 4**.

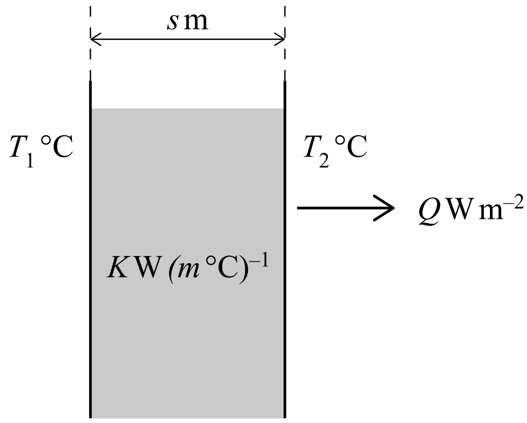

Fig. 4

(i) A solid wall has a thickness of 0.1 m and is made from brick for which $K = 0.4$. The temperature on one side of the wall is 24 °C and on the other side is 12 °C.

Calculate the heat transfer through the surface of the wall.

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(ii) Another wall made of the same brick with a thickness of 0.1 m has a heat transfer of $80\,\mathrm{W\,m^{-2}}$. The temperature on the cooler side of the wall is 10 °C.

Calculate the temperature on the other side of the wall.

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When a wall has n layers, such as an insulated cavity wall, the heat transfer, $Q\,\mathrm{W\,m^{-2}}$, is given by

$$Q = \frac{D}{\sum_{i=1}^{n} \frac{s_i}{K_i}},$$

where s_i and K_i are the thickness and thermal conductivity of layer i respectively.

(iii) A particular insulated cavity wall has three layers; breeze block on the inside, house brick on the outside and polyurethane foam in the cavity. The value of K is K_B for breeze block, K_F for polyurethane foam and 0.4 for brick.

(*A*) When the thickness of breeze block is 0.1 m, the thickness of polyurethane foam is 0.05 m and the thickness of brick is 0.1 m, as shown in **Fig. 5**, the heat transfer though the wall is $10\,\mathrm{W\,m^{-2}}$. For this wall $D = 20$.

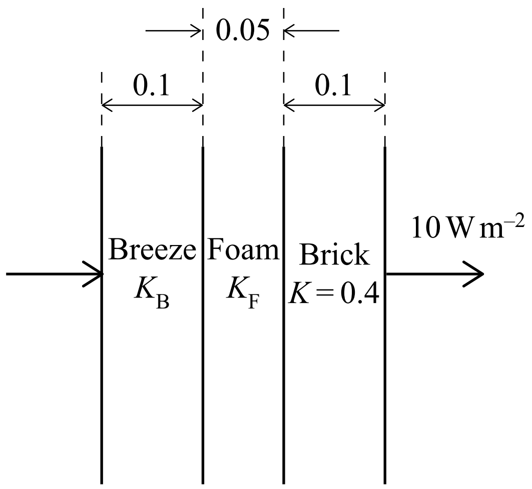

Fig. 5

Show that $\frac{0.1}{K_B} + \frac{0.05}{K_F} = 1.75$.

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(*B*) When the thickness of breeze block is 0.12 m, the thickness of polyurethane foam is 0.03 m and the thickness of brick is 0.1 m, as shown in **Fig 6**, the heat transfer though the wall is 12.5 W m^{-2}. For this wall also $D = 20$.

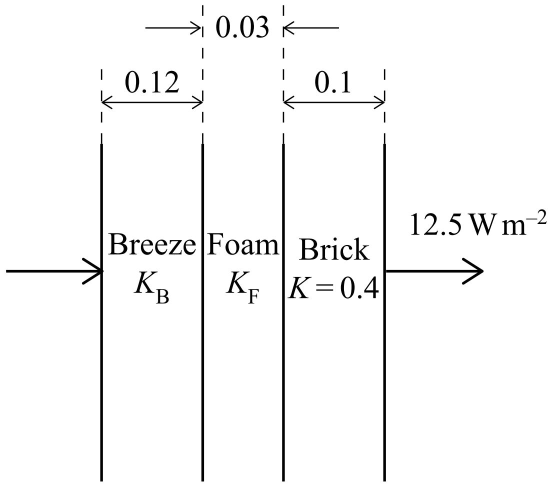

Fig. 6

Show that $\frac{0.12}{K_B} + \frac{0.03}{K_F} = 1.35$.

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(*C*) Calculate the values of K_B and K_F.

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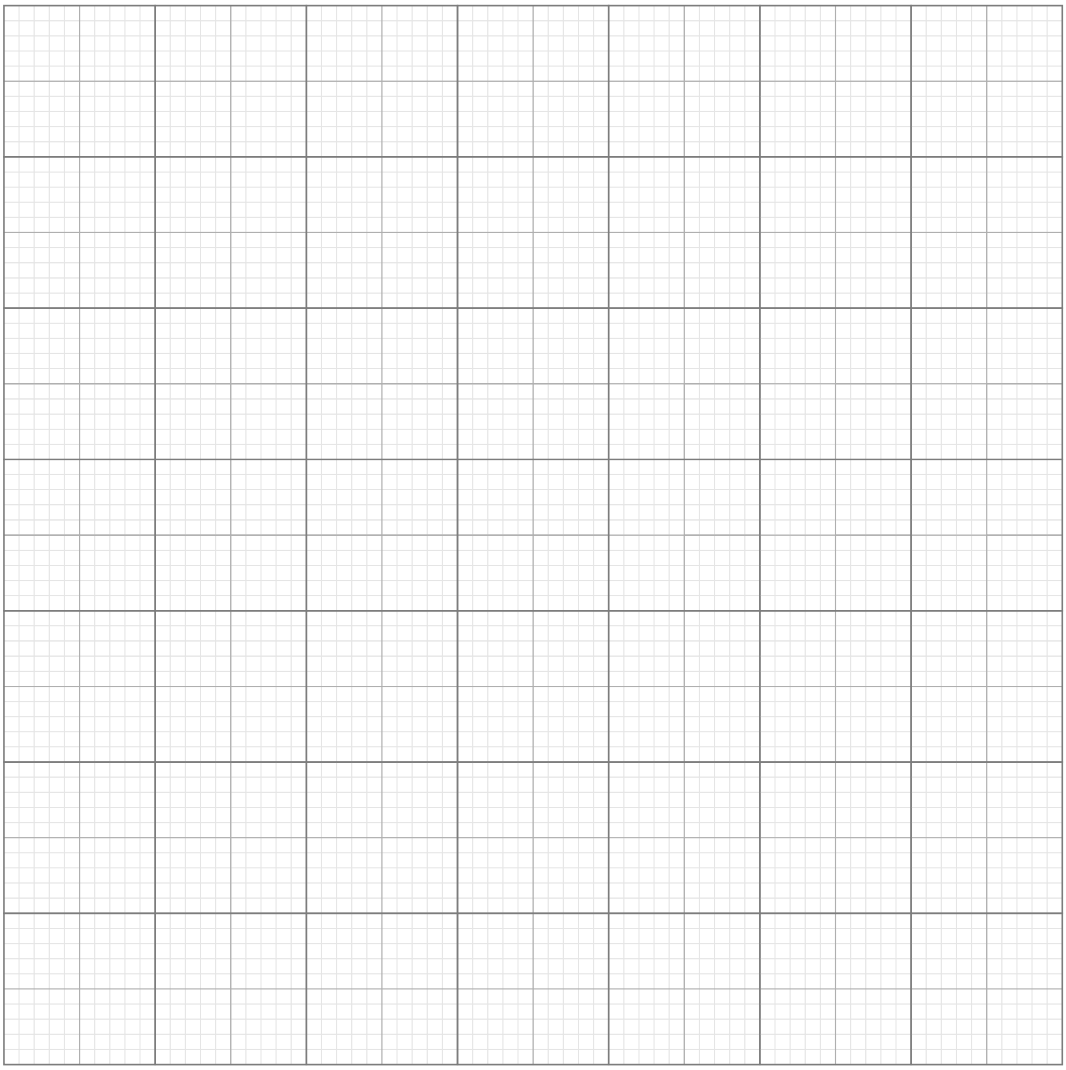
4 A car on a test track accelerates from rest for 10 seconds when it reaches its maximum speed. After that time the car continues with diminishing speed due to reduced engine power. The speed of the car, $v\,\mathrm{m\,s^{-1}}$, during the first 30 seconds of its motion, is modelled by the formula

$$\mathrm{v}(t) = 12t\mathrm{e}^{-\frac{t}{10}},$$

where t is time measured in seconds.

(i) On the grid below, draw a graph of $\mathrm{v}(t)$ against t for $0 \leqslant t \leqslant 30$.



[4]

(ii) For this part of the question you are given the following information:

If the speed, v, of a moving object at time t is given by a formula, $\mathrm{v}(t)$, then the distance, s, that the object travels in the period from t_0 to t_1 is given by

$$s = \int_{t_0}^{t_1} \mathrm{v}(t)\,\mathrm{d}t.$$

Show that the distance the car travels in the 30-second period is $1200(1 - 4\mathrm{e}^{-3})$ m.

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5 **Fig. 7** shows a metal plate in the shape of a triangle OAB. O is the origin of a Cartesian coordinate system, (x, y, z), A has coordinates (2, 4, 5) and B has coordinates (3, 2, 1).

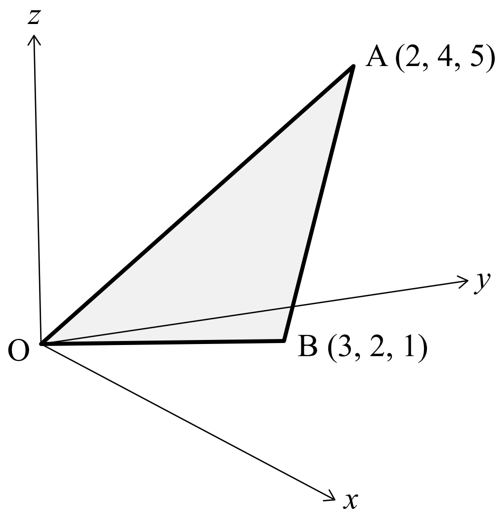

Fig. 7

(i) Associating unit vectors **i**, **j** and **k** with directions x, y and z respectively, define the location of corners A and B as position vectors, $\overline{OA}$ and $\overline{OB}$, in component form, $(a\mathbf{i} + b\mathbf{j} + c\mathbf{k})$.

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(ii) Calculate the vector product $\overline{OA} \times \overline{OB}$, giving your answer as a vector in component form.

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(iii) The magnitude of $\overline{\mathrm{OA}} \times \overline{\mathrm{OB}}$ is twice the area of the plate OAB.

Calculate the area of the plate.

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(iv) Hence or otherwise calculate angle AOB.

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6 A solenoid coil with a resistance of R ohms and an inductance of L henries is connected to an alternating supply voltage, v volts, given by

$v = V\sin(2\pi ft)$,

where V is a constant, f Hz is the frequency of oscillation and t s is time.

The alternating current flowing in the coil, i amps, has the same frequency as the voltage but lags by a phase difference of θ radians and is given by

$i = I\sin(2\pi ft + \theta)$,

where $I = \dfrac{V}{\sqrt{R^2 + (2\pi fL)^2}}$ and $\theta = \tan^{-1}\left(\dfrac{2\pi fL}{R}\right)$.

You are given that $V = 325$, $I = 5$, $f = 50$ and $R = 32.5$.

(i) Calculate the value of L.

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(ii) Calculate the value of θ in radians.

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(iii) For this part of the question you should use

$$\sin A \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}.$$

Using the formulae for v and i given above show that

$$vi = \frac{VI}{2}\left(\cos\theta - \cos\left(4\pi ft + \theta\right)\right).$$

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(iv) The average power consumed by the coil, P W, in the period $0 \leqslant t \leqslant T$ is given by

$$P = \frac{1}{T}\int_0^T vi \, \mathrm{d}t,$$

where $T = \dfrac{1}{f}$.

Calculate P.

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7 A water tank with a uniform cross-section is filled to a height of 0.75 m. A drain valve at the base of the tank is then opened and water flows out at a rate, $Q\,\text{m}^3\,\text{s}^{-1}$. Q is proportional to the height of water, h m and can be modelled by the equation

$$Q = A\frac{\mathrm{d}h}{\mathrm{d}t} = Ch,$$

where $A\,\text{m}^2$ is the area of top surface of the water, t s is the time after the drain valve is opened and $C\,\text{m}^2\,\text{s}^{-1}$ is a constant.

You are given that $C = -0.1$ and $A = 1$.

(i) Find an expression for h in terms of t.

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(ii) Calculate the time at which $h = 0.3$.

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As water flows out of the tank it is allowed to flow into a **second** tank which has a small, open drain valve at its base. This tank is initially empty. It has been found that the height of water, s m, in this tank t seconds after it begins to fill is given by

$$s = 1.5\,(e^{-0.05t} - e^{-0.1t}).$$

(iii) Calculate the maximum height reached by the water in the second tank.

[5]

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional answer space is required, you should use the following lined pages. The question numbers must be clearly shown – for example, 1(i) or 6(iv).