GCE

Further Mathematics A

Y542/01: Statistics

Advanced GCE

Mark Scheme for Autumn 2021

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

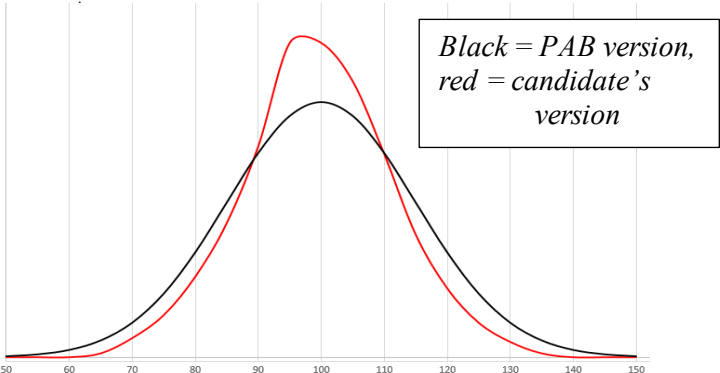Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓ and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

	Question		Answer	Marks	AO	Guidance	
1	**(a)**		$y = 52.7 + 0.251x$	**B1***	1.1	a in range [0.250, 0.251]	
				B1*	1.1	b correct to 3 SF	
				depB1	1.1	Completely correct including letters	
				[3]		SC: Correct formulae used for a and b: M1(A1)A1	
1	**(b)**		This quantity is minimised to find best-fit line	**B1**	2.4	Need "minimised" or "this is its minimum value" OE	
				[1]			
1	**(c)**		$y' = 11.5 + 0.139x$	**M1**	1.1	Apply inverse formula at least once	
			$[y' = \frac{5}{9} \times (\textit{their } a - 32) + \frac{5}{9} \times \textit{their } b]$	**A1ft**	1.1	All correct, any letters, ft on their y	
				[2]			
2			$E(D) = 2 \times 0.1 + 4 \times 0.3 + 6 \times 0.2$	**M1**	2.1	NB: a is not needed by this method	Or change
			$= 2.6$	**A1**	1.1		0, 2, 4, 6 to
							4, 10, 16, 22
			$E(D^2) = 2^2 \times 0.1 + 4^2 \times 0.3 + 6^2 \times 0.2$ $[= 12.4]$	**M1**	1.1	Or $\Sigma(x - \mu)^2 p(x)$	and find a
			$Var(D) = 12.4 - 2.6^2$	**M1**	1.1	$\Sigma p^2 d$ oe gets max M1A1M0M1M1	
			$= 5.64$	**A1**	1.1		
			$Var(3D + 4) = 9 \times Var(D)$	**M1**	3.1a	Allow even if their $Var(D) < 0$	
			$= 50.76$	**A1**	3.4	SC: $\Sigma(x - \mu)^2 p(x)$: M1A1, $a = 0.4$ M1	
				[7]		M1(use this formula), A1M1A1	
3	**(a)**	**(i)**	$P(X \geq 5) - P(X \geq 11) = 0.7^4 - 0.7^{10}$	**M1**	3.1b	Allow 1 term wrong at either end	Or $pq^4 + \ldots + pq^9$
			$= 0.212$	**A1**	3.4	awrt 0.212	
				[2]			
		(ii)	$0.7^{n-1} < \frac{1}{3}$, or $0.103 > 0.1 > 0.072$	**M1**	2.1	Solve $0.3 \times 0.7^{n-1} = 0.1$ or < 0.1, allow inequality error	
			$n_{min} = 5$	**A1**	1.1	5 only	
				[2]		SC: 5 without sufficient justification: B1	
3	**(b)**		$\dfrac{1-p}{p^2} = 42 \Rightarrow 42p^2 + p - 1 = 0$	**M1**	3.1a	Equate correct variance formula to 42	
				A1	1.1	Correct simplified quadratic equation	
			$p = \frac{1}{7}$	**A1**	2.2a		
			Explicitly reject $p = -\frac{1}{6}$	**A1**	2.3	SC: if $-\frac{1}{7}$ and $\frac{1}{6}$, allow A1 for explicitly rejecting $-\frac{1}{7}$	
			$E(X) = 7$	**A1**	2.1		

Question			Answer	Marks	AO	Guidance
				[5]		
4	(a)	(i)	$\hat{\mu} = \bar{x} = 16.8$	**B1** [1]	1.1	Or exact equivalent
		(ii)	$\dfrac{48398}{160} - 16.8^2$ $[= 20.2475]$	**M1**	1.1	If single formula used, full marks if correct; M0M1 if wrong but divisor 159 seen anywhere
			$\times \frac{160}{159}$	**M1**	1.1	
			$= 20.3748\ldots$	**A1** [3]	1.1	Awrt 20.4, www
4	(b)		$\bar{x} \pm z\sqrt{\sigma^2 / 160}$	**M1**	3.3	Any z from Φ^{-1}, 160 needed, allow $\sqrt{}$ errors
			$z = 2.576$	**A1**	1.1	Or better, e.g. 2.575829
			(15.88, 17.72)	**A1** [3]	3.4	Both, 4 sf required by question, www (NB: $\sigma^2 = 20.2475$ gives same end-points to 4 SF but this gets M1A1A0)
4	(c)	(i)	Not needed in **(a)** as E(X) and Var(X) are independent of the distribution	**B1** [1]	2.4	Mention at least one of E(X) and Var(X) explicitly, or "not relevant to $\bar{X}$"
		(ii)	Needed in **(b)** as parent distribution not stated to be normal	**B1** [1]	2.4	Must make it clear that two distributions are involved. "n is large" etc: B0

Question			Answer	Marks	AO	Guidance
5	(a)		The value of Pearson's pmcc would be changed by (most) such changes.	**B1**	2.5	Explain effect on Pearson, *or* not known bivariate normal *or* not testing for *linear* correlation
			The value of Spearman's r_s would not be changed as the ranks remain unchanged.	**B1** **[2]**	2.5	Explain why no effect on Spearman (not "not *likely to be* affected", or "not *much* affected" or "association not correlation"
5	(b)		H_0: no association between ranks of numbers of items H_1: (positive) association between ranks	**B1**	1.1	Don't insist on "population" here, but allow use of ρ_s in both, even if no explanation (not just r_s). Context needed, but don't worry about 1- or 2-tailed here
			Ranks 1 2 3 4 5 6 7 8 9 4 1 3 2 8 5 9 7 6	**M1**	1.1	
			$\Sigma d^2 = 38$	**A1**	1.1	
				M1	1.2	
			$r_s = 1 - \dfrac{6\Sigma d^2}{9(9^2 - 1)}$			
			$= 0.683$	**A1**	1.1	
			< 0.700	**B1**	1.1	Compare TS ($-1 \le$ TS ≤ 1) with 0.7, independent
			Do not reject H_0.	**M1ft**	1.1	ft on TS provided correct formula used, or on CV 0.600
			Insufficient evidence of association between rankings of the items	**A1ft** **[8]**	2.2b	In context, not too positive. FT on TS only SC: 0.600 (2-tailed): B0 M1A0
6	(a)		H_0: Data consistent with N(100, 15^2) H_1: Data not consistent with N(100, 15^2)	**B1** **[1]**	1.1	Allow: "follows N(100, 15^2)" or "can be modelled by". Parameters not needed. No other alternatives seen!
6	(b)		$P(100 \le X < 110) = 0.2475$ **BC** Expected frequency $= 500 \times 0.2475$ [$= 123.754$]	**B1**	3.4	Probability needs to be seen
			$\dfrac{(129 - 123.754)^2}{123.754}$ [$= 0.222\ldots$, **AG**]	**M1** **A1** **[3]**	2.1 2.2a	 Sufficient working to justify **AG**, needs 123.754 at least

Question			Answer	Marks	AO	Guidance
6	(c)		$\Sigma X^2 = 10.5$	B1	1.1	
			$\chi^2(4) = 9.488$ and $10.5 > 9.488$	B1	1.1	Like-with-like comparison needed
			Reject H_0.	M1ft	1.1	FT on TS or CV here. Needn't be stated if next line right
			Significant evidence that data is not consistent with $N(100, 15^2)$.	A1ft	2.2b	FT on TS (but not CV) if method correct.
				[4]		Wrong CV, e.g. 5.991: B1B0M1A0. No ft on H_0/H_1
6	(d)	(i)	E.g. Too few in $X \geq 110$ or in $X \leq 80$, or too many in others, or data truncated, etc	B1 [1]	3.5b	Any relevant point, needn't refer to values of X^2 "Divide into 5 minute groups": B1. "Data discrete": B0. "The variance" (uncalculated): B0
		(ii)	*Black = PAB version, red = candidate's version*	B1 B1 [2]	3.3 3.5c	Deal with aspect identified in (i) Basically correct, areas roughly same
						Examples:
						Uses "data discrete" in (i) B0
						More below 100, so translate to left B2
						More above 110 so translate to right B2
						Divide into 5-minute groups B0
						Variance changed, areas not equal B1
						Data truncated but worse truncation shown B0
7	(a)		H_0: Two samples are from identical populations	B2	1.1	If no reference to "populations", maximum B1
			H_1: Two samples are from populations with different median ratings.		1.1	Allow H_0: "identical population medians", H_1: "not identical populations" or "not identical pop medians"
			$R_m = 1 + 2 + 3 + 4 + 5 + 9 + 10 + 11 \ (= 45)$	M1	1.1	"Pupils' opinions have not changed", etc: B2
			$W = 45$	A1	1.1	If omitted, can still get all other marks
			$8(8 + 8 + 1) - R_m = 91$	B1	2.1	
			$W_{\text{crit}} = 49$	B1	1.1	
			Reject H_0. Significant evidence that there is a difference in median ratings/opinions have changed	M1ft	1.1	FT on TS (< 68) or CV
				A1ft	2.2b	FT on TS only. Allow "increased"
				[8]		SC: Sign or paired-sample test, max B2 (hypotheses)
7	(b)		Eliminate the difference between individual pupils' opinions	B1 [1]	3.5b	"Minimises the difference in tastes" B1 (BOD) Scores arbitrary: B1 (etc). Not "more powerful test".
7	(c)		A paired-sample signed-rank test would have been used	B1	3.5c	Must mention "paired sample" oe – not just "Wilcoxon

Question			Answer	Marks	AO	Guidance
				[1]		signed rank"
7	(d)		0.025×12870	M1	3.1a	$0.05 \times 12870 = 643.5$ M1
			$= 322$	A1	3.2a	321 or 322 or 643 (from 1-tail), must be integer
				[2]		

Question			Answer	Marks	AO	Guidance
8	**(a)**		$f(x) = \frac{1}{2}$	**B1**	3.3	Stated or implied, e.g. on diagram
			$\int_0^2 \frac{1}{2} a\cos(ax)\,dx = 0.3$	**M1**	3.1a	$\int f(x)\, a\cos ax\, dx$ & equated to 0.3
			$\left[\frac{1}{2}\sin(ax)\right]_0^2$	**B1**	1.1	Correct indefinite integral
			$\frac{1}{2}\sin(2a) = 0.3$	**M1**	2.1	Correct limits, solve
			$a = 0.32175\ldots$	**A1**	1.1	Answer, a.r.t. 0.322 (ignore other answers)
				[5]		
8	**(b)**		$F(y) = \frac{1}{2}\,y \qquad [0 \le y \le 2]$	**M1**	3.1a	Use their f(y) to obtain CDF
				A1	1.1	Correct F(y) (range need not be stated explicitly)
			$P(Y^2 \le m) = P(0 < Y \le \sqrt{m})$	**M1**	2.1	Find CDF of Y^2, allow m^2 instead of $\sqrt{m}$, or $\pm\sqrt{m}$, here
			$= F(\sqrt{m}) \qquad [= \frac{1}{2}\sqrt{m}]$	**A1**	1.1	Use F(y) correctly
			$\frac{1}{2}\sqrt{P_{60}} = 0.6$	**M1**	1.1	Equate to 0.6 and solve, need $\sqrt{m}$ here
			$P_{60} = 1.44$	**A1**	2.2a	1.44 or exact equivalent
				[6]		

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Cambridge
Assessment