Qualification Accredited

Oxford Cambridge and RSA

A LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y435/01 Summer 2024 series

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y435 series overview

This was the third full sitting of this paper, which covers the Extra Pure content of H645, A Level Further Mathematics B (MEI), since June 2019.

The paper comprised five compulsory questions, covering almost every area of the specification, and nearly all candidates attempted all of the questions. There was little evidence of time pressure. Overall standards were quite high and, although presentation was variable, solutions were generally set out well.

Throughout the paper it was expected that candidates were able to demonstrate a high level of understanding of the (sometimes abstract) topics. Several questions required proof of a given result and the advice is that candidates should provide a detailed explanation of all their working; the examiner should not need to fill in any gaps of reasoning or calculation even if some of the steps appear obvious.

The following questions were answered well by a majority of candidates:

1 (a), **1 (b), 1 (c)**, **1 (d)**, **2 (a)**, **2 (b)**, **2 (c)**, **4 (a)**, **4 (b) (ii)** and **4 (c) (i)**.

The following questions proved to be more challenging:

2 (d), **3 (a)**, **3 (d)**, **3 (e)**, **4 (c) (ii)**, **5 (a)** and **5 (b)**.

While the majority of candidates appear to do well when tackling most of the specification subject areas, Group Theory (including sets of numbers) still seems to represent a challenge to many. The highly abstract nature of the discipline presents a problem to candidates and most find the requirement to set out a detailed, rigorous explanation or proof, pitched at the correct level, very difficult indeed.

Candidates are also reminded of the importance of putting answers in the correct answer space in the Printed Answer Booklet or, if it is for some reason incorrectly located, clearly and correctly labelling it.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• set out their working neatly, clearly and logically • explained their reasoning rigorously • included every step, even seemingly trivial ones • used correct technical language and notation • answered the question asked in the specified manner • followed standard methodologies • made any necessary corrections to their work neatly and clearly • gave complete but concise explanations, citing known results or theorems in support, as required • did algebraic manipulation step by step • kept things as simple as possible.	• left out their working or presented it in a poorly or illogically structured way • skipped steps or explanations in their reasoning • lacked rigour in their explanations • used technical language or notation imprecisely or incorrectly • used incorrect or inappropriate methodology • did not answer the actual question asked or provide the answer in the required form • did not use the simplest approach to a question but instead overcomplicated • seemed not to have done sufficient practice and so never ironed out issues that inevitably arise when doing actual questions • tried to do too much algebraic manipulation in one go • did not read the questions carefully enough.

Question 1 (a)

1 A surface, S, is defined in 3-D by $z = \mathrm{f}(x, y)$ where $\mathrm{f}(x, y) = 12x - 30y + 6xy$.

(a) Determine the coordinates of any stationary points on the surface. **[5]**

This was a standard question to open the paper and the majority of candidates indeed gained full marks here. A number of candidates correctly found the partial derivates and x and y coordinates of the only stationary point but did not gain the last mark because they did not calculate the corresponding z coordinate. A small number of candidates lost marks either because they did not appear to know the correct condition for the existence of a stationary point or because they did not know the correct notation for the partial derivatives. A few candidates gave stationary point as a vector despite the fact that the question requested coordinates.

Some candidates found $\nabla \mathrm{f}$ rather than $\partial\mathrm{f}/\partial x$ and $\partial\mathrm{f}/\partial y$. These candidates were still able to obtain full marks. However, it is clear that some candidates had some gaps in their understanding of this topic.

Question 1 (b)

(b) The equation $z = \mathrm{f}(x, a)$, where a is a constant, defines a section of S.

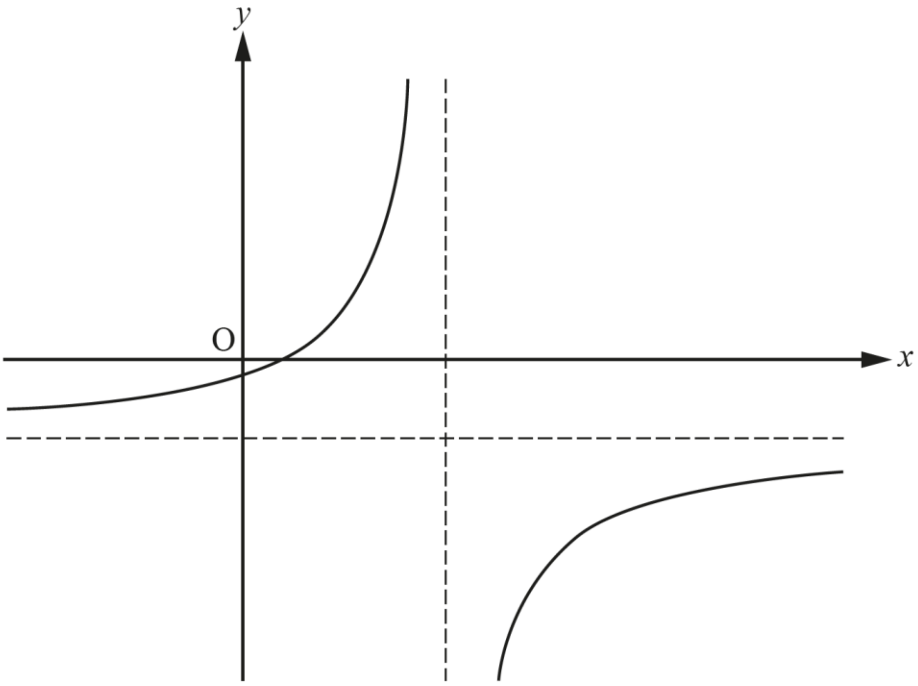
Given that this equation is $z = 24x + b$, find the value of a and the value of b. **[3]**

This question was a reasonably simple test of knowledge of a section of a surface. Again, the vast majority of the candidates showed that they were comfortable with the topic and obtained full marks.

Some candidates used y instead of a, which perhaps indicates a small limitation in their understanding. A small number of candidates derived $24x + b = 12x - 30a + 6ax$ but then did not know how to proceed; this alone was worth no marks; candidates had to show an awareness that the statement must be true for any x value to obtain the first mark.

Question 1 (c)

The diagram shows the contour $z = 12$ and its associated asymptotes.



(c) Find the equations of the asymptotes. **[3]**

Candidates found this question part rather harder than the earlier parts although it was surprising how many of the mistakes were due to basic errors with their algebra and arithmetic. Most candidates understood the basic procedure: replace z with 12 in the function, then rearrange to a useful form and then analyse that form. Most candidates understood that the vertical asymptote could be found by considering which x value caused the y value to be impossible to calculate while the horizontal asymptote could be found either by another such rearrangement and analysis or by considering the limit of y as x tended to infinity; making a second rearrangement of course increases the risk of a rearrangement error.

Candidates were asked for the equations of the asymptotes so a final answer such as '$x \neq 5$' did not receive the accuracy mark.

A small number of candidates used the equation of the contour line in implicit form and found its gradient function. They then considered the value of the gradient function to be either 0 or undefined to derive the required asymptote equations. In this particular case, with the graph shown in the question, this was considered acceptable. In general, however, this would not be considered to be an acceptable method for deriving the equations of the asymptotes, at least not without further analysis or explanation.

A very small number of candidates in effect invented their own form of the chain rule, using the partial derivatives to derive the gradient function. This methodology is incorrect and does not give the gradient function correctly: marks are only awarded 'without wrong working' (www) and so, in effect, any correct answer is a fortuitous coincidence and this approach received no marks. Candidates should be discouraged from inventing rules; any useful rule would have been taught to them.

Misconception

Partial derivatives are not absolute derivatives; rules which apply to the latter do not necessarily apply to the former.

Question 1 (d)

(d) By forming **grad** g, where $g(x, y, z) = f(x, y) - z$, find the equation of the tangent plane to S at the point where $x = 3$ and $y = 2$. Give your answer in vector form. **[3]**

Most candidates were able to form the vector **grad** g successfully. However, the method mark was not given until this vector was used in a manner which indicated that the candidate understood what the vector represented in terms of what was required by the question; in other words, that it is a normal to the required plane. It is all too evident that many candidates have an incomplete or simplistic understanding of what is happening in three dimensions as opposed to two and think that 'differentiating gives the tangent' or similar.

A number of candidates did not give their final answer in the required (vector) form. Other candidates left constant 'dotted' vectors in their final answers. These candidates were not given the final accuracy mark. If there is any doubt about requirements, candidates should ask themselves whether, for example, $y = 2x + 4\times5$ would be considered an acceptable final answer for the equation of a line in two dimensions.

Assessment for learning

Always check the question to make sure that you give what the question asks for and in the form that it requires.

Question 1 (e)

The point (0, 4, −120), which lies on S, is denoted by A.

The plane with equation $\mathbf{r}.\begin{pmatrix}3\\3\\-2\end{pmatrix} = 52$ is denoted by Π.

(e) Show that the normal to S at A intersects Π at the point (−360, 304, −110). **[3]**

Most candidates could again form **grad** g successfully. A small number attempted to use **grad** g from part **(d)** which earned no marks. Once again, the vector had to be used in a correct context before any method marks were given and candidates should be aware that it is understanding of the meaning of **grad** g, rather than simply its formation, which is the focus of assessment of this topic.

Candidates should also be aware that in a 'Show that' question, with the answer given, the onus is on them to demonstrate a clear understanding of *how* to arrive at the answer; solutions which contained no element of explanation, or which jumped from deriving the value of the parameter (i.e., $\lambda = -10$) straight to the given answer without showing any substitution, were not given full marks.

Assessment for learning

In a 'Show that' question, with the answer given, always provide enough detail to demonstrate to someone else exactly what you are doing.

Question 2 (a)

2 **(a)** Determine the general solution of the recurrence relation $2u_{n+2} - 7u_{n+1} + 3u_n = 0$. **[2]**

It was pleasing to note that almost every candidate got this question part fully correct.

Question 2 (b)

(b) Using your answer to part **(a)**, determine the general solution of the recurrence relation $2u_{n+2} - 7u_{n+1} + 3u_n = 20n^2 + 60n$. **[5]**

There was, however, considerably less success with this part than with part (a). While most candidates knew what was required, many candidates incorrectly used $an^2 + bn$ as their trial function, presumably on the basis that the driving function of the recurrence relation was also missing the constant term. Candidates would benefit from practising a good range of questions to experience the full range of eventualities that can arise in this topic.

Once the correct trial function has been specified, a question like this is largely a matter of book-keeping skills. However, many candidates did not go on to derive a fully correct solution. Typically, candidates will try to take too many steps in a single line of working (for example, expanding $(n + 2)^2$, multiplying it by a and expanding an outer bracket with a 7 all in one go). Almost inevitably this leads to error, for example, a missed term, a sign error or forgetting to multiply by one thing or another. Even at the end, after correctly obtaining $a = -10$ and $a - 2c = 0$, a surprisingly large number of such candidates still managed to obtain $c = 5$ rather than $c = -5$.

Some candidates have clearly been taught to consider a general form $u_{n+2} = \alpha u_{n+1} + \beta u_n + \mathrm{g}(n)$ and rearranged the recurrence relation to this form. There is no advantage to this and it merely increases the amount of manipulation required and, hence, the probability of making an error. Related to this is a more serious issue; some candidates have been taught to consider a general form $u_n = \alpha u_{n-1} + \beta u_{n-2} + \mathrm{g}(n)$ and attempted to rearrange the recurrence relation to this form. The problem here is that re-indexing in this manner will cause a change to $\mathrm{g}(n)$ but no candidates took account of this; therefore, candidates going down this path scored very few marks.

When presenting the answer candidates should remember that they have been asked to provide a general solution. Their answer should therefore include "$u_n =$". They should also write it in conventional form (which means, for example, that vulgar fractions being raised to a power should be bracketed).

Question 2 (c)

In the rest of this question the sequence $u_0, u_1, u_2, \ldots$ satisfies the recurrence relation in part **(b)**. You are given that $u_0 = -9$ and $u_1 = -12$.

(c) Determine the particular solution for u_n. **[3]**

Most candidates understood how to respond to this question. However, many candidates made avoidable errors either in substituting values or solving the simultaneous equations.

Some candidates sought to simplify their final answer, for example expressing $6\left(\frac{1}{2}\right)^n$ as $3\left(\frac{1}{2}\right)^{n-1}$. While this is not incorrect, there is no benefit in doing so and candidates are merely increasing the risk of making an error. It should also be pointed out that it is generally easier to deal with a position-to-term rule for u_n, say, if the rule uses n explicitly and consistently, rather than, say, $n - 1$, in one term and n in another.

Question 2 (d)

You are given that, as n increases, once the values of u_n start to increase, then from that point onwards the sequence is an increasing sequence.

(d) Use your answer to part **(c)** to determine, by direct calculation, the least value taken by terms in the sequence. You should show any values that you rely on in your argument. **[2]**

This question part unexpectedly proved to be one of the hardest questions on the paper and not simply because many candidates had not got part 2(c) correct. Many candidates seemed entirely uncertain as to how to go about the question, even though the question itself stated that it should be done 'by direct calculation'. It was intended that candidates were to use their particular solution to start generating subsequent terms of the sequence and to identify the point at which the sequence started to increase.

Of course, if their coefficient of 3^n was negative then the question simply does not work and this should have alerted candidates to a mistake that they had made.

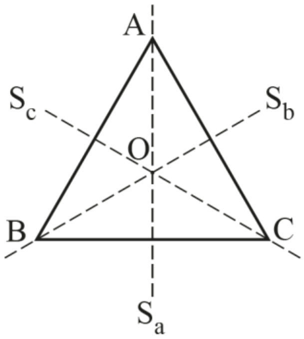
Because of the nature of the question, follow-through was not available for the accuracy mark for this question part but a method mark was still available to a candidate for calculating correctly (or showing a correct calculation) three consecutive terms in their sequence. However, not very many candidates, even including those who got part 2(c) correct, managed to get this mark. Stating the minimum value without sufficient justification was insufficient to score any marks in this part.

Some candidates tried to identify the minimum value by doing some analysis on the sequence; this often included either looking at the differences or using an estimate or differentiating (even though a sequence, by its nature, is not continuous) or deploying some combination of all of these. Candidates should have realised from the tariff ([2]), the available space in the Printed Answer Booklet and, of course, the clear instruction 'by direct calculation' that such approaches were not appropriate.

Question 3 (a)

3 **Fig. 3.1** shows an equilateral triangle, with vertices A, B and C, and the three axes of symmetry of the triangle, S_a, S_b and S_c. The axes of symmetry are fixed in space and all intersect at the point O.

Fig. 3.1

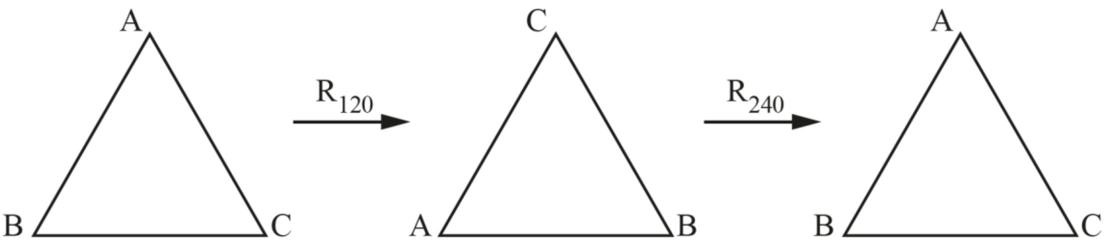


There are six distinct transformations under which the image of the triangle is indistinguishable from the triangle itself, ignoring labels.
These are denoted by I, M_a, M_b, M_c, R_{120} and R_{240} where

- I is the identity transformation
- M_a is a reflection in the mirror line S_a (and likewise for M_b and M_c)
- R_{120} is an anticlockwise rotation by 120° about O (and likewise for R_{240}).

Composition of transformations is denoted by $\circ$.
Fig. 3.2 illustrates the composition of R_{120} followed by R_{240}, denoted by $R_{240} \circ R_{120}$.
This shows that $R_{240} \circ R_{120}$ is equivalent to the identity transformation, so that $R_{240} \circ R_{120} = I$.

Fig. 3.2



(a) Using the blank diagrams in the Printed Answer Booklet, find the single transformation which is equivalent to each of the following.

- $M_a \circ M_a$
- $M_b \circ M_a$
- $R_{120} \circ M_a$

[3]

Again, this question proved to be unexpectedly difficult. The example of 'symmetries of geometric figures' is actually the first mentioned in the spec at Xa2, 'Illustrations of groups, and so candidates should be highly familiar with this kind of scenario.

However, it was very clear that a large number of candidates were not. The most common issue was that many, many candidates seem unaware that $\mathrm{A} \circ \mathrm{B}$ represents transformation B followed by transformation A, exactly as with matrix transformations which candidates should be familiar with from Core Pure. In any case, for the removal of doubt, an example was provided in the question so candidates only had to follow this example through.

Another issue was that some candidates seemed to think that the axes $\mathrm{S_a}$, $\mathrm{S_b}$ and $\mathrm{S_c}$ 'followed' the vertices A, B and C around, even though it is clearly stated in the question that they are 'fixed in space' (not to mention that a, b and c are not A, B and C).

Most candidates found the first mark straightforward. However, only around one third of the candidates were given all 3 marks. It should be noted that the demand of the question is 'using the blank diagrams' and so candidates were expected to complete the blank diagrams in the manner illustrated in the question.

Question 3 (b)

The set of the six transformations is denoted by G and you are given that (G, ○) is a group.

The table below is a mostly empty composition table for ○. The entry given is that for $R_{240} \circ R_{120}$.

First transformation performed is

followed by

○	I	M_a	M_b	M_c	R_{120}	R_{240}
I						
M_a						
M_b						
M_c						
R_{120}						
R_{240}					I	

(b) Complete the copy of this table in the **Printed Answer Booklet**. You can use some or all of the spare copies of the diagram in the **Printed Answer Booklet** to help. **[4]**

It was expected here that most of the table could be filled in by simple deduction and knowledge of such transformations, coupled with the answers to Question 3(a) and knowledge of group properties (for example, the Latin square property). For this part, completion of diagrams was not a requirement and the question merely suggested that they could be used to help.

Most candidates were able to obtain the first mark (for deducing the identity row and column, along with the identities obtained from performing the same reflection twice) and the second mark (for completing the rotation block correctly). However, the last 2 marks proved a lot more elusive and this is clearly a skill which candidates need to practise.

Many candidates did not seem to consider the Latin square property which is a fundamental for a Cayley table; it was not uncommon to see the same transformation occurring twice in a row or column and this should alert candidates to an error.

Question 3 (c)

(c) Explain why there can be **no** subgroup of $(G, \circ)$ of order 4. **[1]**

Most candidates recognised that what was needed here was an argument based on Lagrange's Theorem. However, only just over half of candidates managed to produce such an argument convincingly. Candidates need to recognise that there is a difference between understanding something in one's own mind and conveying this understanding to another person. Lagrange's theorem states that the order of a subgroup must be a factor of the order of a group. If this is accepted as the start point then candidates should realise that to apply it to this situation, they need to state that the order of the group is 6 and that since 4 is not a factor of 6 then 4 cannot be the order of any subgroup of this group. So, all three elements (Lagrange's Theorem, the order of G being 6 and 4 not being a factor of 6) are required. In this regard it was very common for candidates to neglect to mention that the order of G is 6.

The mark was not given here if candidates made a statement that was in some sense not true, for example confusing the words 'factor' and 'multiple' or stating that 'the subgroup must be a factor of the group' or similar.

Approach to 'Explain' questions

Candidates should be encouraged to try to read what they have written as if they were another person.

Exemplar 1

3(c)	By Lagrange's theorem the order of a subgroup must be a factor of the order of the group and 4 is not a factor of 6 so there can't be a subgroup of order 4.

In this example the candidate has not explicitly stated that the order of the group is 6. However, we did allow this to be inferred from the structure of the response; 'the order of the subgroup must be a factor of the order of the group. 4 is not a factor of 6'. However, this might not be the case in subsequent seasons and so candidates should be encouraged to provide all necessary information. An explanation should be complete; it should not be left to the examiner to make important inferences about what is meant.

Question 3 (d)

(d) A student makes the following claim.

"If all the proper non-trivial subgroups of a group are abelian then the group itself is abelian."

Explain why the claim is incorrect, justifying your answer fully. **[3]**

This question proved to be the hardest on the paper with a minority of candidates obtaining full marks on it.

To start with, many candidates appeared to misunderstand the request in the question, answering as if the question had said '**any** of the... subgroups' rather than '**all** of the... subgroups' which is a very different matter.

Many candidates also tried to explain in general how subgroups could be abelian while the main group might not be but such explanations were invariably not rigorous.

Candidates should realise that in a question such as this it is likely that the context of the question will provide the solution and so it proved here since the group G is itself a counter-example to the claim. All that was therefore required was justification and this is reasonably straightforward to provide, if tackled carefully.

Question 3 (e)

(e) With reference to the order of elements in the groups, explain why $(G, \circ)$ is **not** isomorphic to C_6, the cyclic group of order 6. **[1]**

This question, which again proved unexpectedly difficult, demonstrates the importance of reading questions carefully.

The question clearly requires candidates to refer to the **order of elements** in **both** groups, G and C_6. It was therefore actually straightforward to obtain the mark; one simply had to observe that there must be at least one element of order 6 in C_6 while there are none in G. Of course, deeper comparison could be made (for example, tabulating the orders of all elements in both groups). However, the vast majority of candidates did not take this approach, in spite of the demand. It was quite common to see an answer along the lines of 'G has no element of order 6 and so there is no generator'. However, such an answer, while possibly true, does not refer at all to the order of any element in C_6 and so does not fulfil the demand of the question. It should also be clear that it introduces an entirely different (and unexplained) concept, that of a generator, which has not been mentioned in the question part or, indeed, anywhere else in the question.

Question 4 (a)

4 The matrix **P** is given by $\mathbf{P} = \begin{pmatrix} 1 & 7 & 8 \\ -6 & 12 & 12 \\ -2 & 4 & 8 \end{pmatrix}$.

(a) Show that the characteristic equation of **P** is $-\lambda^3 + 21\lambda^2 - 126\lambda + 216 = 0$. **[3]**

This is a very standard question type and one which appears routinely in this paper. The majority of candidates appeared familiar with it and scored all 3 marks here.

Almost all candidates knew which determinant to consider and how to expand this determinant. However, a small number of candidates did not obtain the final mark because they did not write '$= 0$' in their solution, in spite of this being given and being requested to be shown.

Some candidates jumped from the expanded determinant form directly to the given answer. Such candidates have not sufficiently 'shown' what is required and so did not obtain the final mark either. In general, candidates must get to a correct form without brackets before they can be considered to have meet the request.

Assessment for learning

When the answer is given, make sure that your answer matches the given answer exactly.

Dealing with 'Show that' questions

When the answer is given and you are asked to show that it is true, do not jump to the given answer without sufficient justification just because you know it must be true. According to the specification, 'the explanation has to be sufficiently detailed to justify every step of [the] working.'

Question 4 (b) (i)

You are given that the roots of this equation are 3, 6 and 12.

(b) **(i)** Verify that $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ is an eigenvector of $\mathbf{P}$, stating its associated eigenvalue. **[2]**

This was yet another question which candidates, unexpectedly, found hard. The command word here, 'Verify', is critical. Candidates were simply expected to multiply the given vector into the matrix $\mathbf{P}$ and to show that the resulting vector, [3 –6 6] is equal to 3[1 –2 2]. Writing it in this form was sufficient evidence to show that candidates understood that this shows that [1 –2 2] is an eigenvector. However, candidates were expected to state explicitly (or at least indicate somehow) that the eigenvalue is 3, as requested by the question; the embedded form alone was not considered sufficient for the final mark.

Of course, candidates who used the fundamental property of the eigenvector, $\mathbf{e}$, of a matrix, $\mathbf{P}$, that is, that $\mathbf{Pe} = \lambda\mathbf{P}$, were more likely to be successful than those who started from a consequence of this, that $(\mathbf{P} - \lambda\mathbf{I})\mathbf{e} = \mathbf{0}$.

A considerable number of candidates felt that they needed to *derive* the eigenvectors; candidates should have realised that this was not the appropriate approach given the tariff ([2]) and the small amount of space available in the Printed Answer Booklet.

Question 4 (b) (ii)

(ii) The vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is an eigenvector of $\mathbf{P}$ with eigenvalue 6.

Given that $z = 5$, find x and y. **[3]**

Candidates had more success with this question than with 4 (b) (i) with the majority obtaining full marks. Most candidates understood that they had to derive and solve simultaneous equations. No marks were available until candidates had used all the given information (i.e. $\lambda = 6$, $z = 5$ and the fact that $[x\ y\ z]$ is an eigenvector of $\mathbf{P}$).

The most common method error was to set $(\mathbf{P} - 6\mathbf{I})\mathbf{e} = 6\mathbf{e}$ which gained no marks.

A few candidates made algebraic errors which would have easily become apparent had they checked their final answer to see if it really was an eigenvector. Some candidates obtained the correct answer but then claimed that this was equal to [3 1 1], which is not the case. In fact, careful reading of the demand would have shown that all that was actually required was the values of x and y, although in this question embedded answers were accepted.

Question 4 (c) (i)

You are given that $\mathbf{P}$ can be expressed in the form $\mathbf{EDE}^{-1}$, where $\mathbf{E} = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & 2 \end{pmatrix}$ and $\mathbf{D}$ is a diagonal matrix. The characteristic equation of $\mathbf{E}$ is $-\lambda^3 + 7\lambda^2 - 15\lambda + 9 = 0$.

(c) (i) Use the Cayley-Hamilton theorem to express $\mathbf{E}^{-1}$ in terms of positive powers of $\mathbf{E}$. **[2]**

While the majority of candidates managed to obtain both marks for this part many others lost a mark unnecessarily. Candidates should realise that every term in a matrix equation should be a matrix and hence an expression like $15\mathbf{E} + 9$, where $\mathbf{E}$ is a matrix, is meaningless; applying the Cayley-Hamilton theorem is not simply a case of replacing λ with $\mathbf{E}$.

Similarly, some candidates were not given the final mark because they omitted the required $\mathbf{I}$ in the final answer.

A small number of candidates seemed not to understand the presence of $\mathbf{I}$ in the final answer and either replaced it by $\mathbf{E}^0$ (which was allowed, even though it is not a positive power of $\mathbf{E}$) or did some further manipulation to eliminate it. However, candidates should be aware that if they are asked to write something in terms of something else this does not preclude the use of appropriate constants in their answer.

Candidates should also realise that dividing by a matrix is not possible; rather, the equivalent operation is multiplying by the inverse matrix.

Misconception

It is not possible to divide by a matrix.

Question 4 (c) (ii)

(ii) Hence find $\mathbf{E}^{-1}$. **[1]**

This proved to be another question where candidates found it hard to attain the mark. Once again, careful reading of the wording, and particularly the word 'Hence', indicates that simply writing the matrix $\mathbf{E}^{-1}$ (which can, of course, be found using a calculator function) was not sufficient; candidates had to show *how* they could use 4 (c) (i) to find it.

Candidates need to understand that this paper is a test of specification knowledge, rather than calculator usage, and this technique is specifically mentioned in the spec at Xm5; candidates should always be prepared to demonstrate explicitly their knowledge of on-spec techniques even if there is a simpler way of finding what has been requested.

Question 4 (c) (iii)

(iii) By identifying the matrix $\mathbf{D}$ and using $\mathbf{P} = \mathbf{EDE}^{-1}$, determine $\mathbf{P}^4$. **[4]**

Most candidates knew what was required here and, happily, in this question part it was rare for candidates simply to write down the matrix $\mathbf{P}^4$ (which again can be found directly using calculator functions).

Occasionally, candidates were confused and used 1, 3 and 3 (i.e. the eigenvalues of $\mathbf{E}$) as the diagonal entries of $\mathbf{D}$, rather than 3, 6 and 12, the eigenvalues of $\mathbf{P}$. Also, a small number of candidates did not appreciate that these elements had to be in the appropriate order (i.e. the order of the eigenvectors of $\mathbf{P}$ which formed the columns of $\mathbf{E}$; the association between these eigenvectors and eigenvalues was deducible from earlier work in the question).

Some candidates were not given full marks because they did not explicitly identify matrix $\mathbf{D}$, as required by the demand of the question.

Some candidates also did not explicitly show $\mathbf{D}^4$; the command word here is 'determine' so candidates should know that they need to justify their working. The essence of this technique is that powers of a diagonal matrix are easy to find and so candidates should be able to judge that this level of justification is required. Candidates can use calculators in such questions to help with the matrix calculation but again they should bear in mind that they are being tested on specification knowledge and should be able to demonstrate this where required.

Question 5 (a)

5 In this question you may assume that if p and q are distinct prime numbers and $p^{\alpha} = q^{\beta}$ where $\alpha, \beta \in \mathbb{Z}$, then $\alpha = 0$ and $\beta = 0$.

(a) Prove that it is **not** possible to find a and b for which $a, b \in \mathbb{Z}$ and $3 = 2^{\frac{a}{b}}$. **[2]**

It is clear that the majority of candidates find this topic difficult to grasp. This might be because there is less resource material available. It might also be because it is often linked, as with this question, to formal proofs.

Most candidates were not able to produce a convincing proof here. Candidates should be aware that if they are asked to show that something is **not** possible then proof by contradiction is likely to be the 'go-to' method, and so it proves here. The small number who did set up the proof by contradiction properly generally fared well. A common error was to omit to specify that *a* and *b* must be integers; if this was the only error then candidates were given 1 mark rather than 2.

But far too many attempted proofs were simply not rigorous enough; for example, a proof which starts $3 \neq 2^{\frac{a}{b}}$ or leads to $\frac{a}{b} = \log_2 3$ is unlikely to be useful since it does not place much of a restriction on what a and b could be. Finally, many candidates seemed to be under the misapprehension that if *a* and *b* are not integers then *a*/*b* is not a rational number; this is clearly not the case. Candidates must learn the definitions of these types of numbers carefully.

Misconception

The ratio of two non-integers may be rational (either an integer or a non-integer) or irrational.

Question 5 (b)

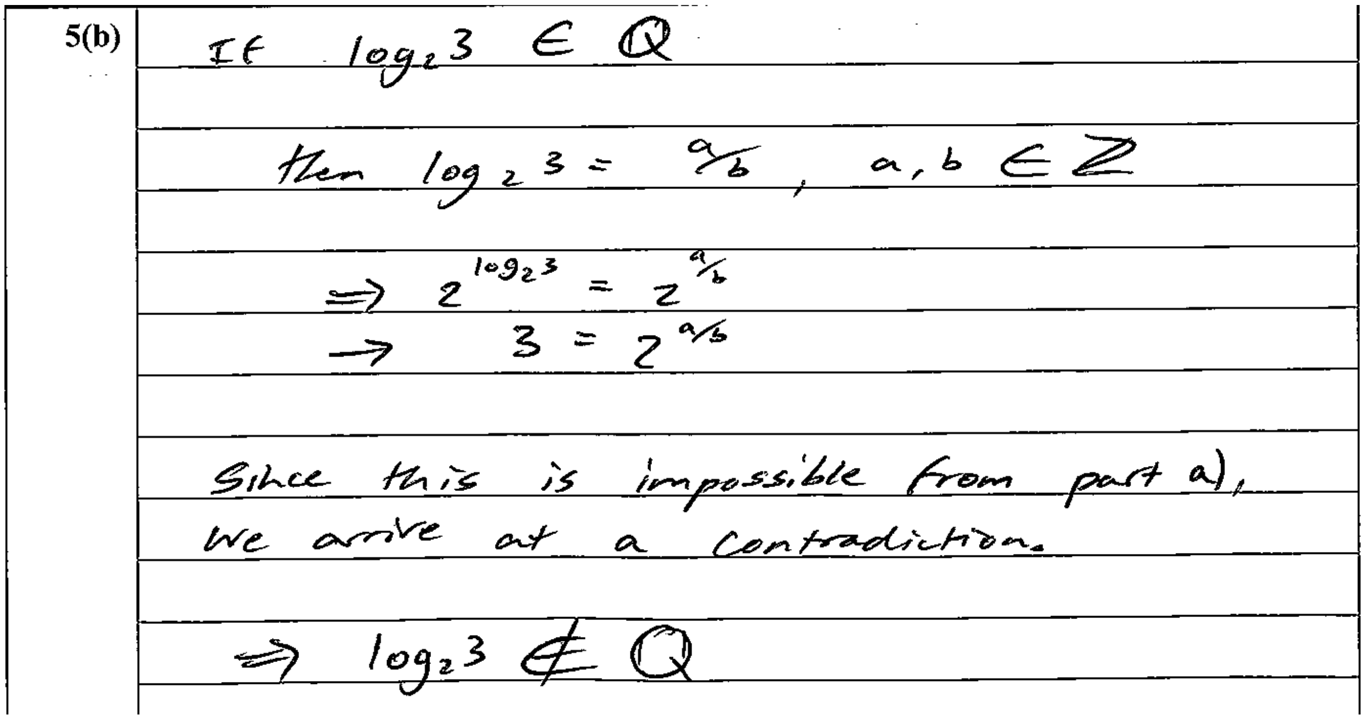
(b) Deduce that $\log_2 3 \notin \mathbb{Q}$. **[2]**

Even without the word 'Deduce' it should have been clear that part (a) would be the starting point to the solution for part (b) and indeed another proof by contradiction using part (a) gave the required solution very quickly; the two parts are really saying the same thing, one in exponential form and the other in logarithmic form. Indeed, slightly more candidates were given full marks for part (b) precisely because of this.

Again, candidates who either did not attempt proof by contradiction or did not set it up correctly did not tend to fare at all well.

Candidates would be well advised to examine the structure of rigorous proofs and to practise carrying them out.

Exemplar 2


5(b)

If $\log_2 3 \in \mathbb{Q}$

then $\log_2 3 = a/b$, $a, b \in \mathbb{Z}$

$\Rightarrow 2^{\log_2 3} = 2^{a/b}$

$\rightarrow 3 = 2^{a/b}$

Since this is impossible from part a), we arrive at a contradiction.

$\Rightarrow \log_2 3 \notin \mathbb{Q}$

This candidate has set out a neat solution for part (a) and then referred to this in their solution to part (b).

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