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A LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y434/01 Summer 2024 series

Contents

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Introduction

Our examiners’ reports are produced to offer constructive feedback on candidates’ performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates’ performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

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Paper 1 series overview

The Numerical Methods minor optional paper counts for 16⅔ % of the qualification OCR A Level Further Mathematics B (MEI) (H645). There is one examination paper which lasts 1 hour 15 minutes. Candidates are expected to know the content of A Level Mathematics and the core Pure mandatory paper (Y420).

Candidates should be able to use the iterative capability of a calculator in the examination. No credit is given for writing down solutions generated by equation solvers.

Candidates should be familiar with the command words used in this specification, specifically, that a question which begins 'determine…' is a signal that sufficient detail of any necessary working must be given to earn full credit. Where a question explicitly requests candidates to 'use …' then there must be sufficient evidence to show that the requested method or expression has been used in the solution.

It is expected that candidates will have routinely used a spreadsheet throughout the course. In the examination candidates will be given output from spreadsheets and may be asked: what certain cells represent; to explain or give formulae for certain cells; to give solutions and justify their accuracy and to comment on errors, convergence, or order of a method.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• made efficient use of the iterative capability of their calculator when implementing iterative formulae, and provided sufficient detail to justify the precision quoted in their answers • knew that a request to determine a result meant that a certain level of detail was required in the response • were able to interpret and give spreadsheet formulae • presented answers to the precision requested • understood the relevance of the ratio of differences in the context of commenting on the order of a method or in terms of developing error analysis through extrapolation • gave precise explanations of why a particular method failed or why it could not be applied.	• were able to use the iterative capability of their calculator when implementing iterative methods, but did not show sufficient detail of their work to justify the precision quoted • knew how to approach a particular problem with a request to determine the outcome, but did not supply sufficient detail to justify their result • understood the processes involved in spreadsheet formulae, but were unable to write them out correctly • presented their answers to a different precision to that requested in the question • had an incomplete understanding of the term ratio of differences and how to interpret it • were not able to articulate their explanations fully.

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Question 1 (a) and (b)

1 The table shows some values of x, together with the associated values of a function, $\mathrm{f}(x)$.

x	1.9	2	2.1
$\mathrm{f}(x)$	0.5842	0.6309	0.6753

(a) Use the information in the table to calculate the most accurate estimate of $\mathrm{f}'(2)$ possible. **[2]**

(b) Calculate an estimate of the error when $\mathrm{f}(2)$ is used as an estimate of $\mathrm{f}(2.05)$. **[2]**

Nearly all candidates earned both marks in part (a) and many went on to do the same in part (b).

Candidates who did less well may have over-complicated things by finding 0.653675 in part (b) and then making a slip.

Some candidates did not appreciate the relevance of their answer to part (a) when answering part (b) and adopted a variety of unsuccessful strategies.

Question 2 (a), (b) and (c)

2 You are given that $a = \tanh(1)$ and $b = \tanh(2)$.

A is the approximation to a formed by rounding $\tanh(1)$ to 1 decimal place.

B is the approximation to b formed by rounding $\tanh(2)$ to 1 decimal place.

(a) Calculate the following.

- The relative error R_A when A is used to approximate a.
- The relative error R_B when B is used to approximate b. **[3]**

(b) Calculate the relative error R_C when $C = \frac{A}{B}$ is used to approximate $c = \frac{a}{b}$. **[2]**

(c) Comment on the relationship between R_A, R_B and R_C. **[1]**

In general candidates answered this question extremely well, with many earning full marks.

Less successful candidates may have made a slip in the arithmetic in one of the calculations or not appreciated the connection between the relative errors when answering part (c). A few candidates appear to have anticipated the result in part (c) and used it to find their answer to part (b).

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Question 3 (a), (b), (c) and (d)

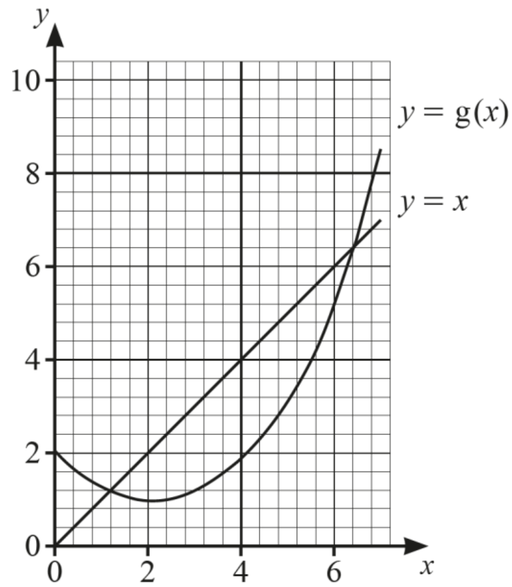
3 The equation $x^2 - \cosh(x-2) = 0$ has two roots, α and β, such that $\alpha < \beta$.

(a) Use the iterative formula

$x_{n+1} = \mathrm{g}(x_n)$ where $\mathrm{g}(x_n) = \sqrt{\cosh(x_n - 2)}$,

starting with $x_0 = 1$, to find α correct to **3** decimal places. **[2]**

The diagram shows the part of the graphs of $y = x$ and $y = \mathrm{g}(x)$ for $0 \leqslant x \leqslant 7$.



(b) Explain why the iterative formula used to find α **cannot** successfully be used to find β, even if x_0 is very close to β. **[1]**

(c) Use the relaxed iteration

$x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n)$,

with $\lambda = -0.21$ and $x_0 = 6.4$, to find β correct to **3** decimal places. **[2]**

In part **(c)** the method of relaxation was used to convert a divergent sequence of approximations into a convergent sequence.

(d) State **one** other application of the method of relaxation. **[1]**

Candidates who answered this question well gave succinct answers to parts (b) and (d) and made efficient and effective use of the iterative capability of their calculator in parts (a) and (c). They may have lost a mark in (a) or (c) by showing an insufficient number of iterates to justify 3 decimal place accuracy.

Candidates who did less well may have given a vague response to part (b) such as 'the gradient is bigger than 1'. In parts (a) and (c) they may have shown only the first two or three iterates and perhaps their last one before stating their answer to 3 decimal places without justification.

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Question 4 (a), (b), (c), (d) and (e)

4 Between 1946 and 2012 the mean monthly maximum temperature of the water surface of a lake in northern England has been recorded by environmental scientists. Some of the data are shown in **Table 4.1**.

Table 4.1

Month	May	June	July	August	September
t = Time in months	0	1	2	3	4
T = Mean temperature in °C	8.8	13.2	15.4	15.4	13.3

Table 4.2 shows a difference table for the data.

Table 4.2

t	T	ΔT	ΔT^2
0	8.8		
1	13.2		
2	15.4		
3	15.4		
4	13.3		

(a) Complete the copy of the difference table in the **Printed Answer Booklet**. **[2]**

(b) Explain why a quadratic model may be appropriate for these data. **[1]**

(c) Use Newton's forward difference interpolation formula to construct an interpolating polynomial of degree 2 for these data. **[4]**

This polynomial is used to model the relationship between T and t. Between 1946 and 2012 the mean monthly maximum temperature of the water surface of the lake was recorded as 8.9 °C for October and 7.5 °C for November.

(d) Determine whether the model is a good fit for the temperatures recorded in October and November. **[2]**

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A scientist recorded the mean monthly maximum temperature of the water surface of the lake in 2022. Some of the data are shown in **Table 4.3**.

Table 4.3

Month	May	June	July	August	September
t = Time in months	0	1	2	3	4
T = Mean temperature in °C	10.3	14.7	16.9	16.9	14.8

(e) Adapt the polynomial found in part **(c)** so that it can be used to model the relationship between T and t for the data in **Table 4.3**. **[1]**

There were many excellent responses to this question, with most candidates earning full marks or close to full marks. They may have made minor slips such as leaving their answers to part (c) and (e) in terms of *x* instead of *t* or by commenting that the second differences are all equal in value, rather than observing that they are approximately equal.

The small number of candidates who were less successful were unable to make correct substitutions into Newton's forward difference interpolation formula and were therefore only able to access the marks to parts (a) and (b).

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Question 5 (a), (b) and (c)

5 The root of the equation $f(x) = 0$ is being found using the method of interval bisection. Some of the associated spreadsheet output is shown in the table below.

	A	B	C	D	E	F
1	a	$f(a)$	b	$f(b)$	c	$f(c)$
2	2	−0.6109	3	6.08554	2.5	1.43249
3	2	−0.6109	2.5	1.43249	2.25	0.17524
4	2	−0.6109	2.25	0.17524	2.125	−0.2677
5	2.125	−0.2677	2.25	0.17524	2.1875	−0.0598
6						

The formula in cell B2 is `=EXP(A2)-A2^2-A2-2` .

(a) Write down the equation whose root is being found. **[2]**

(b) Write down a suitable formula for cell E2. **[1]**

The formula in cell A3 is `=IF(F2<0,E2,A2)` .

(c) Write down a similar formula for cell C3. **[1]**

Candidates who were successful in parts (a) (b) and (c) demonstrated a clear understanding of how to apply and interpret spreadsheet formulae. They may have omitted '= 0' in part (a).

Candidates who did less well may have omitted '=' in parts (b) and (c) or did not recognise 'EXP' in part (a).

Question 5 (d) and (e)

(d) Complete row 6 of the table on the copy in the **Printed Answer Booklet**. **[2]**

(e) **Without** doing any calculations, write down the value of the root correct to the number of decimal places which seems justified. You must explain the precision quoted. **[1]**

Most candidates answered these two parts very well. They completed the table successfully and used the interval quoted in the mark scheme to justify a precision of 1 decimal place.

Candidates who were less successful may have made a slip in the table or quoted an incorrect precision.

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Question 5 (f)

(f) Determine how many more applications of the bisection method are needed such that the interval which contains the root is less than 0.0005. **[3]**

Some candidates interpreted the question to mean how many more applications are needed after row 5 in the spreadsheet rather than after the work they had just completed in row 6. Either interpretation was allowed to score full marks.

Candidates who did well in this question gave sufficient detail of their working to earn full marks.

Candidates who did less well may have adopted one of the approaches outlined in the mark scheme, but given a final answer of 11, or did not show sufficient detail of their reasoning.

Question 6 (a) and (b)

6 **Table 6.1** shows some values of x and the associated values of a function, $y = \mathrm{f}(x)$.

Table 6.1

x	1.5	1	2
$\mathrm{f}(x)$	0.840 89	1	1.189 21

(a) Explain why it is **not** possible to use the central difference method to calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 1$. **[1]**

(b) Use the forward difference method to calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 1$. **[2]**

Candidates who did well in parts (a) and (b) commented that the other two x-values are greater than 1 or that f(0.5) or f(0) are not available. The overwhelming majority of candidates competed part (b) successfully.

Candidates who did less well usually commented that 1.5 and 2 are at different distances from 1.

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Question 6 (c) (i) and (ii)

A student uses the forward difference method to calculate a series of approximations to $\frac{dy}{dx}$ when $x = 2$ with different values of the step length, h.

These approximations are shown in **Table 6.2**, together with some further analysis.

h	0.8	0.4	0.2	0.1	0.05	0.025	0.0125	0.00625
approximation	0.130452	0.138647	0.143381	0.145942	0.147277	0.147959	0.148304	0.148477
difference		0.008195	0.004734	0.002561	0.001335	0.000682	0.000345	0.000173
ratio			0.577633	0.541099	0.521186	0.510762	0.505424	0.502723

(c) **(i)** Explain what the ratios of differences tell you about the order of the method in this case. **[2]**

(ii) Comment on whether this is unusual. **[1]**

Candidates who did well in part (c) commented that the ratio is converging to 0.5 and were able to relate this to the order of the method. They knew that that this was to be expected. They may have lost a mark through not articulating themselves well, for example by commenting that the ratios are all approximately 0.5.

Candidates who were less successful did not realise that the ratios are converging and were usually unable to access any marks in this part.

Question 6 (d)

(d) Determine the value of $\frac{dy}{dx}$ when $x = 2$ as accurately as possible. You must justify the precision quoted. **[4]**

Candidates who answered this question successfully recognised the opportunity to extrapolate to infinity and usually went on to obtain an answer within the specified range. Some candidates lost the final mark by comparing their answer with 0.148477 to give a final answer of 0.15.

Candidates who were less successful adopted the correct approach but made a slip in the calculation – see Exemplar 1 – or subtracted $0.000173 \times \frac{r}{1-r}$ from 0.148477 instead of adding it.

Extrapolation to infinity improves accuracy

In this question, comparison of the estimates of the gradient at $x = 2$ with $h = 0.0125$ and 0.00625 leads to the deduction that $\frac{dy}{dx} \approx 0.15$ to 2 significant figures. Candidates who appreciated that extrapolating to infinity will generally help them to give a more precise approximation than can be obtained by such a comparison generally scored full marks in part (d).

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Exemplar 1

6(d)	extrapolating to infinity:
	$\frac{dy}{dx} = 0.148477 + 0.000173 \times \left(\frac{0.502723}{1-0.502723}\right)$
	$= 0.1486525946$

6(d)	**(continued)**
	comparing to value when h = ~~0.06~~ 0.00625
	0.15 (2dp) seems certain

This candidate writes down the correct extrapolation to infinity for M1M1 but makes a slip in the calculation to generate an answer which lies outside the range to earn A1. In this case the final conclusion would not have earned A1 for an over cautious final answer of 0.15.

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Question 7 (a) and (b)

7 A student is using a spreadsheet to find approximations to $\int_0^1 f(x)\,dx$ using the midpoint rule, the trapezium rule and Simpson's rule. Some of the associated spreadsheet output with $n = 1$ and $n = 2$, is shown in **Table 7.1**.

Table 7.1

n	M_n	T_n	S_{2n}
1	0.612547	1	
2	0.639735		

(a) Complete the copy of **Table 7.1** in the **Printed Answer Booklet**. Give your answers correct to **5** decimal places. **[3]**

(b) State the value of $\int_0^1 f(x)\,dx$ as accurately as possible. You must justify the precision quoted. **[1]**

Most candidates successfully answered part (a) and (b). They successfully completed the table, or they may have lost a mark by giving their answers to a different precision. In part (b) they compared the values of S_2 and S_4 to justify a precision of 1 decimal place.

Candidates who did less well may have slipped up in one or more of the calculations in part (a) or gave their answer to a precision which could not be justified in part (b).

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Question 7 (c)

The student calculates some more approximations using Simpson's rule. These approximations are shown in the associated spreadsheet output, together with some further analysis, in **Table 7.2**. The values of S_2 and S_4 have been blacked out, together with the associated difference and ratio.

Table 7.2

n	S_{2n}	difference	ratio
1	[redacted]		
2	[redacted]	[redacted]	
4	0.674353	−0.0209	[redacted]
8	0.665199	−0.00915	0.438059
16	0.661297	−0.0039	0.426286
32	0.659675	−0.00162	0.415762
64	0.659015	−0.00066	0.406785

(c) The student checks some of her values with a calculator. She does **not** obtain 0.406785 when she calculates $-0.00066 \div (-0.00162)$. Explain whether the value in the spreadsheet, or her value, is a more precise approximation to the ratio of differences in this case. **[2]**

Most candidates answered this question very well. They commented that the spreadsheet stores values to a greater precision than are displayed and went on to say that the spreadsheet uses these values in the calculation whereas the student has used less precise displayed values. So, the value is the spreadsheet is more precise than the value found using the calculator.

Candidates who did less well generally made the first point correctly but were not able to correctly articulate the reason for the value in the spreadsheet being more precise.

Candidates who did not do well in this question usually thought that the calculator value was more precise because it had more digits displayed.

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Question 7 (d) (i), (ii) and (iii)

(d) **(i)** State the order of convergence of the values in the ratio column. You must justify your answer. **[1]**

(ii) Explain what the values in the ratio column tell you about the order of the method in this case. **[2]**

(iii) Comment on whether this is unusual. **[1]**

Candidates who did well in part (i) noted that the values in the ratio column appear to be converging to a constant, which suggests first order convergence, although they may not have articulated this well. In part (ii) they observed that 0.25 < 0.4 < 0.5 so the method is between first and second order in this case. They may have lost a mark because they did not make an explicit comparison, or because they did not explain their observation sufficiently clearly. In part (iii) they either made reference to the convergence ratio when *h* is halved (1/16) or to the usual order of the method when commenting that the observed order is unusual.

Candidates who did not do well were often not able to relate the convergence of the ratio of differences to the order of the method. A number of candidates commented that Simpson's rule is a first or second order method.

Question 7 (e)

(e) Determine the value of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$ as accurately as you can. You must justify the precision quoted. **[4]**

Candidates who did well in this question extrapolated to infinity from S_{128} to obtain a value within the specified range. They earned the final mark by quoting a precision of 4 or 5 decimal places which was justified by commenting on the improvement in accuracy achieved by this method, or by quoting their answer to 3 decimal places and comparing their result explicitly with S_{128}.

Candidates who did less well may have given a suitable precision to their final answer but were unable to justify the precision quoted. Some candidates made a sign error by adding $0.00066\times\frac{r}{1-r}$ to S128 or by rounding their value of r up to a value outside the allowed range. A few candidates carried out a partial extrapolation or used Richardson's extrapolation to earn a special case mark.

A correct extrapolation to infinity may be written in different forms

We expect to see $0.659015 - 0.00066\times\frac{r}{1-r}$ where *r* meets the criteria in the mark scheme. However, sometimes this may be written as $0.659675 - (0.00066 + 0.00066r + 0.00066r^2 \ldots)$ or the simplified result $0.659675 - 0.00066 \times \frac{1}{1-r}$

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