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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y433/01 Summer 2024 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y433 series overview

This is a minor option paper for MEI Further Mathematics. It assesses Modelling with Algorithms under the three strands of algorithms, networks and linear programming.

The evidence suggested that candidates had enough time to attempt all questions in the examination paper. The median mark is slightly higher, with a slightly greater negative skew to the mark distribution, than in 2023.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• attempted all questions and gave responses that were appropriate for the number of marks available • worked neatly and explained their working where appropriate • answered written responses precisely and unambiguously • checked all arithmetic.	• did not give written responses to questions asking for an explanation • worked in a muddled way and misread their own letters or numerical values • did not read the questions carefully enough • did not use all the information provided in the questions.

Assessment for learning

Centres should advise candidates that erased work can sometimes still show through on the scanned copy and may lead to ambiguities especially when it has been overwritten or in diagrams.

Question 1 (a)

1 The values below represent the weights of eleven suitcases, in kg.

23 15 17 10 35 18 12 21 22 11 8

(a) Show the result of applying the first fit algorithm to pack the weights into containers that can hold a maximum of 50 kg. **[2]**

Most candidates were able to apply the first fit algorithm correctly.

A few candidates missed weights or repeated a suitcase.

Question 1 (b)

(b) Show the result of applying the first fit decreasing algorithm to pack the weights into containers that can hold a maximum of 50 kg. **[2]**

Most candidates were able to apply the first fit decreasing algorithm correctly.

A few candidates missed weights or repeated a suitcase.

Assessment for learning

Candidates should make sure that it is clear the order in which items are placed using a packing algorithm.

Question 1 (c)

(c) Give a reason why the number of containers used for the packing of the suitcases in part **(b)** is optimal. **[1]**

Candidates attempted a variety of methods to answer this question. Those that deviated from the given method in the scheme were rarely able to provide a strong enough explanation.

Some candidates calculated a lower bound, but incorrectly reasoned that 3.84 was approximately equal to 4, demonstrating a lack of understanding of their calculation.

Question 2 (a)

2 The table shows the distance, in kilometres, along the direct roads between eight towns, A to H. A dash (-) indicates that there is no direct road linking the towns.

	A	B	C	D	E	F	G	H
A	-	31	21	-	-	-	-	-
B	31	-	8	12	-	-	10	20
C	21	8	-	24	-	35	20	36
D	-	12	24	-	30	-	-	-
E	-	-	-	30	-	12	40	10
F	-	-	35	-	12	-	-	25
G	-	10	20	-	40	-	-	15
H	-	20	36	-	10	25	15	-

(a) Use Dijkstra's algorithm to find the shortest path from A to E for the network shown in the table. **[5]**

Most candidates were able to attempt Dijkstra's algorithm. Candidates were able to update working values, and place labels in the given structure.

Some candidates did not complete the shortest path from A to E, even when they had correctly completed workings.

Question 2 (b) (i)

(b) (i) Apply the **tabular** form of Prim's algorithm on the diagram in the Printed Answer Booklet, **starting at vertex A**, to find the total length of the arcs in the minimum spanning tree for the network shown in the table. **[3]**

Most candidates attempted this question and were able to correctly select the weights in the printed table.

Some candidates did not state the order that the arcs were selected or label them across the top of the table.

Question 2 (b) (ii)

(ii) On the vertices provided in the Printed Answer Booklet draw the minimum spanning tree for this network. **[1]**

Many correct answers, some candidates made at most one error.

Question 2 (c)

(c) Write down the length of the shortest path from A to F which passes through all the towns **except** D. **[1]**

Candidates mostly answered this correctly.

The candidates that were unable to correctly apply Prim's algorithm in part (b) were less likely to achieve this mark.

Question 2 (d)

(d) A computer takes 0.016 secs to solve the minimum connector problem on the network shown in the table using Prim's algorithm.

Approximately how long will it take the computer to solve the minimum connector problem on a network with 1000 edges using Prim's algorithm? **[2]**

Most candidates demonstrated that quadratic order was needed for Prim's algorithm.

Many candidates did not carefully read the information and did not use 16 for the number of arcs in the network shown.

Question 3 (a)

3 A directed network consists of eight nodes, S, A, B, C, D, E, F and T, and seventeen arcs. The arcs represent a system of pipes through which a fluid flows continuously from the source S to a sink T. The weight of each arc is the corresponding capacity of that pipe in litres per minute.

The following LP formulation can be used to find the maximum flow from S to T in this network.

Maximise $SA + SB + SC + SD$

subject to
$$SA + BA - AE - AT = 0$$
$$SB + CB - BA - BE - BF - BT = 0$$
$$SC + DC - CB - CF = 0$$
$$SD - DC - DF = 0$$
$$AE + BE - EF - ET = 0$$
$$BF + CF + DF + EF - FT = 0$$
$$SA \leqslant 25,\ SB \leqslant 61,\ SC \leqslant 37,\ SD \leqslant 35,\ AE \leqslant 25,\ AT \leqslant 38,$$
$$BA \leqslant 39,\ BE \leqslant 40,\ BF \leqslant 17,\ BT \leqslant 49,\ CB \leqslant 12,\ CF \leqslant 26,$$
$$DC \leqslant 28,\ DF \leqslant 19,\ EF \leqslant 25,\ ET \leqslant 27,\ FT \leqslant 32$$

(a) Complete the diagram in the Printed Answer Booklet to show the directed network represented in the above LP formulation. **[2]**

Many correct responses. Some candidates made errors, with either missing arrows or weights.

Question 3 (b)

(b) A cut α partitions the vertices into the sets {S, A, B, C, D}, {E, F, T}.

Calculate the capacity of cut α. **[1]**

There were many correct responses to this question.

Question 3 (c)

(c) By considering the objective function of the LP and the answer to part **(b)**, explain why the maximum flow through the network cannot be equal to the capacity of cut α. **[1]**

Some candidates need to consider the command 'explain'. The numerical value of the cut was not always compared to the value of the objective function and/or the candidates did not explain what this comparison showed.

Question 3 (d)

The LP formulation was run in an online solver and some of the output is shown below.

VARIABLE	VALUE
SA	25.00000
SB	61.00000
AE	25.00000
AT	38.00000
BT	35.00000
CB	12.00000
DC	0.00000
DF	19.00000
EF	0.00000
FT	32.00000

(d) By completing the diagram in the Printed Answer Booklet, determine the maximum value of the flow through the network. **[3]**

The majority of candidates were able to fill in the given weights and directions on the network.

Some candidates were unable to complete the network with consistent flow patterns.

Question 3 (e)

(e) Use a suitable cut to prove that this is the maximum flow. **[2]**

Only a small number of candidates correctly found the minimum cut.

Those candidates that were able to state the correct cut could then compared this to the value in part (d) to prove a flow is maximum.

Misconception

Many candidates incorrectly use the maximal flow diagram instead of the capacity diagram to find the cut.

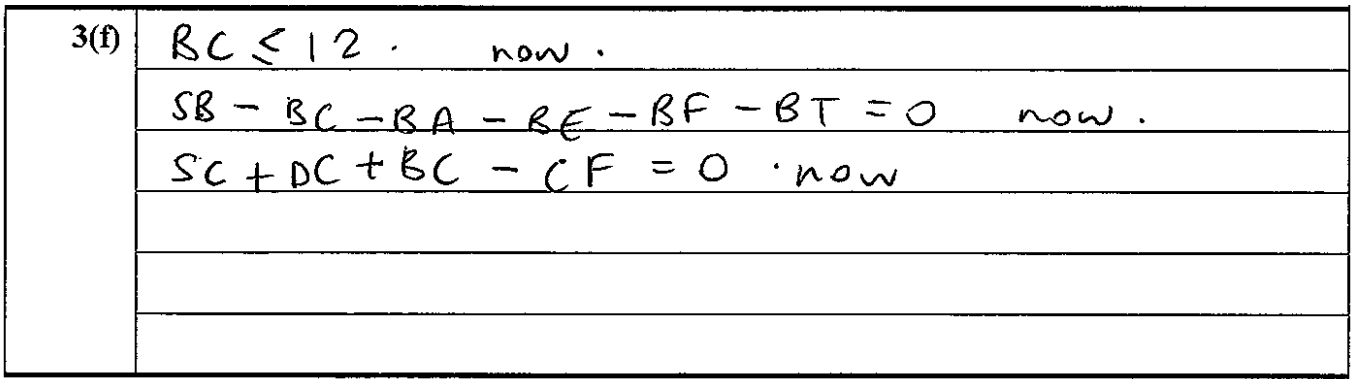
Question 3 (f)

It is decided to reverse the flow in pipe CB so that fluid can flow directly from B to C, instead of C to B. The capacity of the pipe is unchanged. No other pipes are changed.

(f) Explain what changes would be required to the LP formulation due to this modified constraint on the flow in pipe BC. **[2]**

Most candidates were able to state the updated constraints due to the modification but did not give an explanation about what changes would be required.

Exemplar 1



This candidate correctly gave all three updated constraints but gave no explanation about any changes that occurred.

Question 4

4 A student is trying to draw a triangle XYZ subject to the following constraints.

- Angle X must be at most $10°$ smaller than angle Z.
- Angle Y must be at least twice the size of angle Z.

Furthermore, the student wishes to maximise the size of angle Z.

Let x, y, and z represent the size (in degrees) of angle X, Y and Z respectively.

Complete the initial tableau in the Printed Answer Booklet so that the two-stage simplex method may be used to solve this problem in x, y, and z.

Your tableau should only contain **one** surplus variable and **one** artificial variable.

You must show all your working and you are **not** required to solve the problem. **[8]**

Most candidates were able to create two constraints based on information regarding angles X and Z, and angles Y and Z.

Not all candidates used the information regarding angles in a triangle summing to 180^o, and therefore incorrectly used the one required artificial variable stated in the question.

Some candidates misinterpreted the constraints and generated a system of equations with extra artificial variables; however, they were expected to review the full information provided in the question to verify that their system aligns with all the given details. This question is targeting the higher grades, and most of these candidates were able to correct their system to create the correct tableau. Those candidates that ignored the information that there should only be **one** surplus variable and **one** artificial variable and continued with the extra artificial variables could still be given 6 of the available 8 marks.

Most candidates were able to create a correct objective function using P and z but rearranging this objective function to $P - z = 0$ was regularly missed in the candidates working.

The candidates that correctly set up their constraints, were mostly able to form a second objective in A containing one surplus variable, and one artificial variable.

The tableau was generally well completed by candidates.

Question 5 (a)

5 The table in **Fig. 5.1** lists the immediate predecessors for each of the eleven activities in a project.

Fig. 5.1

Activity	Immediate predecessors
A	-
B	-
C	-
D	A
E	A
F	B, E
G	B, C, E
H	D, F
I	D, F, G
J	B, C, E
K	B, C, E

(a) Draw an activity network, using activity on arc, to represent the project. Your network should contain exactly **3** dummy activities. **[3]**

Most candidates attempted an activity network from a single start, 3 dummy activities and a single finish.

Some diagrams were poorly drawn with unclear lines and arrows missing.

Question 5 (b) (i)

Each of the eleven activities in the table requires exactly one worker.

The diagram in **Fig. 5.2** shows how three people can complete the project in the minimum time. Each column in the diagram represent 1 hour. If a box is blank the person is resting for that 1 hour.

Fig. 5.2

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
WORKER 1	A	A	A	E	E	E	E	F	F	F	F	F	F	H	H	H	H	H	H	
WORKER 2	B	B		D	D	D	D	G	G	G	G			I	I	I	I			
WORKER 3	C	C	C	C	C			J	J	J	J	K	K	K	K	K				

(b) (i) State the critical activities of the project. **[1]**

There were many correct responses to this question.

Question 5 (b) (ii)

(ii) State the minimum project completion time. **[1]**

There were many correct responses to this 1-mark question.

Question 5 (c)

For the remainder of this question, it may be assumed that there is no restriction on the number of workers available to complete the project.

(c) Complete the table in the Printed Answer Booklet to show the total float for each activity. **[4]**

Most candidates correctly stated that the critical activities (A, E, F and H) had 0 float, based on their answer to part (b)(i).

This part proved challenging for many candidates since there was no scaffolding to explicitly find the early and late event times. Those candidates that did not find the early and late event times were unable to correctly calculate float for the remaining activities.

Question 5 (d) (i)

A twelfth activity L is added to the project. Activity L can only begin once activity G is completed, and no activity depends on its completion.

(d) It is given that the duration of activity L is 9 hours.

(i) State the new minimum project completion time. **[1]**

Some candidates were able to use figure 5.2 to find the new completion time.

Most candidates who did not achieve a correct answer showed no workings either.

Question 5 (d) (ii)

(ii) State the new critical activities. **[1]**

The candidates that answered part (d) correctly were mostly able to state the new critical activity.

Where an incorrect project completion time was calculated, this impacted the critical activities found.

Question 6 (a)

6 A maximisation LP problem in x, y and z is to be solved.

The tableau after the 1st iteration of the simplex method is shown below.

P	x	y	z	s_1	s_2	s_3	RHS
1	-2	0	-1	0	0	1	20
0	1	0	$\frac{11}{3}$	1	0	$-\frac{1}{3}$	$\frac{130}{3}$
0	2	0	-2	0	1	1	40
0	0	1	$-\frac{2}{3}$	0	0	$\frac{1}{3}$	$\frac{20}{3}$

(a) State the value of the objective function after this 1st iteration. **[1]**

Candidates answered this question well.

Some candidates gave more than the value of the objective function and did not make it clear what they were stating was the value.

Question 6 (b)

(b) Explain how the tableau shows that the pivot value for the 1st iteration came from a value from the y column. **[1]**

Candidates did not use the correct vocabulary well when responding to this question.

Few candidates use the terminology 'basic variable'.

Many candidates attempted to explain using 1s and 0s but did not fully explain the use of a single 1.

Question 6 (c)

(c) By considering the equations in the tableau, formulate the **original** LP problem. You should state the objective and list the constraints as simplified inequalities with integer coefficients. **[5]**

Many candidates were unfamiliar with formulating the original LP problem from a given tableau.

The candidates that started with the fourth row were able to find at least one constraint.

Although the question stated it was a 'maximisation LP problem', some candidates did not fully state their objective function, and many candidates missed constraints that restrict non-negativity.

Question 6 (d)

(d) Perform the 2nd iteration of the simplex method, using the tableau in the Printed Answer Booklet, choosing the pivot from the x column and giving each entry as an exact value. **[3]**

Most candidates correctly used x as their pivot column and divided the third row by 2.

Some candidates made errors when using their pivot row to complete the iteration.

Exemplar 2

P	x	y	z	s_1	s_2	s_3	RHS
1	0	0	−3	0	1	2	60
0	0	[illegible]	14/3	[illegible]	[illegible] −1/2	−5/6	70/3
0	1	0	−1	0	1/2	1/2	20
0	0	1	−2/3	0	0	1/3	40

This candidate incorrectly used the pivot row to change the fourth row although the x value was already 0 after the 1st iteration.

Question 6 (e) (i)

After three iterations of the simplex method an optimal solution to the LP problem is obtained and the following three equations are found.

$$x + \frac{3}{14}s_1 + \frac{11}{28}s_2 + \frac{9}{28}s_3 = 25$$
$$y + \frac{1}{7}s_1 - \frac{1}{14}s_2 + \frac{3}{14}s_3 = 10$$
$$z + \frac{3}{14}s_1 - \frac{3}{28}s_2 - \frac{5}{28}s_3 = 5$$

(e) (i) State the optimal values of x, y and z. **[1]**

There were many correct responses to this question.

Question 6 (e) (ii)

(ii) Determine the corresponding value of the objective function. **[2]**

Candidates were not always able to determine the value, although the value was given. Substitution into a correct objective function, followed by a calculation were sometimes missing.

Assessment for learning

The command word 'Determine' indicates that justification should be given for any result found, including a clear algebraic method where appropriate.

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