Qualification Accredited

Oxford Cambridge and RSA

A LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y421/01 Summer 2024 series

Contents

Introduction

Our examiners’ reports are produced to offer constructive feedback on candidates’ performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates’ performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y421/01 series overview

This is one of the two major examination components for the A Level examination for GCE Further Mathematics B (MEI). It is a 2 hour 15 minute paper consisting of 120 marks. The paper consists of two sections, A and B. Section A has between 25 and 35 marks and comprises more straightforward questions. Section B has between 85 and 95 marks and comprises a mixture of more and less straightforward questions.

Inevitably, the report that follows will concentrate on aspects of the candidates’ performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that many candidates who sat this paper produced successful solutions. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on all the associated mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The specification includes some guidance about the level of written evidence required in assessment question; these were provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed.

The word ‘determine’ in a question does not simply imply that candidates should find the answer but, to quote the specification, ‘this command word indicates that justification should be given for any results found, including working where appropriate.’ This command word featured in Questions 3, 4 (a), 4 (b), 5 (b), 6, 7 (a), 7 (b), 7 (c), 8 (c), 9 (c), 10, 11 (c), 12 (b), 12 (c), 12 (d) and 13 (b).

The phrase ‘show that’ generally indicates that the answer has been given, and that candidates should provide an explanation that has enough detail to cover every step of their working. This command phrase features in Questions 8 (b), 9 (a), 11 (b), 11 (d) and 13 (a).

While there is no specific level of working needed to justify answers to questions which use the command word ‘find’, method marks may still be available for valid attempts that do not result in a correct answer, and standard advice (included in the specification) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation). Regardless of the final required accuracy, candidates should be careful of not rounding prematurely, but also take care to avoid over specifying rounded answers where the context does not support that level of accuracy.

One general point with regards to the answering of certain mechanics questions should be made in this overview: unless told otherwise, the value that candidates should use for the acceleration due to gravity, g, is 9.8 and not 9.81 or 10 (and this value is stated explicitly on the front cover of the examination paper).

Candidates who did well on this paper generally:	**Candidates who did less well on this paper generally:**
• used formal language and notation correctly • defined the letters used for speed, time, etc. in unstructured questions • understood the level of response required for the command words used in the questions • made efficient use of their calculator • read questions carefully and provided the responses that were requested.	• made mistakes in algebraic manipulation • did not give sufficient evidence on 'show that' and 'determine' questions • used imprecise notation or language • did not read the question carefully and gave responses that did not meet the required degree of accuracy (or did not use all the information given in the question).

Section A overview

Section A is designed to provide a gentle introduction to the examination, with short, straightforward questions. Most candidates made a good attempt at these 4 questions, although a small proportion omitted question 3.

Question 1 (a)

1 A car A of mass 1200 kg is about to tow another car B of mass 800 kg in a straight line along a horizontal road by means of a tow-rope attached between A and B. The tow-rope is modelled as being light and inextensible. Just before the tow-rope tightens, A is travelling at a speed of $1.5\,\mathrm{m\,s^{-1}}$ and B is at rest. Just after the tow-rope tightens, both cars have a speed of $v\,\mathrm{m\,s^{-1}}$.

(a) Find the value of v. **[2]**

Most candidates correctly applied the conservation of linear momentum to find the value of v. When errors occurred, they tended to be either sign errors or a mistake in applying one of the values correctly. Occasionally, some candidates tried to use an energy approach believing incorrectly that in this situation kinetic energy was conserved.

Question 1 (b)

(b) Calculate the magnitude of the impulse on A when the tow-rope tightens. **[2]**

Nearly all candidates used the result that 'Impulse = change in momentum' to solve this part. Candidates were approximately equally split between applying this result to either A or B to find the required impulse (although it should be noted that it was slightly easier to apply the result to B due to B being at rest before the tow-rope tightened).

Misconception

? Candidates are reminded that when a question in mechanics asks for the magnitude of a particular quantity the corresponding answer must always be positive.

Question 2 (a)

2 One end of a light spring is attached to a fixed point. A mass of 2 kg is attached to the other end of the spring.

The spring hangs vertically in equilibrium. The extension of the spring is 0.05 m.

(a) Find the stiffness of the spring. **[2]**

Most candidates correctly applied the result that $F = kx$ to find the required stiffness of the spring. The only error seen by examiners was the very occasional use of 2 rather than $2g$ for .

Question 2 (b)

(b) Find the energy stored in the spring. **[2]**

This part was also answered extremely well with most candidates using the result that $E = \frac{1}{2}kx^2$ to find the required energy stored in the spring.

Question 2 (c)

(c) Find the dimensions of stiffness of a spring. **[1]**

Most candidates in this part used the result that $F = kx$ to find the dimensions of stiffness of a spring. Occasionally, candidates used the result $E = \frac{1}{2}kx^2$ and were usually just as successful.

Assessment for learning

Candidates are reminded that if the dimensions of a quantity are explicitly required in a question, then they must be stated using capital letters without any form of brackets. Therefore, in this particular question the only acceptable answer was $[k] = \mathrm{MT}^{-2}$.

Question 2 (d)

A particle P of mass m is performing complete oscillations with amplitude a on the end of a light spring with stiffness k. The spring hangs vertically and the maximum speed v of P is given by the formula

$$v = Cm^{\alpha}a^{\beta}k^{\gamma},$$

where C is a dimensionless constant.

(d) Use dimensional analysis to determine α, β, and γ. **[4]**

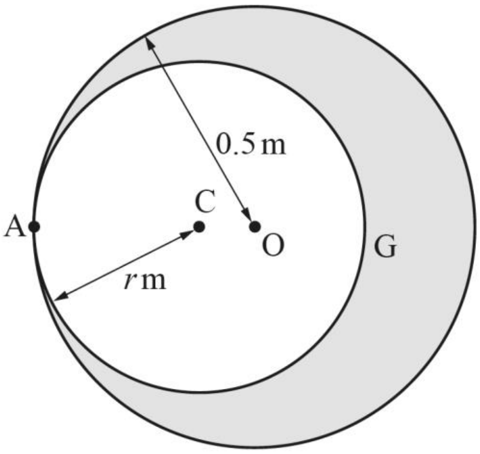
This part was answered extremely well with most candidates correctly using $[v] = \mathrm{LT}^{-1}$, $[m] = \mathrm{M}$, $[a] = \mathrm{L}$ and the correct dimensions of k to find the values of α, β and γ. The most common error was assuming that the a in the question stood for acceleration and not the stated amplitude.

Misconception

The most common misconception in this part was assuming that *k* was dimensionless, even though most candidates had correctly found the dimensions of *k* in the previous part.

Question 3

3



A circular hole with centre C and radius r m, where $r < 0.5$, is cut in a uniform circular disc with centre O and radius 0.5 m. The hole touches the rim of the disc at A (see diagram).

The centre of mass, G, of the remainder of the disc is on the rim of the hole.

Determine the value of r. **[5]**

This question was answered well. Most candidates were aware that they needed to take moments about a point on the diameter through AO and use the given result that the centre of mass of the remainder of the disc was at a distance of $2r$ from A. One common error was to add, rather than subtract, the circular hole that was removed from the disc.

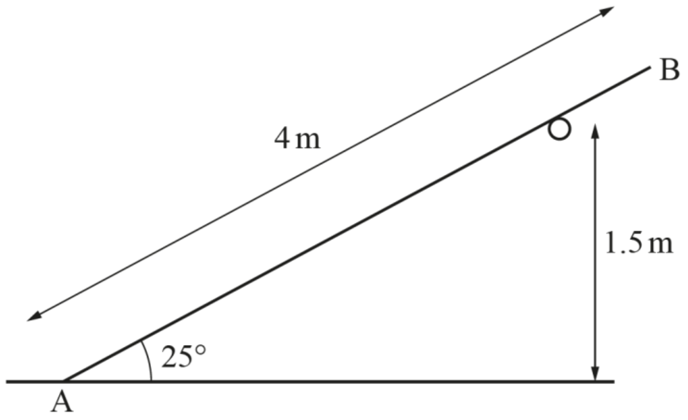
Many candidates found the correct cubic equation for r and then proceeded to solve this algebraically. As the command was to 'determine' the value of r, rather than use 'detailed reasoning', it was sufficient to use a calculator to solve the cubic equation and then state the corresponding value of r. It is good practice to justify why the other roots are not valid solutions, but there was no penalty for not explicitly rejecting roots on this question.

Assessment for learning

In this question there were at least five points that candidates could reasonably have taken moments about to find the required value of r and, at times, it was difficult to follow which point candidates were taking moments about. It is always useful in mechanics questions to either state the principle being applied and/or give further detail about where the principle is being applied (so for example, in this question, it would have been beneficial if candidates had said 'taking moments about A').

Question 4 (a)

4



A uniform rod AB has mass 3 kg and length 4 m. The end A of the rod is in contact with rough horizontal ground.

The rod rests in equilibrium on a smooth horizontal peg 1.5 m above the ground, such that the rod is inclined at an angle of 25° to the ground (see diagram).

The rod is in a vertical plane perpendicular to the peg.

(a) Determine the magnitude of the normal contact force between the peg and the rod. **[3]**

Most candidates realised that to find the magnitude of the normal contact force between the peg and the rod they needed to take moments about point A. The most common errors were not being able to find the distance from A to the peg correctly, or assuming that the direction of the normal contact force at the peg was either horizontal or vertical (rather than being perpendicular to the rod).

Question 4 (b)

(b) Determine the range of possible values of the coefficient of friction between the rod and the ground. **[5]**

Although this part could be done by a variety of methods (for example, taking moments about another point on the rod or resolving parallel/perpendicular to the rod), most candidates opted for resolving horizontally and vertically to find both the normal contact force, and the frictional contact force at A. The most common error was assuming that the normal contact force at A had a magnitude of only $3g$ (and so ignoring the previously found normal contact force from part (a)). Many candidates stated incorrectly that $F_{\mathrm{A}} \leq R_{\mathrm{P}}\sin 25$ which examiners assumed was due to the question asking for the range of possible values of the coefficient of friction. The range of values came from the fact that $F_A = R_P \sin 25$ and

$R_{\mathrm{A}} = 3g - R_{\mathrm{P}}\cos 25$, together with the general result that $F_{\mathrm{A}} \leq \mu R_{\mathrm{A}}$.

Misconception

? A very common misconception was the appearance of an upper limit on the value of the coefficient of friction. It was common to see 0.402 ≤ μ ≤ 1 which is incorrect; unlike the coefficient or restitution, there is no upper limit on the value of μ.

Section B overview

The questions in Section B are designed to be a mix of more and less straightforward questions and to also provided candidates an opportunity to apply their understanding of the mechanics content in a less structured setting.

Question 5 (a)

5 A car of mass 850 kg is travelling along a straight horizontal road. The power developed by the car is constant and is equal to 18 kW. There is a constant resistance to motion of magnitude 600 N.

(a) Find the greatest steady speed at which the car can travel. **[2]**

Most candidates correctly used the result that $P = Dv$ to find the greatest steady speed at which the car could travel.

Question 5 (b)

Later in the journey, while travelling at a speed of 15 m s^{-1}, the car comes to the bottom of a straight hill which is inclined at an angle of $\sin^{-1}\left(\frac{1}{40}\right)$ to the horizontal.

The power developed by the car remains constant at 18 kW. The magnitude of the resistance force is **no longer** constant but changes such that the total work done against the resistance force in ascending the hill is 103 000 J. The car takes 10 seconds to ascend the hill and at the top of the hill the car is travelling at 18 m s^{-1}.

(b) Determine the distance the car travels from the bottom to the top of the hill. **[5]**

Most candidates realised that, due to the variable resistive force, an energy approach needed to be used to find the distance travelled by the car up the hill. Apart from sign errors, the other common errors were forgetting one of the required terms (for example, ignoring the speed at the bottom of the hill or the work done by the car), using the speed found in Question 5 (a), or using the distance travelled up the hill as the change in height for GPE term.

Misconception

Each year, in questions which require the application of the work-energy principle, many candidates include both a 'change in PE' term and a 'work done by gravity' term. These are the same thing and so candidates are in essence adding an extra incorrect term to their equation (and so will not be given the corresponding method mark which is most often given for forming an equation with the correct number of relevant terms).

Question 6

6 In this question you must show detailed reasoning.

In this question, positions are given relative to a fixed origin, O. The unit vectors **i** and **j** are in the x- and y-directions respectively in a horizontal plane. Distances are measured in centimetres and the time, t, is measured in seconds, where $0 \leqslant t \leqslant 5$.

A small radio-controlled toy car C moves on a horizontal surface which contains O.

The acceleration of C is given by $2\mathbf{i} + t\mathbf{j}\ \text{cm s}^{-2}$.

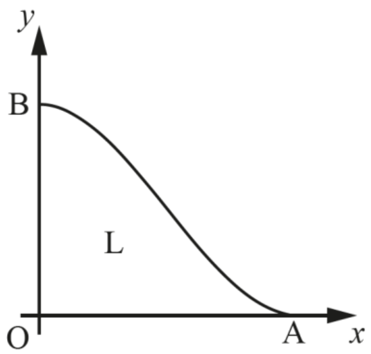
When $t = 4$, the displacement of C from O is $16\mathbf{i} + \frac{32}{3}\mathbf{j}\ \text{cm}$, and the velocity of C is $8\mathbf{i}\ \text{cm s}^{-1}$.

Determine a cartesian equation for the path of C for $0 \leqslant t \leqslant 5$. You are **not** required to simplify your answer. **[6]**

This was one of the best answered questions on the whole paper, with many candidates finding a correct cartesian equation for the path of C. When errors occurred, they were usually (minor) mistakes in calculating the required constants of integration or leaving their final answer in terms of **i** and **j**.

Question 7 (a)

7 The region bounded by the curve $y = x^3 - 3x^2 + 4$, the positive x-axis and the positive y-axis is occupied by a uniform lamina L. The vertices of L are O, A and B, where O is the origin, A is a point on the positive x-axis and B is a point on the positive y-axis (see diagram).



(a) Determine the coordinates of the centre of mass of L. **[5]**

Most candidates correctly found the coordinates of the centre of mass of L. However, it was clear from their working that many spent a considerable amount of time explicitly evaluating the integrals $\int xy\,\mathrm{d}x$ and $\frac{1}{2}\int y^2\mathrm{d}x$ when they could easily have evaluated both of these integrals on their calculators (as once again the command in this part was to 'determine' and 'detailed reasoning' was not required).

Question 7 (b)

The lamina L is the cross-section through the centre of mass of a uniform solid prism M.

The prism M is placed on an inclined plane, which makes an angle of 30° with the horizontal, so that OA lies along a line of greatest slope of the plane with O lower down the plane than A.

It is given that M does **not** slip on the plane.

(b) Determine whether M will topple in this case. Give a reason to support your answer. **[2]**

Most candidates in this part used the result that $\tan\alpha = \frac{\bar{x}}{\bar{y}}$, and compared this value of α to the given angle of 30 to show that M would topple.

Question 7 (c)

The prism M is now placed on the same inclined plane so that OB lies along a line of greatest slope of the plane with O lower down the plane than B.

It is given that M still does **not** slip on the plane.

(c) Determine whether M will topple in this case. Give a reason to support your answer. **[2]**

In a similar approach to Question 7 (b), most candidates used the result that $\tan\alpha = \frac{\bar{y}}{\bar{x}}$, and compared this value of α to the given angle of 30 to show that M, in this case, would not topple.

Question 8 (a)

8 A particle P of mass $3m$ kg is attached to one end of a light elastic string of modulus of elasticity $4mg$ N and natural length 0.4 m. The other end of the string is attached to a fixed point O. The particle P rests in equilibrium at a point A with the string vertical.

(a) Find the distance OA. **[2]**

Most candidates correctly applied Hooke's law in this part to find the extension in the string and then added the natural length (of the string) to find that OA = 0.7 m.

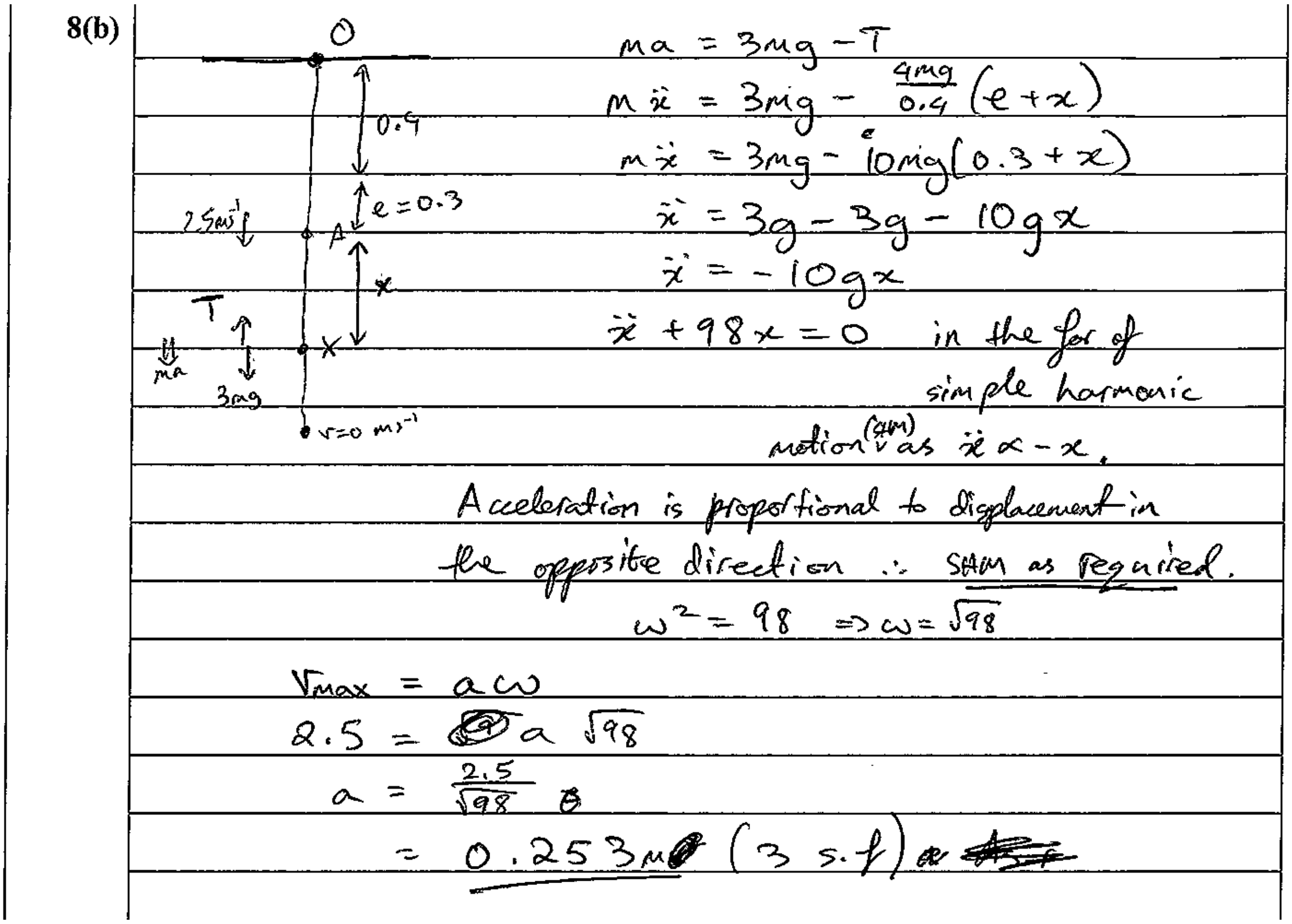
Question 8 (b)

At time $t = 0$ seconds, P is given a speed of $2.5\,\text{m s}^{-1}$ vertically downwards from A.

(b) Show that P initially performs simple harmonic motion with amplitude a m, where a is to be determined correct to **3** significant figures. **[5]**

In this part, many candidates were inconsistent when applying Newton's second law to show that P initially performed SHM. While many correctly stated that the resultant force when P was displaced a distance x from the equilibrium position was given by the expression $3mg - \frac{4mg(0.3+x)}{0.4}$ many then equated this incorrectly to $m\ddot{x}$ forgetting that the mass of P was $3m$ and not m. When it came to finding the corresponding amplitude, many candidates attempted to find this from an expression for x (usually of the form $a\sin\omega t$) instead of simply applying the known result that $v_{\max} = a\omega$ where $v_{\max}$ was given in the question (as 2.5).

Exemplar 1


8(b)

$ma = 3mg - T$

$m\ddot{x} = 3mg - \frac{4mg}{0.4}(e + x)$

$m\ddot{x} = 3mg - 10mg(0.3 + x)$

$\ddot{x} = 3g - 3g - 10gx$

$\ddot{x} = -10gx$

$\ddot{x} + 98x = 0$ in the for of simple harmonic motion (SHM) as $\ddot{x} \propto -x$.

Acceleration is proportional to displacement in the opposite direction ∴ SHM as required.

$\omega^2 = 98 \Rightarrow \omega = \sqrt{98}$

$V_{max} = a\omega$

$2.5 = a\sqrt{98}$

$a = \frac{2.5}{\sqrt{98}}$

$= 0.253$ m (3 s.f)

Exemplar 1 is given 2 out of the 5 marks available and highlights the common (but fundamental) error of the inconsistent use of the mass of P in the two parts of Newton's second law.

Question 8 (c)

(c) Determine the smallest distance between P and O in the subsequent motion. **[3]**

Due to the error mentioned in part (b) (of using m rather than $3m$ for the mass) many candidates believed that P continued to perform SHM throughout its entire motion and so therefore the smallest distance between P and O was given by $0.7 - a$ (which was given no marks). Only the most successful responses could correctly use the conservation of energy (balancing the initial KE and EPE with the final GPE) or another equivalent method to correctly find the shortest distance of 0.231 m.

Question 9 (a)

9 A particle P of mass 5 kg is released from rest at a point O and falls vertically. A resistance of magnitude $0.05v^2$ N acts vertically upwards on P, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P when it has fallen a distance x m.

(a) Show that $\left(\frac{100v}{980-v^2}\right)\frac{\mathrm{d}v}{\mathrm{d}x} = 1$. **[2]**

This part was answered very well with most candidates correctly applying Newton's second law with $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ to derive the given result. The most common (and surprising) error was the number of candidates who stated that $a = \frac{\mathrm{d}v}{\mathrm{d}x}$.

Question 9 (b)

(b) Verify that $v^2 = 980\left(1 - \mathrm{e}^{-0.02x}\right)$. **[4]**

While most candidates did verify this solution by substituting it into the differential equation from Question 9 (a) and checked that when $x = 0$, $v = 0$, several candidates instead solved the given differential equation or did not check the initial conditions.

Misconception

The word 'verify' does not have the same mathematical meaning as 'show that' – many candidates in this part solved the first order differential equation and used the given conditions to calculate the arbitrary constant. While this approach could be given all 4 marks it was a time-consuming way of tackling the problem. Furthermore, the solution to differential equations (apart from those found via SHM) are not required on this paper and it may not always be possible to solve the differential equation that candidates are being asked to verify in this unit.

Question 9 (c)

(c) Determine the work done against the resistance while P is falling from O to the point where P's acceleration is $8.36\,\mathrm{m\,s^{-2}}$. **[5]**

Many candidates appreciated that to determine the work done against resistance, they first needed to find the corresponding values of x and v when $a = 8.36$. Those that did usually then correctly found both the loss in GPE and the gain in KE and used these together to find the required work done. Many candidates attempted to use the result that work done = $\int F\,\mathrm{d}x$, even though this result is not required for this specification. When this result was applied correctly it was given full marks, but many errors were seen such as forgetting the 0.05 or integrating v^2 with respect to v (instead of correctly substituting for the given expression for v^2 and then integrating with respect to x).

Question 10

10 A particle P of mass 2 kg is projected vertically upwards from horizontal ground with an initial speed of $14\,\mathrm{m\,s^{-1}}$. At the same instant a particle Q of mass 8 kg is released from rest 5 m vertically above P. During the subsequent motion P and Q collide. The coefficient of restitution between P and Q is $\frac{11}{14}$.

Determine the time between this collision and P subsequently hitting the ground. **[10]**

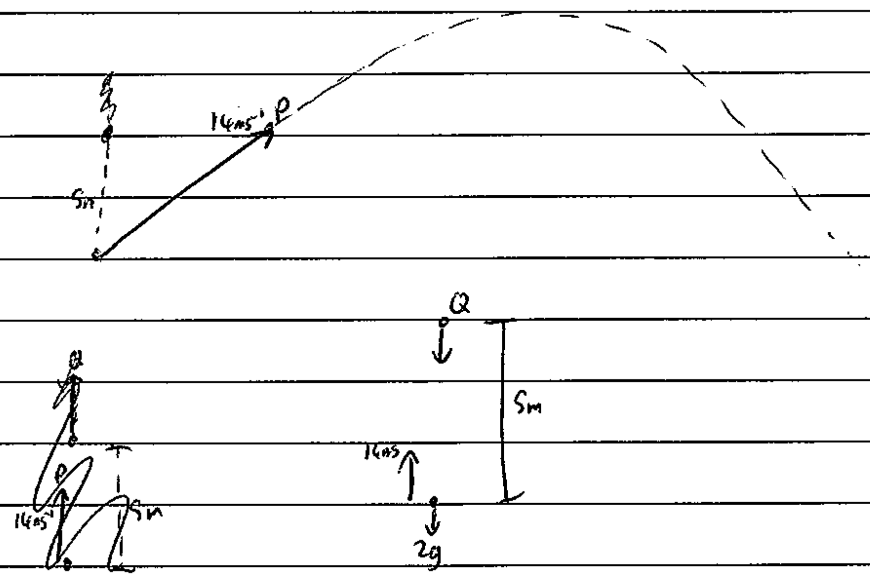
This unstructured direct collision question was generally answered well with most candidates realising the different steps required to find the time for P to hit the ground. Most candidates correctly found both the time for the collision to occur and the height above the ground where this collision took place. Most candidates then correctly calculated the speed of both particles at the point of impact. While most then realised that they needed to apply both the conservation of linear momentum and Newton's experimental law to find the speed of P after impact, many did not apply these equations consistently (with many making sign errors). Of those that did find the correct speed of P after impact, most used the equation $s = ut + \frac{1}{2}at^2$ to find the required time. Some candidates did not read the question carefully and instead found the time from the instant when P first left the ground until the instant when it subsequently hit the ground.

Assessment for learning

When a question involves multiple stages of working that include the need to introduce a certain number of variables that have not been defined in the question, it is the responsibility of the candidate to define the variables they use. In this question, some candidates did not make it explicitly clear which letters were being used for the speeds of the particles at different stages of their motion. In this type of question, candidates may have benefitted from drawing diagrams in the Printed Answer Booklet with all speeds/distances clearly labelled to assist both theirs (and the examiners') understanding.

Exemplar 2

10



① displacement of $Q = 0t + \frac{1}{2}(-9.8)t^2 + 5$

$= -4.9t^2 + 5$

displacement of P is given by: $14t + \frac{1}{2}(-9.8)t^2$

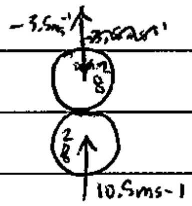
collision occurs when $-4.9t^2 + 5 = 14t - 4.9t^2$

$14t = 5$

$t = 5/14$

velocity of P: $14 - 9.8t = v$ velocity of Q: $v = -9.8t$

so when $t = \frac{5}{14}$ $v = 10.5\ ms^{-1}$ when $t = \frac{5}{14}$ $v = -3.5\ ms^{-1}$



conservation of momentum:

$8(-3.5) + 2(10.5) = 2v_p + 8v_Q$

where v_p = velocity of P after collision and

v_Q = velocity of Q after collision

$-7 = 2v_p + 8v_Q$

coefficient of restitution $= \frac{11}{14}$

$v_Q - v_p = -\frac{11}{14}(-3.5 - 10.5)$

$v_Q - v_p = 11$

$-7 = 2v_p + 8v_Q$ ① $\quad v_Q - v_p = 11$

$2v_Q - 2v_p = 22$ ②

① + ② $\quad 10v_Q = 15$

$v_Q = 1.5$

$1.5 - v_p = 11$

$v_p = -9.5$

distance above ground when P and Q collide

$-9.5 \quad 9g$

$-4.9t^2 + 5 = x$

$t = \frac{5}{14} \qquad x = 4.375$

before hitting ground P

$u = 9.5 \text{ ms}^{-1}$

~~$v = u + at$~~

~~$0 = 9.5 - 9.8t$~~

~~$t = \frac{95}{98}$~~

$s = 4.375$

$a = -9.8$

$4.375 = -9.5t - 4.9t^2$

$v^2 = 9.5^2 - 9.8(2)(4.375)$

$v = \frac{3\sqrt{2}}{2}$

$\frac{3\sqrt{2}}{2} = 9.5 - 9.8t$

$t = \frac{95 - 15\sqrt{2}}{98}$

$t = 0.753\text{s}$

Exemplar 2 is given 9 of the 10 marks available. The candidate makes good use of a number of diagrams and explains their reasoning at each stage to make it clear to the examiner what principles of mechanics are being applied (at each step). The candidate correctly finds the time of impact between P and Q, the distance above ground when this impact occurs, the speeds of P and Q both before and after impact, and then attempts to apply a SUVAT equation to find the required time. The candidate's only error is a sign error when applying $s = ut + \frac{1}{2}at^2$ (in that on the left-hand side the 4.375 should be negative).

Question 11 (a)

11 A particle P of mass 1 kg is fixed to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point O, which is 1.75 m above a horizontal plane. P is held with the string horizontal and taut. P is then projected vertically downwards with a speed of $3.2\,\mathrm{m\,s^{-1}}$.

(a) Find the tangential acceleration of P when OP makes an angle of 20° with the horizontal. **[2]**

It was clear that many candidates were unfamiliar with this relatively straightforward demand to find the tangential acceleration of P with many incorrectly finding (or attempting to find) the radial acceleration instead. The most common error seen was stating that $a = g \sin 20$ rather than the correct $a = g \cos 20$.

Question 11 (b)

The string breaks when the tension in it is 32 N. At this point the angle between OP and the horizontal is θ.

(b) Show that $\theta = 23.1°$, correct to **1** decimal place. **[5]**

Most candidates correctly applied both Newton's second law radially and the principle of conservation of energy to find an equation involving θ only (which if correct was $32 = 29.4 \sin \theta + 20.48$). Many candidates then attempted to use a sign change test to show that $\theta = 23.1$ (correct to **1** decimal place) instead of simply solving this equation algebraically.

Question 11 (c)

Particle P subsequently hits the plane at a point A.

(c) Determine the speed of P when it arrives at A. **[4]**

Candidates either used energy or SUVAT to find the speed of P when it arrived at A. Those using energy were generally more successful, as the majority of candidates who used a SUVAT approach tended to only find the vertical component of the velocity at impact.

Question 11 (d)

(d) Show that A is almost vertically below O. **[5]**

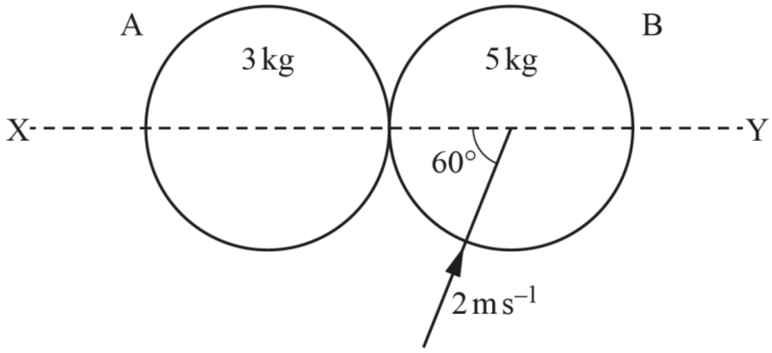
Only the most successful responses were able to complete this part, and many candidates either left this part blank or made little in the way of progress. The most successful responses found the time for P to reach A and then compared the distance travelled using this time with the value of $0.5 \cos 23.1$.

Question 12 (a)

12 Two small uniform discs A and B, of equal radius, have masses 3 kg and 5 kg respectively. The discs are sliding on a smooth horizontal surface and collide obliquely.

The contact between the discs is smooth and A is stationary after the collision.

Immediately before the collision B is moving with speed 2 m s^{-1} in a direction making an angle of $60°$ with the line of centres, XY (see diagram below).



(a) Explain how you can tell that A must have been moving along XY before the collision. **[1]**

This part was answered well with most candidates referencing the fact that as the vertical component of velocity of A was zero after impact, therefore, due to the impulse (at impact) being parallel to the line of centres (XY), the vertical component of velocity must have been zero before the impact, and so A must have been moving along XY before the collision.

Question 12 (b)

The coefficient of restitution between A and B is 0.8.

(b)
- Determine the speed of A immediately before the collision.
- Determine the speed of B immediately after the collision. **[7]**

This part was also answered extremely well with most candidates correctly applying the conservation of linear momentum and Newton's experimental law consistently to derive the correct equations for the speed of A before collision and the velocity component parallel to XY of B after collision. Most candidates then solved these equations and correctly found the speed of A immediately before the collision. The most common error was those candidates who thought that the speed of B immediately after the collision was 6.4, therefore forgetting the component of the velocity perpendicular to XY (which if dealt with correctly would have given the speed after collision as $\sqrt{6.4^2 + (2\sin 60)^2} = 6.63$).

Question 12 (c)

(c) Determine the angle turned through by the direction of B in the collision. **[3]**

While many candidates correctly used the fact that $\tan\theta = \frac{2\sin 60}{6.4}$ to obtain $\theta = 15.1$, not all realised that the angle turned through by the direction of B in the collision was $60 - \theta$.

Question 12 (d)

Disc B subsequently collides with a smooth wall, which is **parallel** to XY. The kinetic energy of B after the collision with the wall is 95% of the kinetic energy of B before the collision with the wall.

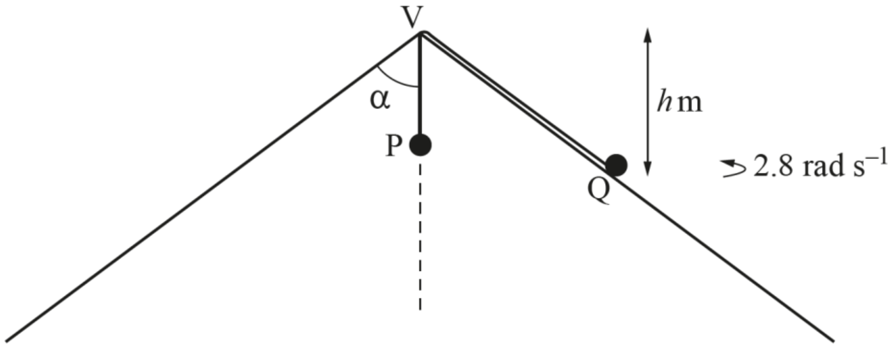
(d) Determine the coefficient of restitution between B and the wall. **[4]**

Even though the word 'parallel' was in bold in the question, many candidates had the wall in this part perpendicular to XY. It was also quite common for candidates to only consider one of the velocity components of B when considering the KE of B either before or after the collision with the wall. Very few candidates correctly obtained the equation

$\frac{1}{2} \times 5 \times (6.4^2 + (e \times 2\sin 60)^2) = 0.95 \times \frac{1}{2} \times 5 \times (6.4^2 + (2\sin 60)^2)$, and even fewer solved this correctly to find the coefficient of restitution between B and the wall.

Question 13 (a)

13



A conical shell, of semi-vertical angle α, is fixed with its axis vertical and its vertex V upwards. A light inextensible string passes through a small smooth hole at V and a particle P of mass 4 kg hangs in equilibrium at one end of the string. The other end of the string is attached to a particle Q of mass 2 kg which moves in a horizontal circle at a constant angular speed 2.8 rad s^{-1} on the smooth outer surface of the shell at a vertical depth h m below V (see diagram).

(a) Show that $k_1 h \sin^2\alpha + k_2 \cos^2\alpha = k_3 \cos\alpha$, where k_1, k_2 and k_3 are integers to be determined. **[7]**

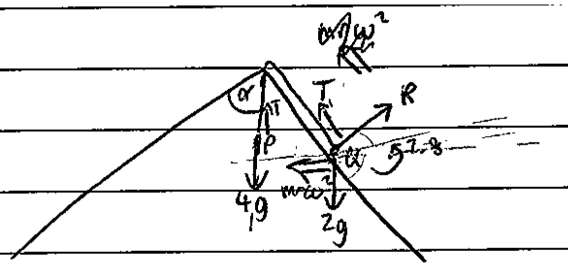
The responses to this unstructured part were very mixed, with some candidates given full marks and others making very little progress (apart from possibly stating that the tension in the string was $4g$). The two main approaches seen by examiners were:

1. Resolving forces vertically at Q **and** also applying N2L horizontally at Q to form two separate equations, then eliminating the normal contact force (at Q) from these two equations.
2. Applying N2L for Q parallel to the surface of the shell.

While many candidates were successful (with applying either approach) there were a number of common errors seen. In the first approach it was common, when resolving vertically, to forget the component of the tension in the string. In the second approach it was common for the acceleration to be given incorrectly as $mr\omega^2$ rather than $mr\omega^2 \sin\alpha$. Errors common to both approaches were: using a wrong expression for r (which if correct should have been $h\tan\alpha$), using an incorrect mass for one (or both) particle(s) and not giving the final answer in the integer form required.

Exemplar 3

13(a)



~~↑~~ Resolving at P $\quad T - 4g = 0$

$T = 4g$

~~Resolving at Q $\quad 4g - 2g\cos\alpha = mr\omega^2$~~

~~where $r = h\tan\alpha$~~

Resolving at Q $\quad T\sin\alpha - R\cos\alpha = mh\tan\alpha\,\omega^2$

Resolving vertically: $T\cos\alpha + R\sin\alpha = 2g$

$R\sin\alpha = 2g - 4g\cos\alpha$

$R = \dfrac{2g - 4g\cos\alpha}{\sin\alpha}$

$4g\sin\alpha - \dfrac{\cos\alpha}{\sin\alpha}(2g - 4g\cos\alpha) = 2h\tan\alpha\,\omega^2$

$4g\sin^2\alpha + 4g\cos^2\alpha - 2g\cos\alpha = \dfrac{2h\sin^2\alpha}{\cos\alpha}(2.8^2)$

$4g - 2g\cos\alpha = \dfrac{15.68h\sin^2\alpha}{\cos\alpha}$

$4g\cos\alpha - 2g\cos^2\alpha = 15.68h\sin^2\alpha$

$15.68h\sin^2\alpha + 2g\cos^2\alpha = 4g\cos\alpha$

$15.68h\sin^2\alpha + 19.6\cos^2\alpha = 39.2\cos\alpha$

$k_1 = 15.68 \quad k_2 = 19.6 \quad k_3 = 39.2$

Exemplar 3 is given 6 of the 7 marks available. The candidate draws a diagram at the beginning of their response, therefore implicitly defining their variables clearly. The candidate's working is easy to follow (as they indicate, at each stage, which mechanical principle(s) they are applying) and they show sufficient working in their attempt to obtain an answer in the form required. Their only error is not giving the three k values as integers (as requested in the question).

Question 13 (b)

(b) Determine the greatest value of h for which Q remains in contact with the shell. **[3]**

This part was a 'stretch and challenge' question with very few candidates making significant progress. Some candidates correctly set an expression for the normal contact force at Q equal to zero and obtained an equation in both α and h. However, it was not common to see the two different equations, containing the normal contact force at Q, used successfully to arrive at the result that

$\cos\alpha = \frac{1}{2} \Rightarrow h_{\text{max}} = \frac{5}{4}$.

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