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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y420 series overview

The Y420 paper is relatively long and presents a challenging test to candidates. Nevertheless, most candidates found all parts of this paper to be accessible and many scored very highly. A small proportion of candidates scored full marks and the median was over 120. It was encouraging to see many high-quality responses with detailed solutions supported by clear structure and detailed working.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• showed all their working and each method they used • set out their working clearly and logically • understood the requirements of the defined command words • gave complete explanations for results they were asked to "show" or "prove" supported by precise language and terminology.	• were not sufficiently rigorous when asked to explain, show or prove a result or worked backwards from the given result • did not show complete methods in questions asking for detailed reasoning and those which used the command words 'determine' and 'show that' • made arithmetic and sign errors and mistakes in algebraic manipulation • were not aware of the formulae provided in the formula booklet and quoted formulae incorrectly or used less efficient methods • did not give answers in the form explicitly requested in the wording of the question • made errors when working with indices.

OCR support

Students should be encouraged to download or bookmark a copy of the Formulae booklet and use this throughout the course so as to become familiar with what needs to be memorised and what is given. They also need to be familiar with where the formulae are placed within the booklet.

Although students do become fluent with many of the formulae during their studies, it is still good practice to refer to the Formulae booklet to avoid the risk of a mistake in recall under exam conditions.

Section A overview

The majority of candidates scored very well on Section A and high marks were seen on the first six questions especially.

Question 7 was the least well answered, in particular part (b).

Question 1

1 By expressing $\frac{1}{r+1} - \frac{1}{r+2}$ as a single fraction, find $\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)}$ in terms of n. **[4]**

This question was very well answered. A small number of candidates did not observe the initial instruction to combine the two fractions explicitly and so could not be given all 4 marks.

Question 2 (a) (i)

2 Two complex numbers are given by $u = -1 + \mathrm{i}$ and $v = -2 - \mathrm{i}$.

(a) **(i)** Find $u - v$ in the form $a + b\mathrm{i}$, where a and b are real. **[1]**

This was answered correctly by almost all candidates.

Question 2 (a) (ii)

(ii) **In this question you must show detailed reasoning.**

Find $\frac{u}{v}$ in the form $a + b\mathrm{i}$, where a and b are real. **[3]**

This question was answered well although some candidates did not show a complete method as was required by the 'detailed reasoning' command. A small number attempted to work with *u* and *v* in modulus-argument form which received no marks.

Question 2 (b)

(b) Express u in exact modulus-argument form. **[3]**

This question was answered correctly by a large majority of candidates. Some candidates had difficulty finding the correct argument, with $-\frac{\pi}{4}$ the most common incorrect value given. A small number of candidates gave their answer in exponential form which could not be given full credit.

Question 3

3 The equation $2x^3 - 2x^2 + 8x - 15 = 0$ has roots α, β and γ.

Determine the value of $\alpha^2 + \beta^2 + \gamma^2$. **[4]**

A large majority of candidates received full marks. The algebra was dealt with well and very few candidates made errors when working with signs and coefficients.

Question 4

4 The equation of a curve is $y = \frac{1}{\sqrt{k^2 + x^2}}$, where k is a positive constant. The region between the x-axis, the y-axis and the line $x = k$ is rotated through 2π radians about the x-axis.

Given that the volume of the solid of revolution formed is 1 unit^3, find the exact value of k. **[4]**

Most candidates provided a fully correct answer, although some forgot to multiply their integral by π and, less commonly, to square y before integrating. A relatively common slip was to inexplicably lose the square on π^2 before reaching the final answer. Candidates should be encouraged to check their work, if they have time, to limit such accuracy errors.

Question 5 (a)

5 **(a)** Given that $\mathbf{u} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{u} \times \mathbf{v} = \begin{pmatrix} 1 \\ b \\ 3 \end{pmatrix}$, find a and b. **[3]**

This question was very well answered. The most common error was incorrectly dealing with the sign of b when finding $\mathbf{u} \times \mathbf{v}$. It was unclear whether this arose from a misunderstanding of how to compute the vector product or from a genuine slip.

Question 5 (b)

(b) Using $\mathbf{u} \times \mathbf{v}$, determine the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$, given that this angle is acute. **[3]**

Of those candidates who followed the explicit instruction to use the vector product, the vast majority went on to provide a fully correct answer. Those who used an alternative method, most commonly using the scalar product, could not receive any marks.

Assessment for learning

When a question specifies a particular method should be used, responses which use an alternative method are likely to receive either no or partial credit, even if the correct final answer is obtained.

Question 6 (a)

6 On separate Argand diagrams, sketch the set of points represented by each of the following.

(a) $|z-1-2\mathrm{i}| \leqslant 4$. **[3]**

Almost all candidates knew to draw a circle and the vast majority indicated its centre. Many candidates did not indicate a region by shading, instead treating the given inequality as an equation. For full marks the radius of the circle had to be clearly seen or implied and this was commonly missed.

Assessment for learning

When asked to sketch, candidates are expected to include the important features of the diagram. More specific details of what is expected in a 'sketch' question are given within section 2b of the specification.

Question 6 (b)

(b) $\arg(z+\mathrm{i}) = \frac{1}{3}\pi$. **[3]**

Most candidates drew a half line with the required angle to the horizontal clearly marked. The most common error was for the half line to start at the wrong point; i was the most common.

Question 7 (a)

7 **(a)** Explain why $\int_1^2 \frac{1}{\sqrt[3]{x-2}}\,\mathrm{d}x$ is an improper integral. **[1]**

A clear explanation, indicating that the integrand was undefined for $x = 2$, was given by most candidates. A notable minority thought that $\sqrt[3]{-1}$ was complex.

Question 7 (b)

(b) In this question you must show detailed reasoning.

Use an appropriate limit argument to evaluate this integral. **[4]**

Most responses correctly completed the first step in the integration and introduced an algebraic limit for 2, although many did not do this using formal limit notation. A common omission was a clear limit argument, explicitly giving the value of $\frac{3}{2}(a-2)^{3/2}$ as $a \to 2$, which meant that the argument was incomplete. Some candidates introduced limit arguments for both limits of integration or considered the value of the integral as $a \to \infty$; others made a sign error when evaluating the final answer or otherwise incorrectly evaluated $(-1)^{2/3}$.

Exemplar 1

7(b)

$$\lim_{\substack{a\to 2 \\ b\to 1}} \int_b^a \frac{1}{\sqrt[3]{x-2}}\,dx$$

$$\lim_{\substack{a\to 2 \\ b\to 1}} \int_b^a (x-2)^{-\frac{1}{3}}\,dx$$

[illegible]

[illegible]

$$\lim_{\substack{a\to 2 \\ b\to 1}} \left[\frac{3}{2}(x-2)^{\frac{2}{3}}\right]_b^a$$

$$=\left[\frac{3}{2}(a-2)^{\frac{2}{3}} - \frac{3}{2}(b-2)^{\frac{2}{3}}\right]$$

as $a \to 2$ $\quad \frac{3}{2}(a-2)^{\frac{2}{3}} \to 0$

as $b \to 1$ $\quad \frac{3}{2}(1-2)^{\frac{2}{3}}$ = not real

$\therefore$ undefined

This response scored 3 marks. An algebraic limit for 2 is introduced in the first step and a clear limit argument is completed after the limits have been substituted. The final answer is incorrect, however, and the candidate has also given an unnecessary limit argument for 1.

Section B overview

Section B contained longer, less structured and less straightforward questions and, as a result, candidate performance was more variable although many candidates were still able to score very highly. Most candidates were able to demonstrate a good understanding of the majority of topics tested and found every question part accessible, including in the final question.

The questions that were answered best were Question 12, Question 14 and Question 16. The least well answered were parts (a) and (d) of Question 8 and Question 17.

Question 8 (a)

8 **(a)** Specify fully the transformation T of the plane associated with the matrix $\mathbf{M}$, where $\mathbf{M} = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}$ and λ is a non-zero constant. **[2]**

Although the majority of candidates recognised that the transformation was a shear, few described the shear with the required precision by giving its fixed line and the image of a point. Common responses were to indicate that a shear was parallel to the x-axis, which does not imply the fixed line, and to use a shear factor to describe the shear. Some candidates confused 'sheaf' with 'shear' and did not score marks.

Misconception

Candidates should not use shear factors to describe a shear transformation. This is stated in the specification. Instead they should give its fixed line and the image of a point. Referring to an invariant line, rather than a fixed line, is insufficient as the fixed line is a line of invariant points instead.

Question 8 (b) (i)

(b) **(i)** Find $\det \mathbf{M}$. **[1]**

Almost all candidates answered this question correctly.

Question 8 (b) (ii)

(ii) Deduce **two** properties of the transformation T from the value of $\det \mathbf{M}$. **[2]**

Most candidates knew the significance of both the size and sign of the determinant to score both marks in this question. Marks were most commonly lost for imprecise explanation, in particular saying that the transformation did not reverse orientation. Examiners did not credit this as it is not strictly equivalent to orientation being unchanged. Some responses referred to a volume scale factor which was inappropriate given the dimensions of the matrix.

Question 8 (c)

(c) Prove that $\mathbf{M}^n = \begin{pmatrix} 1 & n\lambda \\ 0 & 1 \end{pmatrix}$, where n is a positive integer. **[4]**

Most candidates were able to get at least 3 marks on this question, although less than half provided a complete proof by induction. Most commonly the basis step was not shown explicitly. Given the rigorous nature of proof by induction examiners required that $1 \times \lambda$ was seen in order for this to be formally established. When considering $\mathbf{M}^{k+1}$ an intermediate step of working was required; some candidates jumped straight to the required matrix without providing one which meant that the final two accuracy marks were withheld. A number of candidates did not provide a clear or correct conclusion. Most commonly the conclusion was not suitably conditional when considering the truth of the case where $n = k + 1$. Examiners noted that the standard of writing in the conclusion was often poor, with inappropriate word choice and unclear sentence structure leading to imprecise statements which could not be given marks.

Exemplar 2

8(c)

$\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & n\lambda \\ 0 & 1 \end{pmatrix}$ for $n=1$ $\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}^1 = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}$ $\therefore$ LHS=RHS

$\begin{pmatrix} 1 & n\lambda \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}$ true for $n=1$

assume for $n=k$ $\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 1 & k\lambda \\ 0 & 1 \end{pmatrix}$ target: $\begin{pmatrix} 1 & \lambda(k+1) \\ 0 & 1 \end{pmatrix}$

$n=k+1$ $\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & k\lambda \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \lambda k + \lambda \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & \lambda(k+1) \\ 0 & 1 \end{pmatrix}$

as shown true for $n=1$, assumed true for $n=k$ and proven true for $n=k+1$, it's true for all positive integers n by induction

This response's conclusion is too assertive and does not indicate that the truth of the case where $n = k + 1$ is conditional on the truth of the case where $n = k$.

Question 8 (d)

(d) Hence specify fully a **single** transformation which is equivalent to n applications of the transformation T. **[1]**

This question was the least well answered on the paper but approximately 20% of the candidates were able to define the shear correctly. The same errors as in part (a) were seen here, although a large number of candidates rewrote the matrix given in part (c), indicating a failure to distinguish between a transformation and its associated matrix.

Question 9 (a)

9 A curve has polar equation $r = a\sin 3\theta$, for $0 \leqslant \theta \leqslant \pi$, where a is a positive constant.

(a) Sketch the curve. Indicate the parts of the curve where r is negative by using a broken line. **[3]**

A large majority of responses were fully correct. The most common error was to draw an incorrect part of the curve with a broken line or to not draw a broken line at all. Others drew the lower loop above the initial line or rotated their curve so that no loop was in the correct position.

Question 9 (b)

(b) **In this question you must show detailed reasoning.**

Determine the area of one of the loops of the curve. **[5]**

Most candidates received full marks on this question. The most common error was to misidentify the limits required for the area of one loop. Some candidates did not use the double angle formula correctly to integrate, typically making sign errors.

Question 10 (a)

10 **(a)** Write down the first three terms of the Maclaurin series for $\ln(1+x^3)$. **[1]**

Almost all candidates recognised that the most efficient method was to substitute x^3 into the Maclaurin series for $\ln(1 + x)$ and did so correctly.

Question 10 (b)

(b) Use these three terms to show that $\ln(1.125) \approx \frac{n}{1536}$, where n is an integer to be determined. **[3]**

The vast majority of those candidates who answered part (a) correctly were able to correctly substitute into their expansion to obtain the correct answer. A small number of candidates confused the values of x and x^3.

Question 10 (c)

(c) Charlie uses the same first three terms of the series to approximate $\ln 9$ and gets an answer of 147, correct to 3 significant figures. However, $\ln 9 = 2.20$ correct to 3 significant figures.

Explain Charlie's error. **[2]**

Most candidates were able to fully explain the error by clearly identifying a value for x (or x^3) and explaining that it lay outside the range for convergence for the Maclaurin series. The best responses gave this range symbolically. A common error was to misquote this range, typically stating either that x had to satisfy $-1 < x < 1$ or that convergence happened in the case that $|x| \leq 1$. Candidates should be reminded that the correct range is given to them in the formula booklet. Some candidates who gave the range in words were not sufficiently precise for their explanation to be given full marks. Another relatively common error was to refer to the range of convergence for x^3 having stated only the value of x or vice versa.

Question 11 (a)

11 The plane Π has equation $2x - y + 2z = 4$. The point P has coordinates (8, 4, 5).

(a) Calculate the shortest distance from P to Π. **[2]**

Most candidates recognised that the formula for the distance from a point to a plane is given in the formula booklet and used it successfully here. The most common error when using the formula was in either ignoring the 4 in the equation of the plane or giving it the wrong sign. A small number of candidates attempted to derive the distance from first principles and these attempts were usually unsuccessful.

Question 11 (b)

The line L has equation $\frac{x-2}{3}=\frac{y}{2}=\frac{z+3}{4}$.

(b) Verify that P lies on L. **[2]**

Most candidates knew the required method. Some omitted a conclusive statement to indicate that their working verified that the point lies on the line. Others found a value of λ from one coordinate but did not substitute it into both other coordinates.

Question 11 (c)

(c) Find the coordinates of the point of intersection of L and Π. **[3]**

Most candidates understood the required method for finding the point of intersection, although some made arithmetic slips when solving for the value of λ. A common error was to give the position vector of the point of intersection, rather than its coordinates, for which candidates could not receive full credit.

Question 11 (d)

(d) Determine the acute angle between L and Π. **[4]**

This question was answered correctly by most candidates. Most successful responses used the scalar product to find the angle between the line and the normal to the plane before finding the angle between the line and plane, although other successful methods were seen. Some candidates did not continue beyond finding the angle between the line and the normal and so could only get 2 marks.

Question 11 (e)

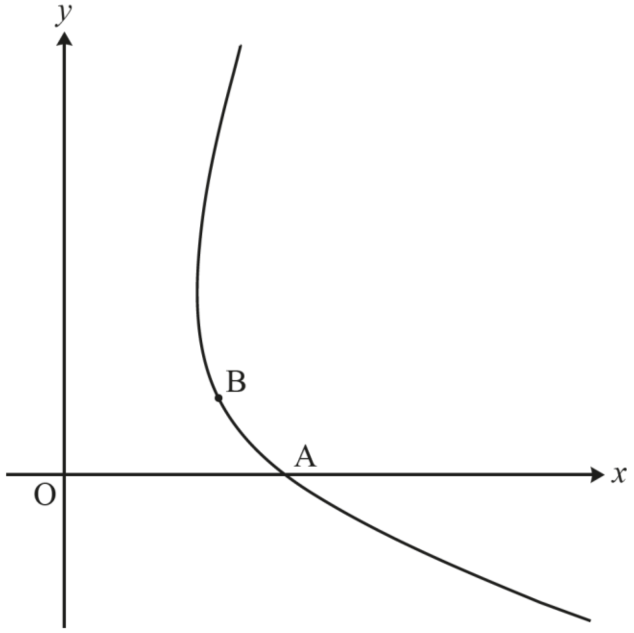
(e) Use the results of parts **(b)**, **(c)** and **(d)** to verify your answer to part **(a)**. **[3]**

This question was more challenging and built on the previous parts of Question 11. Approximately half of candidates achieved full marks. Those candidates who recognised the need to find the distance from P to the point of intersection identified in (c) were generally able to provide a complete method. Some candidates did not attempt this question although the number was relatively low.

Question 12 (a) (i)

12 The diagram shows the curve with parametric equations

$x = 2\cosh t + \sinh t, \; y = \cosh t - 2\sinh t.$



(a) The curve crosses the positive x-axis at A.

(i) Determine the value of the parameter t at A, giving your answer in logarithmic form. **[4]**

The majority of candidates were given full marks for this question. Most used the exponential definitions of the hyperbolic functions either from the outset or after finding the value of $\tanh t$. This approach was less efficient than using the identity for $\tanh t$ and then the formula for $\text{artanh}\, t$ which is given in the formula booklet. This offered less scope for slips and errors. Work with exponentials was often inefficient, even if the correct answer was eventually obtained, for instance square rooting both sides of $e^{2t} = 3$ and only then taking logarithms. In this case some candidates lost the final accuracy mark by leaving their final answer as $\ln(\pm\sqrt{3})$.

Question 12 (a) (ii)

(ii) Find the x-coordinate of A, giving your answer correct to **3** significant figures. **[2]**

Most candidates offered a fully correct answer here but many reached their answers inefficiently. Although candidates should always give their method, the 'find' command does not require detailed reasoning to be given and it was anticipated that candidates would use their calculators from the beginning of the problem to reach the answer here. Many candidates instead used the exponential definitions of the hyperbolic functions and worked towards a simplified exponential expression before using their calculator. Although most did so successfully, it was not uncommon for this method to introduce accuracy errors.

Question 12 (b)

(b) The point B has parameter $t = 0$.

Determine the equation of the tangent to the curve at B. **[6]**

Most candidates received full marks for this question. The most common errors were sign errors when differentiating and using the negative reciprocal of the required gradient rather than the gradient itself. As in the other parts of Question 12, candidates who used the exponential definitions of hyperbolic functions were more prone to introducing accuracy errors.

Question 13 (a) (i)

13 The complex number z is defined as $z = \frac{1}{3}e^{i\theta}$ where $0 < \theta < \frac{1}{2}\pi$.

On an Argand diagram, the point O represents the complex number 0, and the points P_1, P_2, P_3, … represent the complex numbers z, z^2, z^3, … respectively.

(a) Write down each of the following.

(i) The ratio of the lengths $OP_{n+1} : OP_n$ **[1]**

Many candidates got the correct answer. Many candidates left exponential terms in their ratio which could not be given credit as it was not simplified. Others wrote a fraction rather than a ratio.

Assessment for learning

It is stated in section 2b of the H645 specification that candidates are expected to simplify algebraic and numerical expressions when giving their final answers, even if the examination question does not explicitly ask them to do so.

Question 13 (a) (ii)

(ii) The angle $P_{n+1}OP_n$ **[1]**

Many candidates got the correct answer in this question. Incorrect answers most commonly gave a numerical value for the angle.

Question 13 (b) (i)

(b) **(i)** Show that $(3-e^{i\theta})(3-e^{-i\theta}) = a + b\cos\theta$, where a and b are integers to be determined. **[2]**

This part was better answered. Some candidates lost the accuracy mark after making sign slips, particularly when factorising after the initial expansion. Others omitted i from $e^{i\theta}$ and $e^{-i\theta}$ or, more commonly, $i\sin\theta$ if using modulus-argument form.

Question 13 (b) (ii)

(ii) By considering the sum to infinity of the series $z + z^2 + z^3 + \ldots$, show that

$$\tfrac{1}{3}\sin\theta + \tfrac{1}{9}\sin 2\theta + \tfrac{1}{27}\sin 3\theta + \ldots = \frac{3\sin\theta}{10-6\cos\theta}.$$ **[6]**

Those candidates who recognised the series was a geometric one generally performed well on this question. Most were able to correctly rewrite their sum so that the denominator was a real expression. The most common difficulties came when attempting to use $e^{i\theta} = \cos\theta + i\sin\theta$ in the final step where arithmetic slips and bracketing errors often resulted in the final 2 marks not being awarded. It was not uncommon for the final answer to be derived immediately from a term with an exponential numerator but this could gain 4 marks at most: as this question requested candidates to 'show that' the result held, examiners required that all steps in the method were given.

Assessment for learning

The command words 'show that' indicate that a given result must be achieved from the starting information. Because the result has been given, the explanation has to be sufficiently detailed to cover every step of the working.

Question 14 (a)

14 **(a)** Find the general solution of the differential equation $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 12e^{-x}$. **[7]**

A large majority of candidates provided a fully correct answer to this question. Very few did not derive the correct complementary function. Some considered the wrong form for the particular integral, most commonly Cxe^x rather than Ce^x which meant that at most 4 marks could be given.

Question 14 (b)

You are given that y tends to zero as x tends to infinity, and that $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$.

(b) Find the exact value of x for which $y = 0$. **[5]**

Although there were many fully correct answers seen, candidates were less successful on the whole on this part of Question 14. In order to make significant progress towards the final answer candidates had to recognise that the coefficient of their e^{x} term had to be 0 and failing to do this limited them to at most 2 marks. Those candidates who did recognise this were generally able to reach a fully correct final answer. A small number of responses did not include an attempt to solve $y = 0$ after a correct particular solution for the differential equation was given.

Question 15 (a)

15 Three planes have equations

$$\begin{aligned} x + ky + 3z &= 1, \\ 3x + 4y + 2z &= 3, \\ x + 3y - z &= -k, \end{aligned}$$

where k is a constant.

(a) Show that the planes meet at a point except for one value of k, which should be determined. **[4]**

The majority of candidates made good progress on this question, although a significant number were not awarded the final mark. This resulted from either an incorrect or incomplete conclusion. Candidates were expected to state that the planes met at a point for all values of k other than -1 but, more often than not, they concluded only that the planes did not meet at a point when $k = -1$ – saying nothing about the other values – or did not state what they had shown at all. Others exclusively gave the condition for their matrix being non-singular without referring to the intersection of the planes at all.

Exemplar 3

15(a)

$$\begin{pmatrix}1 & k & 3\\ 3 & 4 & 2\\ 1 & 3 & -1\end{pmatrix} = A$$

$$\det A = 1\begin{vmatrix}4 & 2\\ 3 & -1\end{vmatrix} - k\begin{vmatrix}3 & 2\\ 1 & -1\end{vmatrix} + 3\begin{vmatrix}3 & 4\\ 1 & 3\end{vmatrix} =$$

$$-10 - k(-5) + 3(5) = 0$$

$$-10 + 15 + 5k = 0$$

$$5k = -5$$

$$k = -1$$

at $k = -1$ det $A = 0$ so no unique point of intersection.

In this response the candidate works with the correct determinant and, although it is not explicitly given, it is implied by $k = -1$. The candidate has said that the planes do not meet at a point for this value of k but they have not stated that they do for all other values and so the final accuracy mark could not be given.

Question 15 (b)

(b) Show that, when the planes do meet at a point, the y-coordinate of this point is independent of k. **[6]**

Most successful responses attempted to use an inverse matrix to find the value of *y*. This was generally done well, although some arithmetic and sign slips were seen which meant the final accuracy mark could not be given. Those who found the *x*- and *z*-coordinates, which were not required, were more likely to make such errors. A small number of candidates recognised that only the second row of the inverse matrix needed to be found which significantly increased the efficiency of their answer. Those candidates who attempted to work with simultaneous equations were less likely to provide a fully correct answer as the additional algebraic burden of their method meant that slips were far more common. Regardless of method, some candidates did not make clear what they had shown – that the *y*-coordinate was independent of *k* – and so lost the final accuracy mark.

Assessment for learning

Work with matrices often requires significant amounts of numerical and algebraic manipulation. Students should be encouraged to check their workings carefully for sign slips and other numerical errors as these can occur regularly.

Question 16

16 In this question you must show detailed reasoning.

Show that $\int_0^1 \frac{1}{\sqrt{x^2+x+1}}\,dx = \ln\left(\frac{a+b\sqrt{3}}{c}\right)$, where a, b and c are integers to be determined. **[6]**

This question was very well answered with a large majority of candidates achieving full marks. Examiners noted that most candidates integrated immediately after completing the square without using a substitution to do so. Less successful candidates made errors in working with the logarithmic definition of the arsinh function.

Question 17 (a) (i)

17 In an industrial process, a container initially contains 1000 litres of liquid. Liquid is drawn from the bottom of the container at a rate of 5 litres per minute. At the same time, salt is added to the top of the container at a constant rate of 10 grams per minute. After t minutes the mass of salt in the container is x grams, and you are given that $x = 0$ when $t = 0$.

In modelling the situation, it is assumed that the salt dissolves instantly and uniformly in the liquid, and that adding the salt does not change the volume of the liquid.

(a) (i) Show that the concentration of salt in the liquid after t minutes is $\frac{x}{1000-5t}$ grams per litre. **[1]**

Approximately two thirds of candidates were able to show the given result. Successful responses started with a correct and labelled expression for the volume of liquid left in the container before reaching the given answer in a separate subsequent step.

Question 17 (a) (ii)

(ii) Hence show that the mass of salt in the container is given by the differential equation

$$\frac{dx}{dt} + \frac{x}{200-t} = 10.$$ **[3]**

Successful responses to this question started with an unsimplified expression for $\frac{dx}{dt}$ in terms of the input and output of salt into the process before working towards the given answer. Some candidates were not clear on where to start, instead trying to work backwards from the equation they were asked to derive.

Question 17 (b)

(b) Show by integration that $x = 10(200-t)\ln\left(\frac{200}{200-t}\right)$. **[8]**

Over half of responses were given full marks for this question. Most candidates were able to find the correct integrating factor and to multiply each term in the equation by it. When writing the left-hand side of the equation as an exact derivative a number of candidates incorrectly used $\frac{\mathrm{d}}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}t}$ for $\frac{\mathrm{d}}{\mathrm{d}t}$ which led to the final accuracy mark being withheld. Some sign errors were seen when integrating and finding the value for the constant of integration. The most common difficulty was in successfully combining log terms; those candidates who multiplied through by $(200 - t)$ before combining log terms were most prone to bracketing errors which, without clear recovery, led to the last 2 marks being withheld.

Question 17 (c) (i)

(c) (i) Hence determine the mass of salt in the container when half the liquid is drawn off. **[2]**

This question was well answered.

Question 17 (c) (ii)

(ii) Determine also the time at which the mass of salt in the container is greatest. **[5]**

This question was found to be one of the most challenging on the whole paper. Most candidates were able to get the first mark for giving the condition for x to be maximised but less than half were able to progress further. The most common approach was to differentiate the expression for x found in (b) but this was often done incorrectly, meaning that no further marks could be given. The few candidates who used the results from (a) and (b), together with $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$, were far more regularly able to progress to the correct final answer and generally did so.

Question 17 (d)

(d) When the process is run, it is found that the concentration of salt over time is higher than predicted by the model.

Suggest a reason for this. **[1]**

Approximately half of candidates were able to identify that a relevant modelling assumption was unrealistic. Less successful responses restated the premise of the question – that salt was added to the top and liquid was drawn from the bottom of the container – or instead focused on factors affecting the chemical nature of the process, such as saturation, temperature and evaporation.

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