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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y545/01 Summer 2024 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y545/01 series overview

This summer candidates found the paper overall challenging, compared to previous years. Nevertheless, there were some very well answered questions, for instance Question 2 on determining the equation of a tangent plane and Question 6 on finding the coordinates of stationary points of a 3-D surface and determining their nature. Given that these topics were newly introduced in the reformed A Level Further Mathematics A specification, it is pleasing to see that they have become well embedded in candidates' knowledge and skills. Other questions which showed solid performance were Question 3, on simultaneous linear congruences (again a newly introduced topic) and Question 5 on modelling, which saw an improvement compared to previous years.

As is often the case, candidates found problem solving questions and proofs challenging, especially Question 1 on number bases (including part (b) with an algebraic twist) and Question 8 with an (abstract) cyclic group of order 12 and an unfamiliar context in the second part. Question 4 also represented a hurdle, although the first part becomes straightforward when basic facts on the vector product are recalled.

There was evidence to suggest that some candidates struggled with time management and either rushed or omitted some of the parts on Question 8.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• read the questions carefully to make sure that they used the requested method and that they had satisfied all requirements • were able to use and apply standard techniques with confidence • communicated well using mathematical language correctly • used their calculator efficiently • had practised past paper questions on topics that are regularly assessed.	• made arithmetic or algebraic errors which prevented them from gaining the easiest marks in the paper • produced unclear explanations or incomplete mathematical arguments • struggled with problem solving elements of questions • found it challenging to start/complete proofs • did not organise their time well.

Question 1 (a), (b)

1 (a) The number N has the base-10 form $N = abba\ abba\ \ldots\ abba$, consisting of blocks of four digits, as shown, where a and b are integers such that $1 \leqslant a < 10$ and $0 \leqslant b < 10$.

Use a standard divisibility test to show that N is always divisible by 11. **[3]**

(b) The number M has the base-n form $M = cddc\ cddc\ \ldots\ cddc$, where $n > 11$ and c and d are integers such that $1 \leqslant c < n$ and $0 \leqslant d < n$.

Show that M is always divisible by a number of the form $k_1 n + k_2$, where k_1 and k_2 are integers to be determined. **[3]**

The vast majority of candidates gained either 2 or 3 marks in part (a). They knew the standard divisibility test for number 11 and began their work well and gained the first M mark. Many also went on to gain the first A mark by considering all the blocks in number *N* (or via the even/odd-numbered places sum route). However, when considering the whole number *N*, responses often lacked detail in their argument. Detail in their argument is especially important in this question, where AO2.1 (construct rigorous mathematical arguments, including proofs) and AO2.2a (make deductions) are assessed.

Part (b) was generally poorly answered or not attempted. Many found this to be one of the most challenging questions in the paper. Exemplar 1 is an example of a response that gained all 3 marks. In this part question, AO2.1 (construct rigorous mathematical arguments, including proofs), AO3.1a (translate problems in mathematical contexts into mathematical processes) and AO2.4 (explain their reasoning) are assessed.

Exemplar 1

1(b)

$M = c + d \times n + dn^2 + cn^3 + \dots +$
$cn^{4k-3} + dn^{4k-2} + dn^{4k-1} + cn^{4k}$
for some $k \in \mathbb{Z}^+$

consider $M \bmod (n+1)$

$n \equiv -1 \bmod (n+1)$ ⇒ so M can be reduced to
$n^2 \equiv 1 \bmod (n+1)$
$n^3 \equiv -1 \bmod (n+1)$ $\equiv c - d + d - c + c - d + \dots + c - d + d - c$
$\vdots$ $= 0 \bmod (n+1)$
$n^{4k-1} \equiv -1 \bmod (n+1)$ so $(n+1) \mid M$ always
$n^{4k} \equiv 1 \bmod (n+1)$ so $k_1, k_2 = 1$

This response gains full marks in part (b). The candidate chooses to work mod (n+1), which is equivalent to using the Factor Theorem. First, each block is expressed in base n (first M mark); the expression is then evaluated for $n = -1$ and proved zero (second M mark). Finally, a conclusion is drawn.

Assessment for learning

More practice with abstract base work is recommended.

Question 3

3 Determine all integers x for which $x \equiv 1 \pmod{7}$ and $x \equiv 22 \pmod{37}$ and $x \equiv 7 \pmod{67}$.
Give your answer in the form $x = qn + r$ for integers n, q, r with $q > 0$ and $0 \leqslant r < q$. **[6]**

In this question a good number of candidates gained either 5 or 6 marks. Here a variety of AOs are tested:

AO3.1a translate problems in mathematical contexts into mathematical processes

AO2.2a make deductions

AO2.4 explain their reasoning.

Simultaneous linear congruences allow for a rich variety of methods, which are difficult to encapsulate in commonly observed responses. Notable features of common responses are combining two bases into one variable, which the vast majority of candidates accomplished, and then combining the third base, which again many achieved. In the mark scheme only one justification in terms of hcf was required, to ensure the existence of a solution, but it was often omitted. A good proportion of candidates chose to use the Chinese Remainder Theorem. Arithmetic errors were frequent but not terribly costly.

Question 4 (a)

4 The vectors **a** and **b** are given by $\mathbf{a} = \begin{pmatrix} p-1 \\ q+2 \\ 2r-3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2p+4 \\ 2q-5 \\ r+3 \end{pmatrix}$, where p, q and r are real numbers.

(a) Given that **b** is **not** a multiple of **a** and that $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, determine all possible sets of values of p, q and r. **[3]**

Exemplar 2

4(a)	since b is not a multiple of a, b and a are not parallel so angle between vectors is not 0, so either a or b or both are the zero vectors.
	if a = 0 $p-1=0$ $p=1$ $(p,q,r)=(1,-2,\frac{3}{2})$
	$q+2=0$ $q=-2$
	$2r-3=0$ $r=\frac{3}{2}$
	if b = 0 $2p+4=0$ $p=-2$ $2q-5=0$ $q=\frac{5}{2}$
	$r+3=0$ $r=-3$ $(p,q,r)=(-2,\frac{5}{2},-3)$

Exemplar 2 illustrates a concise and well-crafted response addressing the main point of the question, namely that for the vector product of two vectors to be **0** either the two vectors are parallel or one of them is **0.**

Question 4 (b)

(b) You are given instead that $\mathbf{b} = \lambda\mathbf{a}$, where λ is an integer with $|\lambda| > 1$.

By writing each of p, q and r in terms of λ, show that there is a unique value of λ for which p, q and r are all integers, stating this set of values of p, q and r. **[7]**

The candidates that did not answer part (a) correctly did not consider the route that **a** = **0** or **b** = **0**. Many calculated $\mathbf{a} \times \mathbf{b}$ in terms of λ, equated it to **0** and went into lengthy algebraic manipulations to produce simultaneous equations with unknowns *p*, *q* and *r*. None of them were able to determine the correct values of *p*, *q* and *r* following this route. Of those who answered the part question successfully, a good proportion arrived at the correct solution by considering the magnitude of $\mathbf{a} \times \mathbf{b}$ and then stating that either |**a**| = **0** or |**b**| = **0**, as $\sin\theta \neq 0$ since the vectors are not parallel.

Responses to part (b) were generally not successful and a number of candidates did not give a response. Of those who attempted the part question, most were able to obtain the first 2 marks by expressing *p*, *q* and *r* in terms of λ. After that, many found $\lambda = 5$ and subsequently found the values of *p*, *q* and *r* by inspection, so gaining 2 special case B marks. Others attempted to determine λ using the division algorithm or the hcf of numerator and denominator of a fraction, earning 1 or 2 method marks. Few candidates found all the possible values of λ allowed by this method and rejected all but $\lambda = 5$ with justification. In this part question, in which AO2 is assessed, constructing a rigorous mathematical argument to reach the possible value of λ was essential. AO3.1a (translate problems in mathematical contexts into mathematical processes) and AO3.2a (interpret solutions to problems in their original context) were also tested.

Assessment for learning

Further practice with determining the value of a parameter for which an algebraic fraction (where numerator and denominator are linear functions of the parameter) reduces to an integer is recommended.

Misconception

A significant number of candidates thought that, as $\mathbf{a}$ and $\mathbf{b}$ are not parallel and $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, then $\mathbf{a} = \pm\,\mathbf{b}$, disregarding the given information that $|\lambda| > 1$. It is essential to understand that equal or opposite vectors are also parallel.

Question 5 (a), (b)

5 In a conservation project in a nature reserve, scientists are modelling the population of one species of animal.

The initial population of the species, P_0, is 10 000. After n years, the population is P_n. The scientists believe that the year-on-year change in the population can be modelled by a recurrence relation of the form
$P_{n+1} = 2P_n(1 - kP_n)$ for $n \geqslant 0$, where k is a constant.

(a) The initial aim of the project is to ensure that the population remains constant.

Show that this happens, according to this model, when $k = 0.000\,05$. **[2]**

(b) After a few years, with the population still at 10 000, the scientists suggest increasing the population. One way of achieving this is by adding 50 more of these animals into the nature reserve at the end of each year.

In this scenario, the recurrence system modelling the population (using $k = 0.000\,05$) is given by
$P_0 = 10\,000$ and $P_{n+1} = 2P_n(1 - 0.000\,05P_n) + 50$ for $n \geqslant 0$.

Use your calculator to find the long-term behaviour of P_n predicted by this recurrence system. **[1]**

This modelling question, in which both AO2 (reasoning) and AO3 (problem solving) are assessed, was overall answered well. In part (a), almost all candidates earned at least 1 mark. The majority interpreted the command word 'Show that' correctly, found an equation in k by substituting both P_n and P_{n+1} with 10000 and calculated the required value for k. However, many only verified that $k = 0.00005$ and that P_0=10000 yields P_1=10000 and so on, thereby not earning both marks.

In part (b), about a half of the candidates gained the mark for stating 10049 (or 10050) animals as the value the recurrence system converges to long term; 10050 was allowed (provided there was evidence that it was arrived at by iteration and was not just the value of P_1), although 10049 was the desired result. Non-integer values were not accepted as they are not in context (there are no part animals).

Question 5 (c) (i), (ii), (iii), (iv)

(c) However, the scientists decide **not** to add any animals at the end of each year. Also, further research predicts that certain factors will remove 2400 animals from the population each year.

(i) Write down a modified form of the recurrence relation given in part **(b)**, that will model the population of these animals in the nature reserve when 2400 animals are removed each year and no additional animals are added. **[1]**

(ii) Use your calculator to find the behaviour of P_n predicted by this modified form of the recurrence relation over the course of the next ten years. **[1]**

(iii) Show algebraically that this modified form of the recurrence relation also gives a constant value of P_n in the long term, which should be stated. **[3]**

(iv) Determine what constant value should replace 0.000 05 in this modified form of the recurrence relation to ensure that the value of P_n remains constant at 10 000. **[2]**

Part (c) (ii), on the behaviour of a sequence, was answered correctly by slightly over a half of the candidates. As AO2.5 (use mathematical language and notation correctly) is assessed, a statement that P_n is decreasing was required here, while the absence of the qualifier 'monotonically' was condoned.

The majority of candidates gained either 2 or 3 marks in part (c) (iii). Most arrived at 6000 and 4000 as the constant value(s) of P_n via the 'fixed-term' quadratic equation, but not all of them were able to justify selecting 6000 (thus rejecting 4000). A justified conclusion was necessary as this mark assesses AO2.3 (assess the validity of mathematical arguments).

Finally, part (c) (iv) was done generally well, with most candidates determining the value of k correctly via a linear equation, although some errors in rearranging were observed (AO3.5c, explain how to refine models).

Assessment for learning

Candidates should be reminded that the command word 'Show that' signposts that explanations have to be sufficiently detailed to cover every step of their working.

A full list of command words can be found on pages 9-11 in the specification. There is also a poster that can be printed and displayed in the classroom.

OCR support

For support material on sequences and series, see OCR Delivery Guides Section 8.01 Sequences and Series, check-in tests and activities.

Question 6 (a), (b), (c)

6 The surface C is given by the equation $z = x^2 + y^3 + axy$ for all real x and y, where a is a non-zero real number.

(a) Show that C has two stationary points, one of which is at the origin, and give the coordinates of the second in terms of a. **[6]**

(b) Determine the nature of these stationary points of C. **[5]**

(c) Explain what can be said about the location and nature of the stationary point(s) of the surface given by the equation $z = x^2 + y^3$ for all real x and y. **[2]**

Parts (a) and (b) of this question on stationary points of a 3-D surface and their nature had many successful responses, with most candidates achieving all the marks available. The only issue in part (a) was that candidates often forgot to calculate the z-coordinate of the stationary point other than the origin. In part (c), many candidates did not identify the single stationary point at the origin for the simplified equation. Most evaluated the determinant of the Hessian matrix and correctly stated that they could not determine the nature of the stationary point if the determinant was zero. Those who stated that the single stationary point is a saddle point, often provided too vague justifications. This question assessed two strands of AO2 (make deductions and explain reasoning).

Assessment for learning

Candidates should be reminded that stationary points of 3-D surfaces need to be stated as points in three dimensions, by means of three coordinates.

Question 7 (a), (b), (c)

7 Let $I_n = \int_0 \frac{x^n}{\sqrt{x^3+1}}\,\mathrm{d}x$ for integers $n > 0$.

(a) By considering the derivative of $\sqrt{x^3+1}$ with respect to x, determine the exact value of I_2. **[2]**

(b) Given that $n > 3$, show that $(2n-1)I_n = 3 \times 2^{n-1} - 2(n-2)I_{n-3}$. **[5]**

(c) Hence determine the exact value of $\int_0 x^5\sqrt{x^3+1}\,\mathrm{d}x$. **[3]**

Part (a) of this question had many successful responses, with only a few candidates not heeding the instruction to determine the required integral by inspection through the derivative and using a substitution instead. In part (b) candidates tended to struggle to start their response (obtaining no marks at all). Of those who started, they found it challenging to correctly identify u and v in their integration by parts and to arrive at the required reduction formula in order to gain all available marks. More than a third of the candidates expediently rewrote $(x^3+1)^{\frac{1}{2}}$ as $(x^3+1)(x^3+1)^{-\frac{1}{2}}$, which allowed them to express I_n in terms of I_{n-3}.

Part (c) was found challenging by most of the candidates. Only approximately one third achieved full marks in this part question. Those who realised that the required integral is I_5+I_8 (AO3.1a: translate problems in mathematical contexts into mathematical processes) were likely to determine its correct value by using the (given) reduction formula twice. Some errors in substituting values in the reduction formula(e) lead to the loss of the accuracy mark.

Assessment for learning

Candidates should practise setting up and solving integrals using integration by parts when proving a reduction formula. It is important that they carefully check every algebraic manipulation.

Question 8 (a) (i), (ii), (iii)

8 The group G is cyclic and of order 12.

(a) **(i)** State the possible orders of all the proper subgroups of G. You must justify your answers. **[2]**

(ii) List all the elements of each of these subgroups. **[5]**

(iii) Explain why G must be abelian. **[1]**

Candidates demonstrated comprehensive knowledge of Lagrange's Theorem concerning the order of a subgroup of a finite group and the vast majority gained 2 marks in part (a) (i).

In part (a) (ii) they tended to either achieve full marks (or 4 out of 5), if they understood that a cyclic group is generated by 'powers' of a single element (generator), or not to make any progress at all.

In part (a) (iii), few candidates achieved the available mark. Most of them stated that the group G is abelian since it is cyclic, neglecting the instruction to give an explanation (AO2.4: explain their reasoning).

Assessment for learning

Candidates would benefit from practising working with abstract cyclic groups, especially proving their properties.

OCR support

For support material on sequences and series, see OCR Delivery Guides Section 8.03 Groups, check-in tests and activities.

Question 8 (b) (i) and (ii)

The group $\mathbb{Z}_k$ is the cyclic group of order k, consisting of the elements $\{0, 1, 2, \ldots, k-1\}$ under the operation $+_k$ of addition modulo k.

The coordinate group C_{mn} is the group which consists of elements of the form (x, y), where $x \in \mathbb{Z}_m$ and $y \in \mathbb{Z}_n$, under the operation $\oplus$ given by $(x_1, y_1) \oplus (x_2, y_2) = (x_1 +_m x_2, y_1 +_n y_2)$. For example, for $m = 5$ and $n = 2$, $(3,0) \oplus (4,1) = (2,1)$.

(b) **(i)** List all the elements of $J = C_{34}$. **[1]**

(ii) Show that G and J are isomorphic. **[1]**

Both parts (b), (i) and (ii), were less successfully answered. Successful responses in part (i) mainly saw a list of the twelve elements of C_{34}. Unsuccessful ones often included extraneous elements or featured missing elements. In part (ii), candidates commonly stated that J and G are isomorphic as they are both cyclic but did not show that J is cyclic (by providing a generator, i.e. (1,1)). Part (b) (ii) assessed AO2.4, explain reasoning.

Misconception

A few candidates thought that C_{34} meant a cyclic group of order 34 and went on to find subgroups of order 2 and 17 when discussing isomorphisms.

Question 8 (c) (i), (ii), (iii), (d)

There is a second coordinate group of order 12; that is, $K = C_{mn}$, where $1 < m < n < 12$ but neither m nor n is equal to 3 or 4.

(c) **(i)** State the values of m and n which give K. **[1]**

(ii) Hence list all of the elements of K. **[1]**

(iii) Explain why K must be abelian. **[1]**

(d) Show that G and K are **not** isomorphic. **[2]**

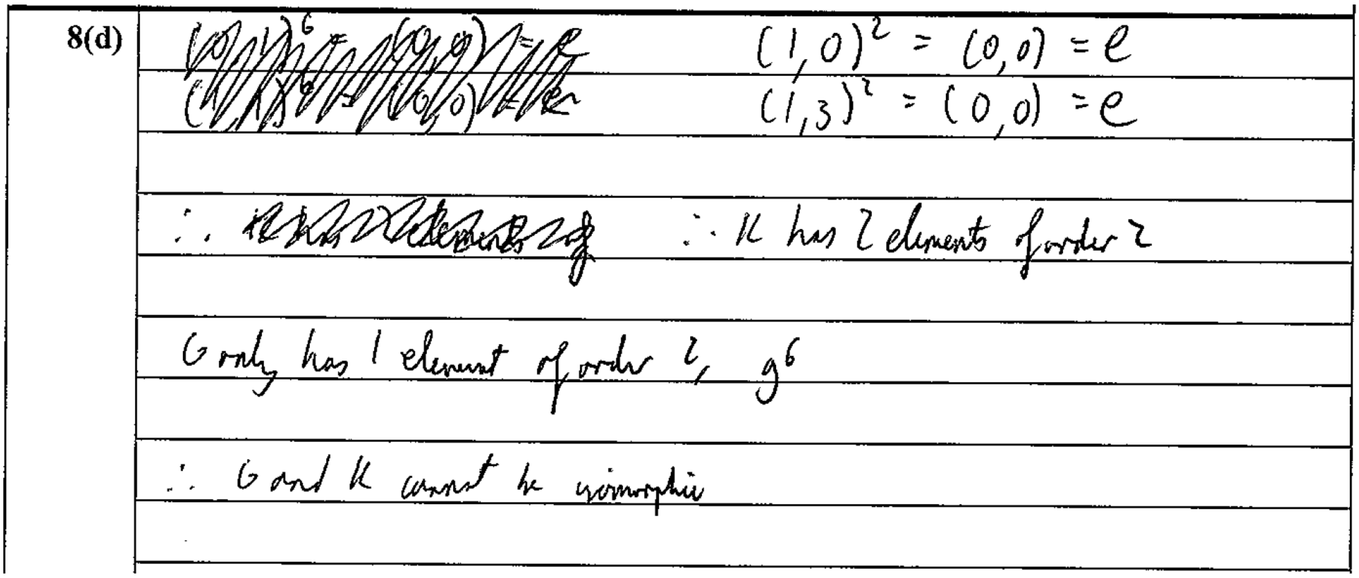
It seemed like significant number of candidates had not managed their time well and run out of time at this point, as many did not attempt parts (c) and (d). Many of those who answered part (c) (i) gained the available mark for stating $m=2$ and $n=6$.

Part (c) (ii) was generally done well if attempted, although extraneous elements or missing elements were seen.

In part (c) (iii), candidates often referred to commutativity of modular addition in $\mathbb{Z}_2$ and $\mathbb{Z}_6$ to justify that K is abelian but did not address the fact that the two components do not interfere with each other and therefore did not earn the mark. The AO assessed in part (c) (ii) is 2.5, use mathematical language and notation correctly.

Finally, part (d) was attempted by few candidates who would generally obtain at least the method mark for identifying an aspect in which G and K differ. Typically, the property which was analysed was cyclicity, even though evidence that K is not cyclic was seldom present. In part (d), AO2.1 (construct rigorous mathematical arguments, including proofs) and AO2.4 (explain their reasoning) were tested, thus the need for a complete argument.

Exemplar 3


8(d)

$(1,0)^2 = (0,0) = e$

$(1,3)^2 = (0,0) = e$

∴ K has 2 elements of order 2

G only has 1 element of order 2, g^6

∴ G and K cannot be isomorphic

Exemplar 3 for part (d) gains both available marks. The non-matching property considered here is the number of self-inverse elements in each group. The candidate shows that there are (at least) two elements of K of order 6, while G has only one.

Assessment for learning

Candidates would benefit from being repeatedly exposed to practise problems involving group properties and isomorphisms, with emphasis on the need of clear and logical reasoning when proving group properties and the existence of isomorphisms.

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