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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y544/01 Summer 2024 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y544 series overview

This is an option paper for Further Mathematics. It assesses Discrete Mathematics through graphs and networks, algorithms, linear programming and the application of these to problems including critical path analysis and game theory.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• worked neatly and explained their working where appropriate • attempted all questions and gave responses that were appropriate for the number of marks available.	• did not read the questions carefully enough • worked in a muddled way and misread their own letters or numerical values • did not use the algorithms given in the formulae booklet.

Question 1 (a)

1 At the end of each year the workers at an office take part in a gift exchange.
Each worker randomly chooses the name of one other worker and buys a small gift for that person.

Each worker's name is chosen by exactly one of the others.
A worker cannot choose their own name.

In the first year there were four workers, A, B, C and D.
There are 9 ways in which A, B, C and D can choose the names for the gift exchange.
One of these is already given in the table in the Printed Answer Booklet.

(a) Complete the table in the Printed Answer Booklet to show the remaining 8 ways in which the names can be chosen. **[2]**

Most candidates were able to list the remaining derangements although a few duplicated one or gave an arrangement that was not a derangement in at least one position.

Question 1 (b)

During the second year, worker D left and was replaced with worker E.
The organiser of the gift exchange wants to know whether it is possible for the event to happen for another 3 years (starting with the second year) with none of the workers choosing a name they have chosen before, assuming that there are no further changes in the workers.

(b) Classify the organiser's problem as an existence, construction, enumeration or optimisation problem. **[1]**

This was an existence problem and almost all of the candidates recognised this. A few suggested that it was a construction problem. Very few thought that it was an enumeration or an optimisation problem.

Question 1 (c)

After the second year, the organiser drew a graph showing who each worker chose in the first two years of the gift exchange.
None of the workers chose the same name in the first and second years.

The vertices of the graph represented the workers, A, B, C, D and E, and the arcs showed who had been chosen by each worker.

(c) Explain why the graph must be a digraph. **[1]**

Most candidates knew that a digraph is a directed graph and were able to explain why this graph must be directed, usually in terms of who was giving and who was receiving a gift. A few confused 'digraph' with 'bipartite graph' or talked about what happened in each year.

Question 1 (d)

(d) State the number of arcs in the digraph that shows the choices for the first two years. **[1]**

Most candidates got the answer 8, some forgot that D is replaced by E and gave an answer of 10. Other answers were seen, but these were all allowed follow through in part (e).

Question 1 (e)

(e) Assuming that the digraph created in part **(d)** is planar, use Euler's formula to calculate how many regions it has. **[2]**

The biggest issues here were candidates who thought that there were only 4 (or sometimes 6) vertices and candidates who did not show that they had substituted $V = 5$, and their 8 into Euler's formula to achieve their answer for the number of regions.

Some candidates had values that could not possibly have corresponded to planar graphs, but this was not penalised.

Question 2 (a)

2 A linear programming problem is

Maximise $P = 2x - y + z$

subject to

$$3x - 4y - z \leqslant 30$$
$$x - y \leqslant 6$$
$$x - 3y + 2z \geqslant -2$$
and $x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$

(a) Complete the table in the Printed Answer Booklet to represent the problem as an initial simplex tableau. **[3]**

Most candidates were able to deal with the objective row (top row) but several did not transform $x - 3y + 2z \geq -2$ into an inequality that needed a slack variable, or just assumed that the inequality was $\leq$ and not $\geq$ without considering that the value on RHS was negative.

Question 2 (b)

(b) Carry out one iteration of the simplex algorithm. **[3]**

Almost all candidates were able to identify an appropriate pivot element, although some chose an entry with a negative value in RHS column or a negative (or zero or infinite) ratio. Most candidates were able to apply the algorithm, as given in the formulae booklet, to perform the pivot operations and several were able to achieve a tableau that had the right structure.

Assessment for learning

Here is a quick bullet point guide for students when carrying out this process:

After each iteration there should be

- a full set of basis columns (including P)
- no negative values in the RHS column
- the value of P (RHS of objective row) should have increased (or remained the same in a degenerate case).

Question 2 (c)

(c) State the values of x, y and z that result from your iteration. **[1]**

Most candidates knew how to read off the current solution from their simplex tableau. Some did not know how to deal with non-basis variables, and some gave the values of s, t, and u as well.

Question 2 (d)

After two iterations the resulting tableau is

P	x	y	z	s	t	u	RHS
1	0	0	−2	0	2.5	0.5	16
0	0	0	−2	1	−2.5	0.5	16
0	1	0	−1	0	1.5	0.5	10
0	0	1	−1	0	0.5	0.5	4

The boundaries of the feasible region are planes, with edges each defined by two of x, y, z, s, t, u being zero.
At each vertex of the feasible region there are three basic variables and three non-basic variables.

(d) Interpret the second iteration geometrically by stating which edge of the feasible region is being moved along. As part of your geometrical interpretation, you should state the beginning vertex and end vertex of the second iteration. **[2]**

This question tested the geometric interpretation of the iteration. The vertex at the end of the first iteration could be read from their answer to part (c), provided it had three basic variables (apart from P) and the vertex at the end of the second iteration could be read from the given tableau. A few candidates ignored the tableau given in the stem to this question and wrote down the vertices corresponding to the first iteration instead of the second iteration.

The question told candidates that the edges are defined by two of the variables being zero, so they needed to identify the two variables that had the value 0 in both vertices.

Exemplar 1

2(d) Move along edge y = 0, z = 0

From x = 6, y = 0, z = 0, s = 12, t = 0, u = 8

to x = 10, y = 4, z = 0, s = 16, t = 0, u = 0

This candidate has the correct vertices, but the edge should be z = 0, t = 0. This can be seen by looking for columns with value 0 in both vertices.

Question 3 (a)

3 Amir and Beth play a zero-sum game.

The table shows the pay-off for Amir for each combination of strategies, where these values are known.

		Beth X	Beth Y	Beth Z
Amir	P	2	−3	c
	Q	−3	b	4
	R	a	−1	−2

You are given that $a < 0 < b < c$.

Amir's play-safe strategy is R.

(a) Determine the range of possible values of a. **[3]**

Many candidates found the row minima for rows P and Q as -3, using the fact that b and c are both > 0. The row minimum for row R is either -2 or a, depending on the value of a (which is < 0). This is the maximin, so min(a, -2) > -3 and hence -3 < a < 0.

The inclusion of unknown entries confused some candidates.

Question 3 (b)

Beth's play-safe strategy is Y.

(b) Determine the range of possible values of b. **[3]**

Some candidates found the column maxima correctly and used the fact the column Y gives the minimax to determine that 0 < b < 2.

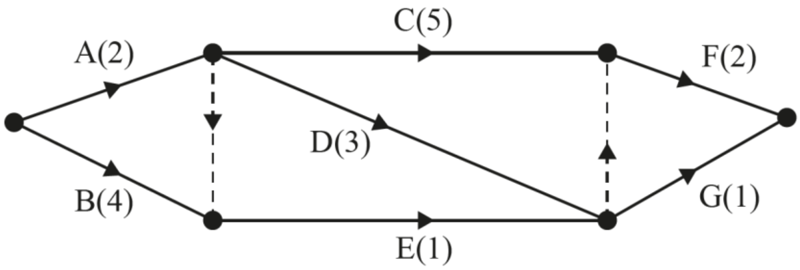
Question 3 (c)

(c) Determine whether or not the game is stable. **[3]**

There were several approaches used to show that the game is not stable. Some candidates used row maximin and column minimax values, others considered whether a player would want to change from their current position. The candidates who just compared the values of a and b had not quite done enough because a is not necessarily the row maximin.

Question 4 (a)

4 A project is represented by the activity network below.
The activity durations are given in hours.



(a) By carrying out a forward pass, determine the minimum project completion time. **[2]**

Most candidates were able to complete this part correctly, some did not show their forward pass (early event times).

Question 4 (b)

(b) By carrying out a backward pass, determine the (total) float for each activity. **[4]**

Some candidates did not deal with the dummy activities correctly and claimed, for example, that the late event time at the start of G is 9 – 1 = 8, instead of (9 – 2) – 0 = 7 – 0 = 7. Many candidates were able to use their early and late event times to calculate the float for each activity.

Question 4 (c)

(c) For each non-critical activity, determine the independent float and the interfering float. **[3]**

Many candidates knew how to use early and late event times to find the independent float and hence the interfering float. Some did not know how to deal with activity E where $\text{EET}_j - \text{LET}_i < \text{D}_{ij}$.

Assessment for learning

'Determine' is a command word and means that justification should be given for any results.

A full list of command words can be found on pages 9–11 in the specification. There is also a poster that can be printed and displayed in the classroom https://www.ocr.org.uk/Images/533967-a-level-maths-command-words-poster-a4-size.pdf

Question 4 (d)

(d) Construct a cascade chart showing all the critical activities on one row and each non-critical activity on a separate row, starting at its earliest start time, and using dashed lines to indicate (total) float. You may not need to use all the grid. **[3]**

Most candidates were able to show the critical activities correctly, many also showed the non-critical activities correctly but there were some errors, and a few candidates did not show the float for every non-critical activity.

Question 4 (e)

Each activity requires exactly one worker.

(e) Construct a schedule to show how exactly two workers can complete the project as quickly as possible. You may not need to use all the grid. **[2]**

This was usually done well, with correct activity durations but sometimes activity F or G was shown starting before D and E were both finished. A few candidates tried to split an activity between the workers, this is not appropriate unless specified in the question.

Question 4 (f)

Issues with deliveries delay the earliest possible start of activity D by 3 hours.

(f) Construct a schedule to show how exactly two workers can complete the project with this delay as quickly as possible. You may not need to use all the grid. **[2]**

Most candidates were able to construct a schedule with activity D starting at time 5, many were able to deal with the precedences appropriately, although a few overlapped F or G with D, and some started C later than was necessary which caused the project to overrun.

Question 5 (a)

5 **(a)** Write down a way in which the nearest neighbour method can fail to solve the problem of finding a least weight cycle through all the vertices of a network. **[1]**

The nearest neighbour method can fail by stalling (reaching a position where the path cannot be continued). Some candidates described specific situations that would cause this to happen, such as a network with a vertex of degree 1. Some candidates worried about whether the network was complete, or not, without quite getting to the issue. The candidates who realised that the network needed to be connected had the neatest responses.

Question 5 (b)

(b) Explain why, when trying to find a least weight cycle through all the vertices of a network, an ad hoc method may be preferable to an algorithmic approach. **[1]**

Any appropriate reason for why an ad hoc method may be preferable to an algorithmic approach, either in general or in this specific case, was accepted.

Question 5 (c)

The distance matrix below represents a network connecting six viewpoints A, B, C, D, E and F.
The distance matrix shows the direct distances between each pair of viewpoints where a direct route exists.
The distances are measured in km.
A blank shows that there is no direct route between the two viewpoints.

	A	B	C	D	E	F
A		6		4		
B	6		5	2	9	
C		5		15	7	6
D	4	2	15		5	
E		9	7	5		
F			6			

(c) Draw the network on the vertices given in the Printed Answer Booklet. **[2]**

Most candidates scored full marks here, although some positioned their arc weights ambiguously and tripped themselves up later because of this.

Question 5 (d)

(d) Apply the nearest neighbour method, starting from A. **[2]**

Most candidates were able to apply the nearest neighbour method, although some went past the point where the method stalled.

The application of the nearest neighbour method should result in a path or cycle, and not just a sum of arc weights. Here the method gives $A – D – B – C – F$ and then stalls without getting to E.

Question 5 (e)

A hiker wants to travel between the six viewpoints, starting and finishing at A.
The hiker must visit every viewpoint at least once, but may visit a viewpoint more than once.

(e) Show that the hiker does not need to travel as far as 50 km. **[2]**

Most candidates gave a suitable route, usually of length either 39 km or 45 km. A few candidates adapted their response to part (d), which was fine provided they either referred to part (d) or wrote out the resulting route.

Question 5 (f)

(f) Use an appropriate algorithm to find the shortest distance from F to each of the other viewpoints. **[5]**

Candidates needed to tell us that they were using Dijkstra's algorithm and then apply it, in any form, to find the shortest distances from F. This meant starting Dijkstra's algorithm from vertex F and updating at vertex D. Some candidates showed all the temporary labels, whether they were improvements or not.

Some candidates used the box notation on the network from part (c), this was fine especially if they said that the working was shown in part (c).

Some candidates chose Kruskal's algorithm, but while a minimum spanning tree can be used to find the shortest distances from F to A, B, C and D it would give FE = 18 instead of 13.

Question 5 (g)

(g) Complete the table in the Printed Answer Book to show the shortest distance between each pair of viewpoints. **[2]**

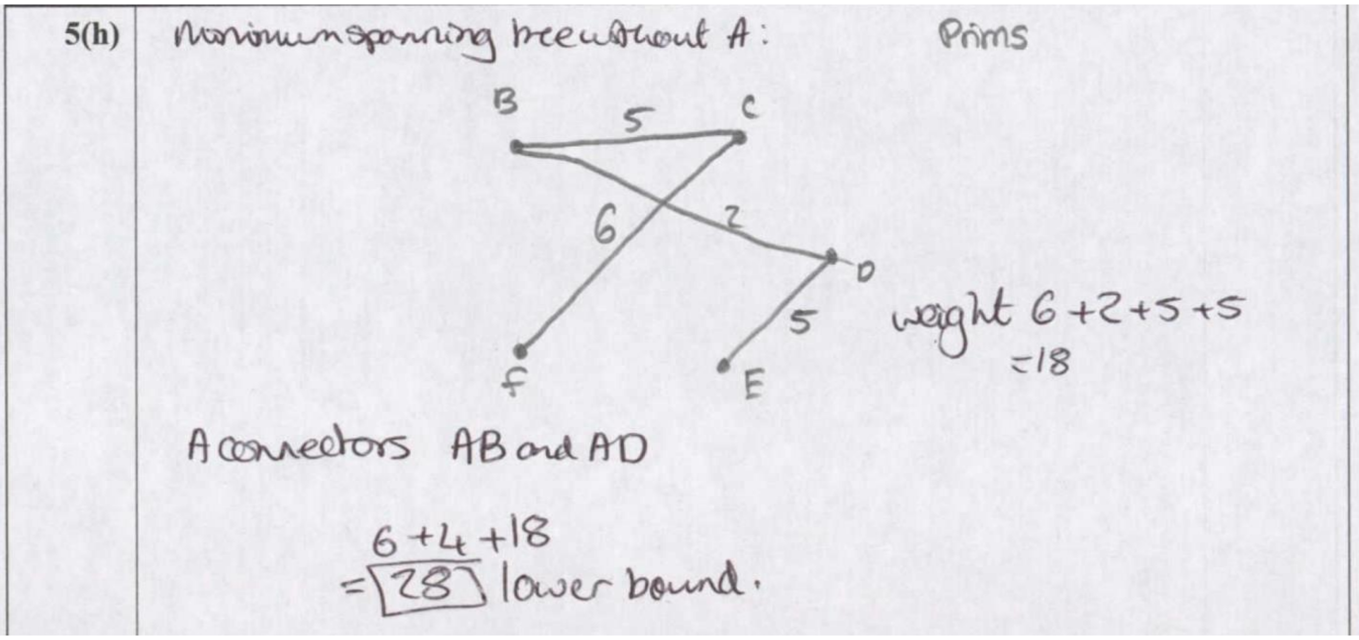
Having found the shortest distances from F the shortest distances from C were then easy to find. Several candidates missed at least one of the instances where the direct route was longer than an indirect route (BE, CD) and some miscalculated the shortest distances where no direct route was available (AC, AE)

Question 5 (h)

(h) Use your answer to part **(g)** to find a lower bound for the distance the hiker must travel by initially deleting vertex A. **[3]**

Usually done well, although some candidates did not show which arcs they were including in their spanning tree.

Exemplar 2



This candidate has drawn the minimum spanning tree. The spanning tree could be drawn, shown by listing arcs or shown by ringing entries in a table. At this point spanning trees that were not the correct minimum spanning tree were allowed.

The candidate has shown that they have added the two least weight arcs from A to get the lower bound as 28 km.

Question 6 (a)

6 Sasha is making three **identical** bead bracelets using amber, brown and red coloured beads. Sasha has 20 amber beads, 12 brown beads and 10 red beads.

Each bracelet must use exactly 12 beads.

The profit from selling a bracelet is 6 pence for each amber bead used plus 2 pence for each brown bead used plus 3 pence for each red bead used.

Sasha wants to maximise the total profit from selling the three bracelets.

(a) Express Sasha's problem as a linear programming formulation in **two** variables a and b, where a represents the number of amber beads in **each** bracelet and b represents the number of brown beads in **each** bracelet. **[5]**

Most candidates realised how to express the number of red beads used in terms of a and b, and were able to find an appropriate expression for the objective, to be maximised.

Some candidates only made one bracelet or made three bracelets that were not identical.

Some candidates tried to use proportional reasoning with the total number of beads.

The number of beads of each colour needed to be integer valued, so, for the non-trivial constraints, as well as $b \leq 4$ we needed $a \leq 6$ rather than $a \leq 20/6$ and to make sure that the number of red beads was non-negative we needed $a + b \geq 9$.

Question 6 (b)

(b) Determine how many beads of each colour will be used in each bracelet. **[3]**

Many candidates did not attempt this question, of those who did, several were able to find a feasible, or near feasible solution for *a* and *b* and used it to calculate the number of red beads used. Often this was done using informal reasoning or trying out cases, although some candidates chose a graphical approach and a few tried to use Simplex. Good candidates were able to use the upper bounds for the variables to determine the feasible options.

Question 6 (c)

(c) By listing all the feasible solutions, identify an aspect of the optimal solution, other than the profit, that is different from all the other feasible solutions. **[2]**

Many candidates did not attempt this question, and of those who did, many did not answer the question which they had been asked, often only giving the values for a and b and sometimes repeating the solution already given in part (b). The question asked candidates to list all the feasible solutions, this means the number of beads of each colour (including red) for the three feasible solutions and to identify an aspect that was different for the optimal solution compared with each of the non-optimal solutions.

Question 6 (d)

The beads that are not used in making the bracelets can be sold.

The profit from selling each amber bead is k pence, where k is an integer, but nothing for each brown or red bead sold.

All the previous constraints still apply.

Instead of maximising the profit from the bracelets, Sasha wants to maximise the total profit from selling the bracelets and any left over beads.

You are given that the optimal solution to the earlier problem does not maximise the total profit from selling the bracelets and any left over beads.

(d) Determine the least possible value of Sasha's maximum total profit. **[5]**

Most candidates struggled with this part, finding it challenging to handle the constraints in the question, particularly the constraint that the three bracelets needed to be identical. However, there were some excellent solutions. Some responses were incomplete but it was clear that the candidate had understood what was involved.

Exemplar 3

6(d)

Now profit > ~~168p~~ 153p + 2k.

To maximise profit, use ~~the~~ maximum number of red and brown beads to leave as many amber left over

∴ Each bracelet contains 5 amber, 4 brown, 3 red

, so total bracelet profit = $3(5\times6+4\times2+3\times3)$

= 141

Extra profit = 5k

$141+5k > 153+2k$

$3k > 12$

$k > 4$

$\Rightarrow k \geq 5$

So least possible value = $141+5\times5 = 166$p

Least possible value of Sasha's maximum total profit is ~~...~~ 166p.

This candidate has correctly identified the number of surplus amber beads in each case and made an appropriate comparison to determine the minimum value of k for which the previous optimum no longer maximises the profit, and hence the least possible profit in this situation.

Question 6 (e)

(e) Why might Sasha not achieve this maximum profit? **[1]**

Generally, candidates who answered this part were successful as it was a straightforward mark at the end of the paper.

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