Qualification Accredited

OCR
Oxford Cambridge and RSA

A LEVEL

Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y543/01 Summer 2024 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y543 series overview

This paper provided candidates a good opportunity to demonstrate their understanding of the specification. The most successful candidates confidently tackled questions involving energy consideration, and calculation of unknown forces and/or velocities. Standard calculations were structured effectively with formal algebraic methods seen. A number of candidates are showing a lack of understanding of the command words, showing insufficient detail in their solution to 'show that' or 'determine' questions. Question 6 (b), Question 7 (c) and Question 8 (b) were less accessible to candidates, proving to be a good test of problem solving skills.

Candidates who did well on this paper generally:	**Candidates who did less well on this paper generally:**
• used clear, labelled diagrams to support their solutions • set work out with concise and accurate use of algebra • considered their answers in the framework of the real-life context given.	• confused key formulae, for example in the context of energy versus force calculations • missed out steps in 'determine' and 'show that' questions • confused directions when resolving forces/velocities • omitted multiple questions.

Question 1 (a)

1 A particle P of mass 12.5 kg is moving on a smooth horizontal plane when it collides obliquely with a fixed vertical wall.

At the instant before the collision, the velocity of P is $-5\mathbf{i}+12\mathbf{j}\ \mathrm{m\,s^{-1}}$.

At the instant after the collision, the velocity of P is $\mathbf{i}+4\mathbf{j}\ \mathrm{m\,s^{-1}}$.

(a) Find the magnitude of the momentum of P **before** the collision. **[2]**

This question was answered effectively by almost all candidates, with clear working shown. The most common error was neglecting to multiply the magnitude of velocity by the mass to obtain momentum.

Question 1 (b)

(b) Find, in vector form, the impulse that the wall exerts on P. **[2]**

Candidates answered this question well, with most gaining both marks. The most common errors seen here, were those candidates who chose to calculate the impulse P exerts on the wall, subtracting the velocities in the incorrect order. A small minority of candidates misread the question, considering either momentum or velocity here, or incorrectly stated that no impulse would be felt.

Question 1 (c)

(c) State, in vector form, the impulse that P exerts on the wall. **[1]**

The majority of candidates were able to effectively reverse the direction of their answer to Question 1 (b) in this part.

Question 1 (d)

(d) Find in either order.

- The magnitude of the impulse that the wall exerts on P.
- The angle between $\mathbf{i}$ and the impulse that the wall exerts on P. **[3]**

Almost all candidates gained the mark for effective calculation of the magnitude of the impulse exerted on P. Candidates who drew a diagram were typically successful in accurately calculating the angle between I and the impulse. The dot product formulae and right angled trigonometry were equally effective when used with a diagram. Common errors seen included calculating the angle between the impulse and one of the velocities rather than **i**, or finding the complementary angle to the angle asked for.

Question 2 (a)

2 One end of a light elastic string of natural length 1.4 m and modulus of elasticity 20 N is attached to a small object B of mass 2.5 kg. The other end of the string is attached to a fixed point O.

Object B is projected vertically upwards from O with a speed of $u\,\text{m s}^{-1}$.

(a) State **one** assumption required to model the motion of B. **[1]**

Almost all candidates answered this effectively, with common correct answers commenting on air resistance being neglected or the lack of consideration of the dimensions of the object. A minority of candidates incorrectly stated that the object was modelled as if it had no mass or that the elastic string had been modelled as an inextensible string.

Question 2 (b)

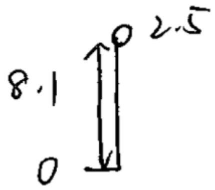
The greatest height above O achieved by B is 8.1 m.

(b) Determine the value of u. **[4]**

Those candidates who constructed a clear conservation of energy equation and then substituted values in typically obtained full marks for an efficient and effective response. A few candidates attempted to break the motion into two stages, considering the point when the string was taut as a separation point between stages. This overly-complex approach gave further scope for errors to be introduced and increase the time spent on this question. Some candidates did not square the extension in their expression for elastic potential energy, with a small minority assuming that the object was projected downwards, rather than upwards.

Exemplar 1

2(b)



$\frac{\lambda x^2}{2l} = \frac{20(8.1-1.4)^2}{2\times 1.4}$

$= 320.6428571$

Initial:

GPE $= mgh = 0$

EPE $= 0$

KE $= \frac{1}{2}mv^2 = \frac{1}{2}\times 2.5\times u^2$

Final:

GPE $= 2.5\times g\times 8.1$

EPE $= 320.64$

KE $= 0$

$\frac{1}{2}\times 2.5u^2 = 2.5g\times 8.1 + 320.64$

$u^2 = 415.27$

$u = 20.378$

$u \approx 20.4$ m/s.

This candidate has demonstrated a clear and logical solution that has been given full marks. They have considered all initial and final energy terms, showing clear substitution of any quantities specific to the question and combining these correctly in a single conservation of energy equation.

Question 3 (a)

3 The mass of a truck is 6000 kg and the maximum power that its engine can generate is 90 kW. In a model of the motion of the truck it is assumed that while it is moving the total resistance to its motion is constant.

At first the truck is driven along a straight horizontal road. The greatest constant speed that it can be driven at when it is using maximum power is $25\,\mathrm{m\,s^{-1}}$.

(a) Find the value of the resistance to motion. **[2]**

Almost all candidates calculated the driving force effectively from the maximum power and velocity. For the final mark, candidates needed to clearly state that resistance is equal to that driving force at constant speed. Some candidates were unclear in their statement of which quantity they had calculated, losing the final mark. Those candidates who made use of a diagram were likely to have answered this clearly.

Question 3 (b)

The truck is being driven along the horizontal road with the engine working at 60 kW.

(b) Find the acceleration of the truck at the instant when its speed is $10\,\mathrm{m\,s^{-1}}$. **[2]**

Candidates answered this question well, constructing an effective Newton's 2nd Law equation. A minority of candidates incorrectly used 90000W instead of 60000W.

Question 3 (c)

The truck is now driven **down** a straight road which is inclined at an angle θ below the horizontal. The greatest constant speed that the truck can be driven at maximum power is $40\,\mathrm{m\,s^{-1}}$.

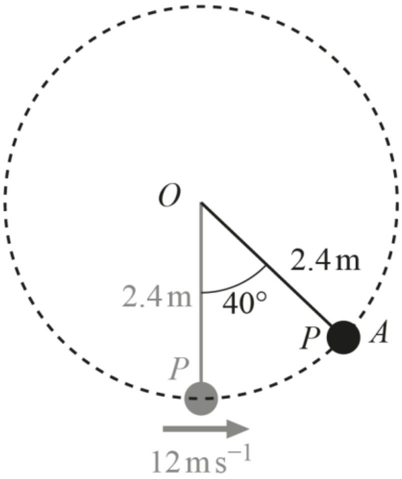
(c) Determine the value of θ. **[3]**

The majority of candidates who had been successful in Question 3 continued their performance in this part, making use of the acceleration of 0 to construct an effective equation. Some candidates incorrectly used 60000W rather than returning to the 90000W capability of the engine. A small minority of candidates resolved incorrectly when attempting to find the component of weight or misread and assumed the truck was travelling upslope.

Question 4 (a)

4 A particle, P, of mass 6 kg is attached to one end of a light inextensible rod of length 2.4 m. The other end of the rod is smoothly hinged at a fixed point O and the rod is free to rotate in any direction.

Initially, P is at rest, vertically below O, when it is projected horizontally with a speed of $12\,\text{m s}^{-1}$. It subsequently describes complete vertical circles with O as the centre.



The angle that the rod makes with the downward vertical through O at each instant is denoted by θ and A is the point which P passes through where $\theta = 40°$ (see diagram).

(a) Find the tangential acceleration of P at A, stating its direction. **[2]**

Many candidates answered this question well, accurately calculating the tangential acceleration. Some candidates lost the final mark by attempting to describe the direction using a complicated combination of angles from an ambiguous set of axes, or describing it using words that imply turning such as 'clockwise'.

Question 4 (b)

(b) Determine the radial acceleration of P at A, stating its direction. **[6]**

In this detailed reasoning question, candidates needed to show the derivation of their expression for v, by considering conservation of energy in order to gain all marks. Some candidates neglected to state the direction of the acceleration as part of their final answer. Those that did, typically unambiguously stated 'towards 0'. Most candidates were able to correctly calculate GPE with a minority using change in height using $\Delta h = 2.4\cos(40°)$ rather than $2.4(1 - cos(40°))$.

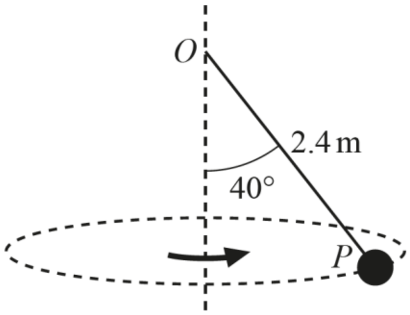
Question 4 (c)

(c) Find the magnitude of the force in the rod when P is at A, stating whether the rod is in tension or compression. **[2]**

This question was completed more variably by candidates, with many gaining at least 1 mark for consideration of Newton's 2nd Law parallel to the rod. Some candidates were uncertain if the rod was under compression or tension, again, a diagram was helpful here to support their understanding.

Question 4 (d)

The motion is now stopped when P is at A, and P is then projected in such a way that it now describes horizontal circles at a constant speed with $\theta = 40°$ (see diagram).



(d) Find the speed of P. **[4]**

This was an accessible question for candidates, with many able to effectively resolve forces horizontally and vertically for P, resulting in an accurate velocity. Common errors seen include resolving tension as $T = 6gcos(40)$, calculating the radius for P as 2.4 rather than $2.4 \sin\theta$, or confusing $\sin\theta$ and $\cos\theta$ when resolving. Some candidates incorrectly transferred the tension from Question 4 (c) to this part, even though the direction of circular motion had changed.

Question 4 (e)

(e) Explain why, wherever P's motion is initiated from and whatever its initial velocity, it is **not** possible for P to describe horizontal circles at constant speed with $\theta = 90°$. **[1]**

Just under half of candidates answered this effectively, explaining that there is no vertical component of tension to balance the weight of P, so it would experience movement downwards. Unsuccessful candidates tended to state that tension itself was zero, rather than the vertical component, implying no circular motion.

Question 5

5 In this question you may assume that if x and y are any physical quantities then $\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right] = \left[\frac{y}{x}\right]$.

A machine drives a piston of mass m into a vertical cylinder. The equation below is suggested to model the power developed by the machine, P, while it is not doing any other external work.

$$P = k_1 mv\frac{\mathrm{d}v}{\mathrm{d}t} + k_2 mgv + k_3 E$$

in which

- v is the velocity of the piston at a given time,
- g is the acceleration due to gravity,
- E is the **rate** at which heat energy is lost to the surroundings,
- k_1, k_2 and k_3 are dimensionless constants.

Determine whether the equation is dimensionally consistent. Show **all** the steps in your argument. **[6]**

Many candidates gained 4 or 5 out of the 6 marks available. Successful candidates showed formal derivation of the dimensions of each term of the formula to gain the correct dimension. A significant number missed out considering the constant k, or the combination of the terms on the right-hand side of the equation to give a term of equivalent dimension, leading to the loss of the final mark. Candidates that were less successful did not identify which term they were attempting to derive dimensions for or listed known dimensions for different parts of each term without giving a clear combination to obtain an answer.

Question 6 (a)

6 Two identical spheres, A and B, each of mass m kg, are moving directly towards each other along the same straight line on a smooth horizontal surface until they collide. Just before they collide, the speeds of A and B are $20\,\mathrm{m\,s^{-1}}$ and $10\,\mathrm{m\,s^{-1}}$ respectively. The coefficient of restitution between A and B is e.

(a) By finding, in terms of e, an expression for the velocity of B after the collision, show that the direction of motion of B is reversed by the collision. **[5]**

Candidates generally completed this part well. Clear diagrams were typically included. If errors were seen they tended to be either in using the correct signs when combining the speeds of the two particles, or when combining the simultaneous equations to obtain v_B. Candidates should write a formal conclusion for this 'show that' question, showing clear consideration of the velocity before and after collision.

Question 6 (b)

After the collision between A and B, which is **not** perfectly elastic, B goes on to collide directly with a fixed, vertical wall. The coefficient of restitution between B and the wall is $\frac{2}{5}e$. After the collision between B and the wall, there are no further collisions between A and B.

(b) Determine the range of possible values of e. **[7]**

This question was completed variably. Many candidates were able to find the velocity of A and B after the collision between B and the wall. However, the altered direction of both particles caused some confusion when candidates attempted to compare their velocities to fulfil the condition that there were no further collisions. A clear diagram would have supported candidates to consider this more effectively. Candidates who constructed a correct inequality comparing the velocities, tended to proceed effectively to a solution, with a few neglecting to show the solution to their inequality, or not excluding perfectly elastic collisions in their final solution.

Exemplar 2

6(b)

6(b) (continued)

$e \neq 1$

$V_{\text{after wall}} = -\frac{2}{5} e V_B$

$-\frac{2}{5} e V_B \geq V_A$

$V_A = V_B - 30e \qquad V_B = 5 + 15e$

$V_A = 5 - 15e$

$-\frac{2}{5} e (5 + 15e) = -2e - 6e^2$

$-2e - 6e^2 > 5 - 15e$

$0 > 6e^2 - 13e + 5$

$0 > (2e - 1)(3e - 5) \qquad e = \frac{1}{2}, \frac{5}{3}$

$\therefore e \leq \frac{1}{2} \vee e \geq \frac{5}{3} \qquad$ but $0 < e \leq 1$

$\therefore 0 < e \leq \frac{1}{2}$

This candidate has made an effective start to the question with correct velocities from for A and B. A correct comparison is made. It would have been beneficial for the candidate to sketch the obtained quadratic in *e*, in order to select the correct range of solutions, enabling them to proceed to the correct final answer.

Assessment for learning

It is good practice to encourage candidates to sketch multiple diagrams in multi collision questions, clearly identifying the chosen direction of calculated velocities at each stage. The total net momentum of a situation should be taken into account to make sure that candidates are not selecting impossible directions in relation to the initial velocities. Candidates can be encouraged to perform this as a standalone task, to emphasise the importance of this interpretation in context.

Question 7 (a)

7 A body B of mass 1.5 kg is moving along the x-axis. At the instant that it is at the origin, O, its velocity is $u\,\mathrm{m\,s^{-1}}$ in the positive x-direction.

At any instant, the resistance to the motion of B is modelled as being directly proportional to v^2 where $v\,\mathrm{m\,s^{-1}}$ is the velocity of B at that instant. The resistance to motion is the only horizontal force acting on B.

At an instant when B's velocity is $2\,\mathrm{m\,s^{-1}}$, the resistance to its motion is 24 N.

(a) Show that B's motion can be modelled by the differential equation $\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} = -4$. **[3]**

Candidates showed clear formal reasoning in the majority of cases, deriving the constant of proportionality and proceeding to set up an effective equation using $v\frac{dv}{dx}$ directly for acceleration. A few candidates used a longer method, showing the derivation of this expression from $\frac{dv}{dt}$ via the chain rule. This is not necessary and recall of this expression would save time. In this detailed reasoning question, candidates were expected to show clear rearrangement of their Newton's 2nd Law equation to obtain the final mark.

Question 7 (b) (i)

(b) (i) Solve the differential equation in part **(a)** to find the particular solution for v in terms of x and u. **[4]**

Candidates found this part very accessible with almost all gaining at least 2 marks, and the majority of those gained all 4 marks for correct use of limits to obtain a final expression for *v*. Most candidates showed formal use of initial conditions to find the particular solution, with those that were less successful neglecting this step, and deriving an integration constant with an incorrect sign.

Question 7 (b) (ii)

(ii) By considering the behaviour of v as $x \to \infty$ describe **one** feature of the model that is not realistic. **[1]**

Few candidates were able to answer this question effectively. Those that did gave clear consideration of the fact that the model predicts that the velocity will tend towards but never reach 0, stating that this is not realistic as the body will stop. Common incorrect answers included suggesting that the velocity would tend towards the initial velocity, or implied that the model predicts that the body will stop.

Question 7 (c) (i)

At the instant when B reaches the point A, where $x = X$, its speed is $V \mathrm{~m\,s^{-1}}$. The work done by the resistance as B moves from O to A is denoted by W J.

(c) (i) Use the formula $W = \int F \, \mathrm{d}x$ to determine an expression for W in terms of X and u. **[3]**

This question was relatively accessible to candidates, with correct use of limits in their definite integral proceeding smoothly to the answer. Those who chose to perform an indefinite integration first sometimes found it harder to make effective use of the conditions given to arrive at a final solution. Candidates needed to take care to include the negative sign in their integral and to make clear use of capital letters for the quantities given. A minority of candidates converted their integral to an integration with respect to *v*. Effective use of the limit, *V*, enabled candidates to achieve all marks with this alternative method.

Question 7 (c) (ii)

(ii) Explain the relevance of the sign of your answer in part **(c)(i)**. **[1]**

Candidates found this part challenging, with few formally considering the sign of their expression for W when X>0. Those who were successful proceeded to state that the particle was doing work against the resistance. More commonly, candidates incorrectly stated that 'work is being done against motion' showing an insufficiently specific understanding of the situation.

Question 7 (c) (iii)

(iii) By writing your answer to part **(c)(i)** in terms of V and u show how the quantity W relates to the energy of B. **[2]**

Rigorous candidates did well here, starting with their expression for W and clearly substituting V. Less successful solutions made ambiguous use of signs and capitalisation, with conclusions needing greater clarity.

Question 8 (a)

8 A shape, S, is formed by attaching a particle of mass $2m$ kg to the vertex of a uniform solid cone of mass $8m$ kg. The height of the cone is h m and the radius of the base of the cone is 1.1 m.

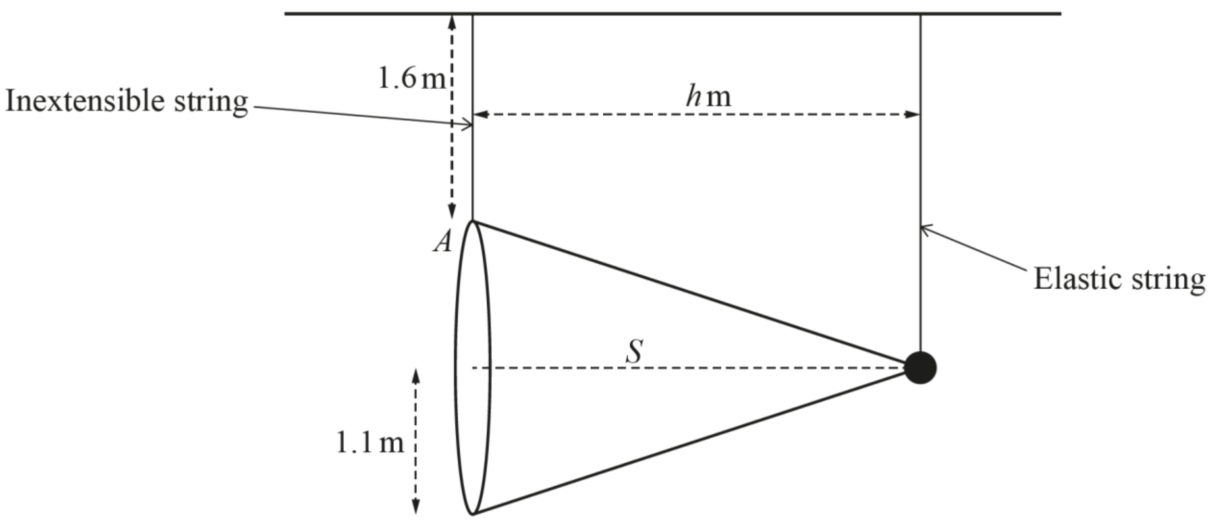
(a) Explain why the centre of mass of S must lie on the central axis of the cone. **[1]**

This mark was given to almost all candidates, who gave a clear explanation that this was due to symmetry.

Question 8 (b)

Two strings are attached to S, one at the vertex of the cone and one at A which is a point on the edge of the base of S. The other ends of the strings are attached to a horizontal ceiling in such a way that the strings are both vertical. The string attached to S at A is inextensible and has length 1.6 m. The string attached to S at the vertex is elastic with modulus of elasticity $8mg$ N.

Shape S is in equilibrium with its axis horizontal (see diagram).

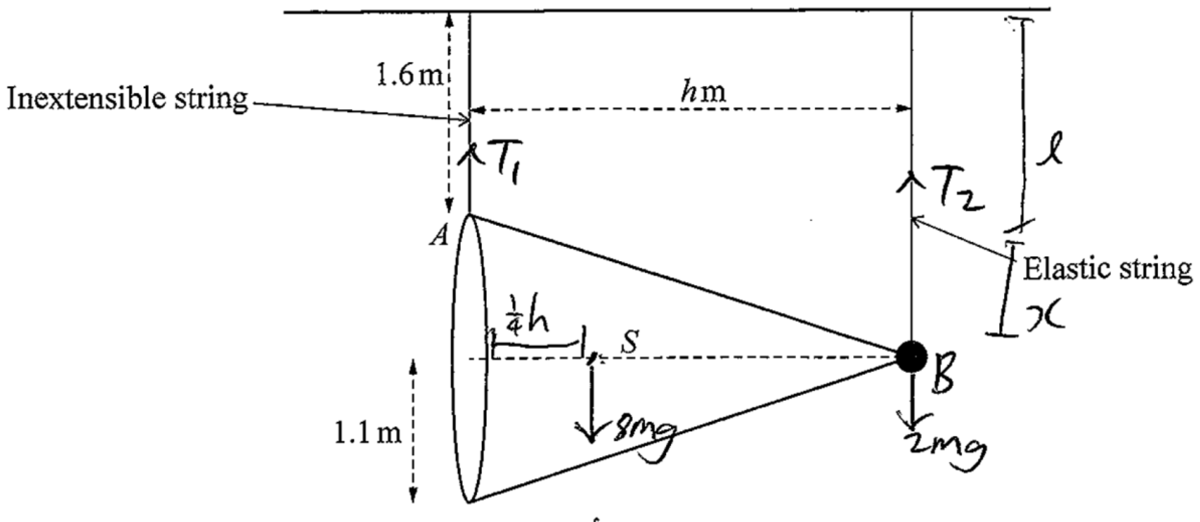


(b) Determine the natural length of the elastic string. **[7]**

Many candidates were able to answer this question effectively and efficiently, supported by a clear diagram with labelled tensions and weights. A concise method comprised a few lines of working with complete consideration of moments around the base leading directly to the tension in the elastic string combined with the substitution of Hooke's law to find the natural length. A number of candidates were able to find the centre of mass of the compound object correctly, but then used mass in their calculation of some moments rather than weight, leaving them unable to proceed further. Other common errors included the use of the centre of mass for a hollow cone, miscalculating the distances used in the moments equation, or using Hooke's law on the inextensible string.

Exemplar 3

8(b)



Centre of mass of S : $\frac{1}{4}h$ from base

$(N2\downarrow)$: $8mg + 2mg = T_1 + T_2$ $\quad 10mg = T_1 + T_2$

B : $8mg \times \frac{3}{4}h = T_1 \times h$

$T_1 = 6mg$

$10mg = (6mg) + T_2 \quad \therefore \quad T_2 = 4mg$

$4mg = \frac{8mg}{l} \times x \qquad l + x = 1.6 + 1.1$

$l + x = 2.7$

$l = 2x \qquad 2x + x = 2.7$

$3x = 2.7$

$l = 1.8 \qquad x = 0.9$

This candidate has made good use of moments equations to concisely find the tension of the inextensible string and thus the other unknown tension. At each stage they have clearly indicated the type of equation they are constructing, and the positive direction selected where appropriate. They have also indicated all quantities clearly on the diagram. This has enabled them to tackle the question efficiently. It would have been swifter still to take moments around the base of the cone initially, avoiding the need to calculate the tension of the inextensible string.

Supporting you

Teach Cambridge

Make sure you visit our secure website **Teach Cambridge** to find the full range of resources and support for the subjects you teach. This includes secure materials such as set assignments and exemplars, online and on-demand training.

Don't have access? If your school or college teaches any OCR qualifications, please contact your exams officer. You can forward them this link to help get you started.

Reviews of marking

If any of your students' results are not as expected, you may wish to consider one of our post-results services. For full information about the options available visit the OCR website.

Access to Scripts

We've made it easier for Exams Officers to download copies of your candidates' completed papers or 'scripts'. Your centre can use these scripts to decide whether to request a review of marking and to support teaching and learning.

Our free, on-demand service, Access to Scripts is available via our single sign-on service, My Cambridge. Step-by-step instructions are on our website.

Keep up-to-date

We send a monthly bulletin to tell you about important updates. You can also sign up for your subject specific updates. If you haven't already, sign up here.

OCR Professional Development

Attend one of our popular professional development courses to hear directly from a senior assessor or drop in to a Q&A session. Most of our courses are delivered live via an online platform, so you can attend from any location.

Please find details for all our courses for your subject on **Teach Cambridge**. You'll also find links to our online courses on NEA marking and support.

Signed up for ExamBuilder?

ExamBuilder is a free test-building platform, providing unlimited users exclusively for staff at OCR centres with an Interchange account.

Choose from a large bank of questions to build personalised tests and custom mark schemes, with the option to add custom cover pages to simulate real examinations. You can also edit and download complete past papers.

Find out more.

Active Results

Review students' exam performance with our free online results analysis tool. It is available for all GCSEs, AS and A Levels and Cambridge Nationals (examined units only).

Find out more.

You will need an Interchange account to access our digital products. If you do not have an Interchange account please contact your centre administrator (usually the Exams Officer) to request a username, or nominate an existing Interchange user in your department.

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on
01223 553998

Alternatively, you can email us on
support@ocr.org.uk

For more information visit
- **ocr.org.uk/qualifications/resource-finder**
- **ocr.org.uk**
- **facebook.com/ocrexams**
- **twitter.com/ocrexams**
- **instagram.com/ocrexaminations**
- **linkedin.com/company/ocr**
- **youtube.com/ocrexams**

We really value your feedback

Click to send us an autogenerated email about this resource. Add comments if you want to.
Let us know how we can improve this resource or what else you need. Your email address will not be used or shared for any marketing purposes.

I like this **I dislike this**

Please note – web links are correct at date of publication but other websites may change over time. If you have any problems with a link you may want to navigate to that organisation's website for a direct search.

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT