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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y540/01 Summer 2024 series

Version 1

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y540/01 series overview

This paper, along with Y541/01, assesses the compulsory core content of the A Level Further Mathematics qualification. Questions in each paper can assess any part of the core specification. This paper had a number of straightforward questions, and most candidates had a good attempt at these questions. However, there were some unfamiliar problem-solving aspects which looked daunting, and some candidates struggled to process the issues that the questions posed in order to approach their response in a productive way. Most candidates appeared to be able to complete the paper in the time available.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• showed a secure grasp of all standard techniques • communicated well using mathematical language correctly • applied the breadth of their mathematical knowledge to find ways to tackle problems unfamiliar to them • made minimal arithmetic and algebraic errors • organised their time well so that they had time to check (and possibly correct) their work.	• had gaps in their knowledge of standard techniques • produced unclear or incomplete mathematical arguments, often resulting in algebraic and arithmetic errors • could not find routes into problems unfamiliar to them.

Question 1

1 Given that $y = \sin^{-1}(x^2)$, find $\dfrac{dy}{dx}$. **[3]**

This question was answered very well with most candidates able to write down the correct derivative with minimal working. The two most common errors were failing to apply the chain rule correctly (so forgetting to multiply by $2x$) or forgetting to square the x^2 term in the denominator. Several candidates, probably unfamiliar with the standard results given in the H245 Formula Booklet, tackled this problem by first rewriting as $x^2 = \sin y$ and then differentiated implicitly; these attempts were not as successful as those who simply used the result that $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$.

Assessment for learning

Candidates need to be familiar with the results that appear in the H245 Formula Booklet and, unless a question says otherwise, they may quote these results without proof.

Question 2 (a)

2 The locus C_1 is defined by $C_1 = \left\{z : 0 \leqslant \arg(z + \mathrm{i}) \leqslant \frac{1}{4}\pi\right\}$.

(a) Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_1. **[2]**

Most candidates answered this part correctly and drew a half-line from the point $(0, -1)$ at an angle of $\frac{1}{4}\pi$ to the horizontal and shaded accordingly. The most common errors were lines either beginning at the wrong point or not at the correct angle.

Question 2 (b)

(b) Determine whether the complex number $1.2 + 0.8\text{i}$ is in C_1. **[2]**

Examiners noted many different (although valid) methods for tackling this part. However, several candidates did not give a correct justification as to why the given complex number was in C_1. For example, many candidates calculated a correct angle of 0.9827… (from considering $\tan^{-1}\frac{1.8}{1.2}$) but then did not explicitly compare this to $\frac{1}{4}\pi$. The other common method seen was to state the Cartesian equation of the half-line as $y = x - 1$, substitute for $x = 1.2$ and then compare the answer of 0.2 with 0.8.

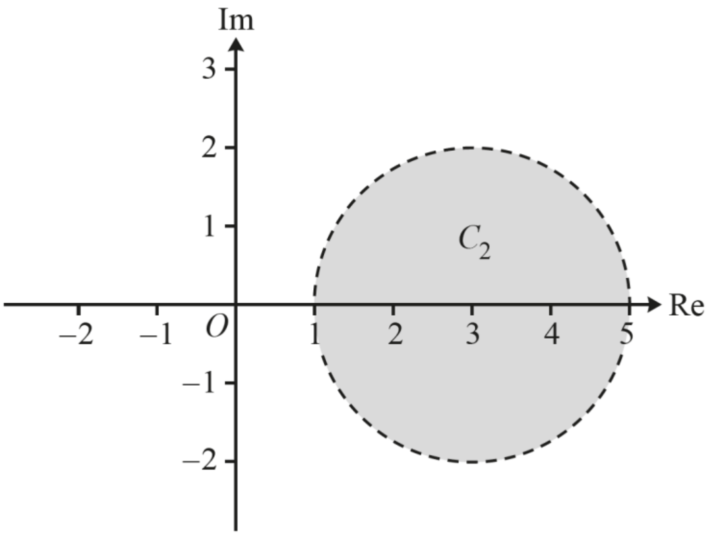
Assessment for learning

Many questions in this paper asked candidates to show detailed reasoning, for example, 'show that' questions, or used the command word 'determine'. In these questions, candidates are required to demonstrate understanding of the topic being assessed and this requires them to show all the steps in their working and/or give sufficient reasoning for their answers. Candidates would benefit from understanding these commands.

A full list of command words can be found on pages 9–11 in the specification. There is also a Teach Cambridge poster that can be printed and displayed in the classroom.

Question 2 (c)

The locus C_2 is the set of complex numbers represented by the interior of the circle with radius 2 and centre 3. The locus C_2 is illustrated on the Argand diagram below.



(c) Use set notation to define C_2. **[2]**

While most candidates correctly understood that the locus of C_2 involved both $|z-3|$ and 2, many used either a wrong inequality sign (so included the boundary of this region) or did not give the answer in set notation; therefore, the correct answer of $\{z: |z-3| < 2\}$ was seen only in around half of the responses.

Question 2 (d)

(d) Determine whether the complex number $1.2 + 0.8\text{i}$ is in C_2. **[2]**

The responses to this part were similar to those seen in part **(b)** in that a number of candidates did not give sufficient reasoning to convey the fact that this time the given complex number was in C_2. The most common method employed by candidates was to consider the value of $|1.2 + 0.8\text{i} - 3|$ and then compare this value (which if correct was 1.969…) to 2.

Question 3 (a)

3 A transformation T is represented by the matrix $\mathbf{N} = \begin{pmatrix} a & 4 & 2 \\ 5 & 1 & 0 \\ 3 & 6 & 3 \end{pmatrix}$, where a is a constant.

(a) Find $\mathbf{N}^2$ in terms of a. **[3]**

This question part was answered very well, with almost all candidates correctly finding $\mathbf{N}^2$ in terms of a. When errors occurred, it was usually a mistake in a single term.

Question 3 (b)

(b) Find det $\mathbf{N}$ in terms of a. **[2]**

Similar to part **(a),** this part was answered extremely well. However, examiners noted that some candidates misread the question and attempted to calculate the determinant of $\mathbf{N}^2$ (rather than $\mathbf{N}$) or incorrectly stated the reciprocal of the correct determinant.

Question 3 (c)

The value of a is 13 to the **nearest integer**.

A shape S_1 has volume 11.6 to 1 decimal place. Shape S_1 is mapped to shape S_2 by the transformation T.

A student claims that the volume of S_2 is less than 400.

(c) Comment on the student's claim. **[3]**

The responses to this problem-solving part were mixed. Most candidates scored two marks for correctly finding a case in which the volume of S_2 was either above or below the value of 400. However, many did not consider that as there were situations in which the volume could be **both** above **and** below this value (for example, the UB was 401.925 and the LB was 363.825), therefore the student's claim (that the volume is less than 400) was not **necessarily** true. Most candidates either gave a definitive answer that the claim was either true or false without realising that the claim could be correct but not necessarily so (so there was some inference rather than a clear deduction happening here).

Question 4

4 In this question you must show detailed reasoning.

The equation $2x^3+3x^2+6x-3=0$ has roots α, β and γ.

Determine a cubic equation with integer coefficients that has roots $\alpha^2\beta\gamma, \alpha\beta^2\gamma$ and $\alpha\beta\gamma^2$. **[3]**

Although there were two equally valid methods for solving this problem very few candidates used the more elegant (and less time consuming) method of substitution. In this case, the substitution that gives the required cubic equation was $w=\frac{3}{2}x$ (based on the fact that the product of the three roots α,β,γ was $\frac{3}{2}$). Most candidates instead opted for using the result that the new roots would satisfy the cubic equation

$w^3-(\alpha^2\beta\gamma+\alpha\beta^2\gamma+\alpha\beta\gamma^2)w+(\alpha^3\beta^3\gamma^2+\alpha^2\beta^3\gamma^3+\alpha^3\beta^2\gamma^3)w-\alpha^4\beta^4\gamma^4=0$ but these attempts were not as successful due to sign errors in evaluating the required coefficients. Regardless of the method used, many candidates did not give their final answer (as requested) with integer coefficients or did not give their answer as an equation (so did not include an '= 0').

Question 5

5 Express $\dfrac{12x^3}{(2x+1)(2x^2+1)}$ using partial fractions. **[5]**

This question had fewer strong responses, with most candidates scoring less than half the marks available. The main issue was that many candidates did not realise that the fraction was improper and so incorrectly stated that $\frac{12x^3}{(2x+1)(2x^2+1)} = \frac{A}{2x+1} + \frac{Bx+C}{2x^2+1}$. Of those that did recognise that the fraction was improper, many made errors using long division to obtain a fraction which was not improper. Those that started by stating that $\frac{12x^3}{(2x+1)(2x^2+1)} = A + \frac{B}{2x+1} + \frac{Cx+D}{2x^2+1}$ were usually the most successful and the majority went on to score all 5 marks.

Assessment for learning

In Further Maths A Level, candidates can be asked to express both proper and improper algebraic fractions in partial fraction form. Therefore, before beginning the problem, candidates should pause and establish beyond doubt which of the two cases is being considered (as incorrectly thinking that a fraction is not improper is a fundamental error which will mean the loss of a significant number of the marks available).

Misconception

Candidates are reminded that an algebraic fraction of the form $\frac{\mathrm{f}(x)}{\mathrm{g}(x)}$ where $\mathrm{f}(x)$ is a polynomial expression of degree m and $\mathrm{g}(x)$ is a polynomial expression of degree n is said to be improper if $m \geq n$ and not just if $m > n$.

Question 6

6 **In this question you must show detailed reasoning.**

Determine the exact value of $\displaystyle\int_9^{\infty} \frac{18}{x^2\sqrt{x}}\,\mathrm{d}x$. **[4]**

Many responses to this question were extremely pleasing with most candidates integrating correctly and dealing with the infinite limit of this improper integral correctly. Most candidates correctly replaced the infinite limit with some letter, say k, and then considered the limit as $k \to \infty$. Examiners were pleased to note that expressions of the form $\frac{1}{\sqrt{\infty^3}} = 0$ were relatively rare.

Question 7 (a)

7 **(a)** By using the definitions of $\cosh u$ and $\sinh u$ in terms of e^u and e^{-u}, show that $\sinh 2u \equiv 2\sinh u \cosh u$. **[2]**

Although most candidates derived the given identity correctly, a number of candidates had working that was either very difficult to follow, or they did not begin their 'show that' with stating one side of the equation explicitly and then, at the end of their derivation, stating the other side of the equation explicitly. Those candidates who started with the right-hand side, for example, $2\sinh u \cosh u = 2\left(\frac{e^u - e^{-u}}{2}\right)\left(\frac{e^u + e^{-u}}{2}\right)$ were usually the most successful. Examiners noted that occasionally candidates opted for a hybrid of u's and x's or incorrectly wrote sinh as sin or cosh as cos.

Question 7 (b)

The equation of a curve, C, is $y = 16\cosh x - \sinh 2x$.

(b) Show that there is only one solution to the equation $\frac{d^2y}{dx^2} = 0$ **[4]**

Most candidates scored the first three marks in this part for correctly differentiating y with respect to x twice and then setting the correct second derivative equal to zero. Unfortunately, many candidates then went directly from $8\cosh x\,(2 - \sinh x) = 0$ to $\sinh x = 2$ without giving any indication as to why $\cosh x$ could not equal zero.

Misconception

There seemed to be some confusion regarding the reasons why some candidates rejected the equation $\cosh x = 0$ with many stating that $\cosh x \geq 0$ or $\cosh x > 1$ or the imprecise statement that '$\cosh x = 0$ has no solutions'. Candidates are reminded that $\cosh x \geq 1$ for all values of x.

Question 7 (c)

You are now given that C has exactly one point of inflection.

(c) Use your answer to part **(b)** to determine the exact coordinates of this point of inflection. Give your answer in a logarithmic form where appropriate. **[3]**

While many candidates correctly stated the x-coordinate of the point of inflection as $\ln\left(2 + \sqrt{5}\right)$ only the stronger candidates were able to correctly determine the y-coordinate in a suitable form, (namely $12\sqrt{5}$) using either the identity $\cosh^2 x - \sinh^2 x \equiv 1$ or the result that $\cosh\left(\ln\left(a + \sqrt{b}\right)\right) \equiv \frac{1}{2}\left(a + \sqrt{b} + \frac{1}{a+\sqrt{b}}\right)$.

Question 8

8 Prove by induction that $11 \times 7^n - 13^n - 1$ is divisible by 3, for all integers $n \geqslant 0$. **[5]**

There were many responses to this proof by induction question that were extremely pleasing. Most candidates scored the first two method marks for forming the correct inductive hypothesis that $11 \times 7^k - 13^k - 1 = 3m$ (for some integer m) and then continuing by considering the expression $11 \times 7^{k+1} - 13^{k+1} - 1$ in an attempt to use the inductive hypothesis to show that this expression was therefore divisibly by 3 too. The most common errors were not considering the correct base case (with many considering the case when $n = 1$ rather than $n = 0$) or not including all the required parts in their conclusion of the proof. It should be noted that the mark for the conclusion is the final mark of the proof and is dependent on all previous marks being given (therefore a 'perfect' conclusion that follows an incorrect (or incomplete) proof will not gain any credit).

Exemplar 1

8

$11 \times 7^n - 13^n - 1$

(1) consider $n = 1$

$11 \times 7^1 - 13^1 - 1$

$= 77 - 14 = 63 = 3(21)$ ∴ divisible by 3 for $n = 1$

(2) Assume true for $n = k$

$11(7^k) - 13^k - 1 = 3m$

(3) consider case $n = k+1$

$11(7^{k+1}) - 13^{k+1} - 1 = 7(11)(7^k) - 13(13)^k - 1$

$= 6(11)(7^k) - 12(13)^k$
$+ 11(7^k) - 13^k - 1$

$= 6(11)(7^k) - 12(13)^k + 3m$

$= 3\left(2(11)(7^k) - 4(13^k) + m\right) \Rightarrow$ divisible by 3

$\Rightarrow$ If true for $n = k$, then true for $n = k+1$, since true for $n = 1$, then true for all $n \in \mathbb{Z}^+$

This exemplar highlights the common error of considering the incorrect base case of $n = 1$ rather than the correct $n = 0$. This response scored all but the final accuracy mark (so scored 4 out of the 5 marks available).

Question 9 (a)

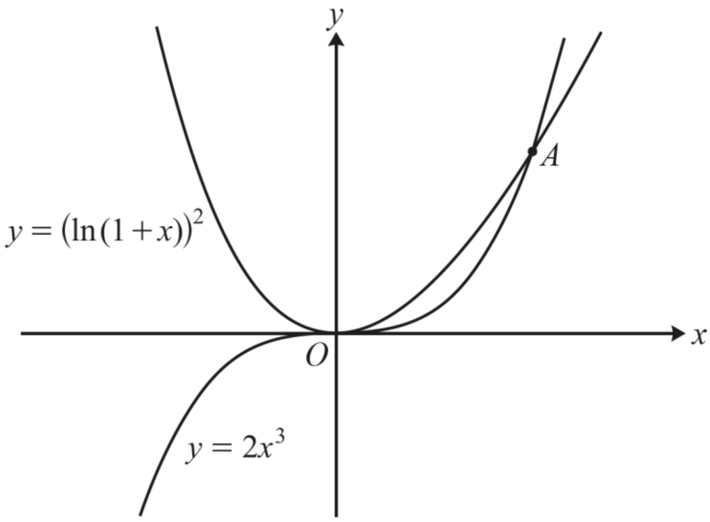
9 **(a)** Find the Maclaurin series of $(\ln(1+x))^2$ up to and including the term in x^4. **[3]**

As in Question 1, candidates must be familiar with, and therefore may use without proof, the results given in the H245 Formula Booklet (in this case on page 2 under the section **Maclaurin series**). Those candidates who stated that $(\ln(1+x))^2 = \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots\right)^2$ and expanded were much more likely to obtain the correct answer than those who attempted to differentiate $(\ln(1+x))^2$ four times. This differentiation method likely took up a large amount of time compared to the relatively low number of marks available. The most common error from candidates who did attempt to use the series expansion for $\ln(1+x)$ was to only consider the first two terms and therefore not realising that the cubic term would have an impact on the x^4 term in the required expansion.

Question 9 (b)

The diagram below shows parts of the graphs of the curves with equations $y = (\ln(1+x))^2$ and $y = 2x^3$.

The curves intersect at the origin, O, and at the point A.



(b) In this question you must show detailed reasoning.

Use your answer to part **(a)** to determine an approximation for the value of the x-coordinate of A. Give your answer to **2** decimal places. **[3]**

Although most candidates correctly set their expansion from part **(a)** equal to $2x^3$ and obtained the correct quadratic equation for the approximate x-coordinate at A (which, if correct, was $1 - 3x + \frac{11}{12}x^2 = 0$), many did not appreciate the command to show 'detailed reasoning' and simply said that $x = 0.38$ because it was the 'smallest' of the two roots without appreciating that the series expansion for $\ln(1+x)$ was only valid for $-1 < x \leq 1$ (which again was given to candidates in the Formula Booklet). Examiners also noted that for every candidate who stated the value was the 'smaller' of the two roots, there was another candidate who argued for the larger of the two roots to be the required coordinate.

Question 10 (a)

10 A particle B, of mass 3 kg, moves in a straight line and has velocity $v\,\text{ms}^{-1}$.

At time t seconds, where $0 \leqslant t < \frac{1}{4}\pi$, a variable force of $-(15\sin 4t + 6v\tan 2t)$ Newtons is applied to B. There are no other forces acting on B. Initially, when $t = 0$, B has velocity $4.5\,\text{ms}^{-1}$.

The motion of B can be modelled by the differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} + P(t)v = Q(t)$ where $P(t)$ and $Q(t)$ are functions of t.

(a) Find the functions $P(t)$ and $Q(t)$. **[2]**

Apart from sign errors, this part (a) was answered extremely well, with most candidates correctly setting up the correct differential equation for the motion of B and then correctly obtaining the correct expressions for both P and Q.

Question 10 (b)

(b) Using an integrating factor, determine the first time at which B is stationary according to the model. **[8]**

The responses to part (b) were mixed. Although most candidates could set up the correct expression for the integrating factor, $\mathrm{e}^{\int 2\tan 2t\,\mathrm{d}t}$, many struggled to evaluate this correctly as $\sec 2t$. Those that had the wrong integrating factor $\mathrm{I}(t)$ soon gave up (or made little tangible progress) as they then struggled to evaluate their $v \times I(t) = -5\int \sin 4t \times \mathrm{I}(t)\mathrm{d}t$. Of those that did have the correct integrating factor, it was pleasing that most recognised that $\sin 4t = 2\sin 2t\cos 2t$ and correctly obtained $v\sec 2t = 5\cos 2t + c$. Those candidates who had made it this far usually applied the correct initial conditions, set $v = 0$ and then correctly solved the equation $\cos 2t = 0.1$ to find the first time when B was stationary.

Exemplar 2

10(b)

$I = e^{\int 2\tan 2t \, dt}$

$2\int \tan 2t \, dt$ let $u = \tan 2t$, $du = 2\sec^2 2t$

$\int \cos^2 t = \int \frac{\cos 2t - 1}{2} = \frac{-\sin 2t}{4} - \frac{t}{2}$

$\int \tan 2t = \int \frac{\sin 2t}{\cos 2t}$ let $u = \cos 2t$, $du = 2\sin 2t$

$= \frac{1}{2}\int \frac{1}{u}\, du$

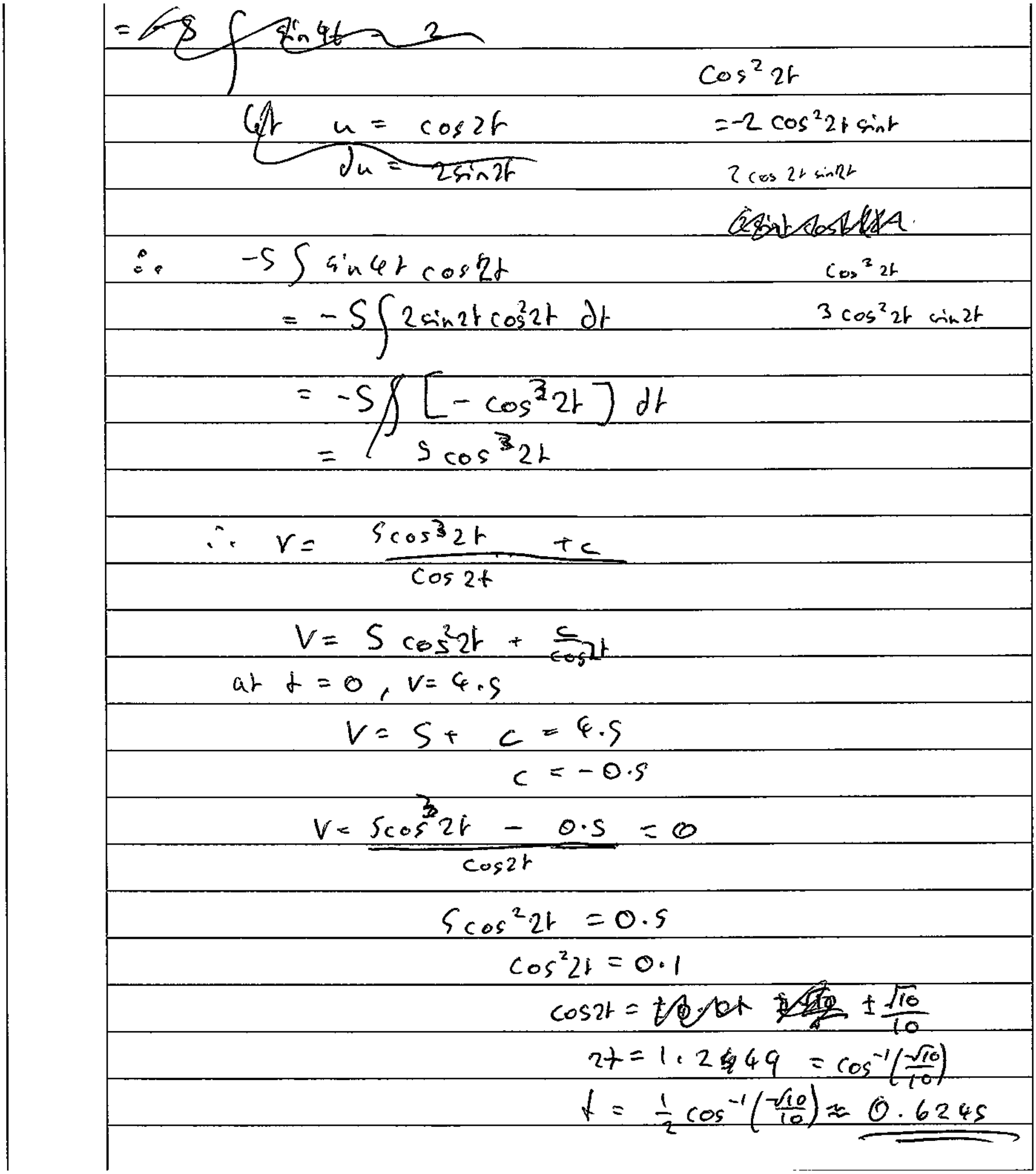
$= \frac{1}{2}\ln(\cos 2t)$

$\therefore I = e^{\ln \cos 2t} = \cos 2t$

$\therefore \cos 2t \, y = -5\int \sin 4t \cos 2t \, dt$

$= -5\int \sin 4t(1 - 2\sin^2 t)\, dt$

(answer space continued on next page)


$\cos^2 2t$

$u = \cos 2t$ $\quad = -2\cos^2 2t \sin t$

$du = 2\sin 2t$ $\quad 2\cos 2t \sin 2t$

$\therefore \quad -5\int \sin 4t \cos 2t$ $\quad \cos^3 2t$

$= -5\int 2\sin 2t \cos^2 2t \, dt$ $\quad 3\cos^2 2t \sin 2t$

$= -5\int [-\cos^3 2t] \, dt$

$= 5\cos^3 2t$

$\therefore \quad V = \frac{5\cos^3 2t}{\cos 2t} + c$

$V = 5\cos^2 2t + \frac{c}{\cos 2t}$

at $t = 0$, $V = 4.5$

$V = 5 + c = 4.5$

$c = -0.5$

$V = \frac{5\cos^3 2t - 0.5}{\cos 2t} = 0$

$5\cos^2 2t = 0.5$

$\cos^2 2t = 0.1$

$\cos 2t = \pm\frac{\sqrt{10}}{10}$

$2t = 1.249 = \cos^{-1}\left(\frac{-\sqrt{10}}{10}\right)$

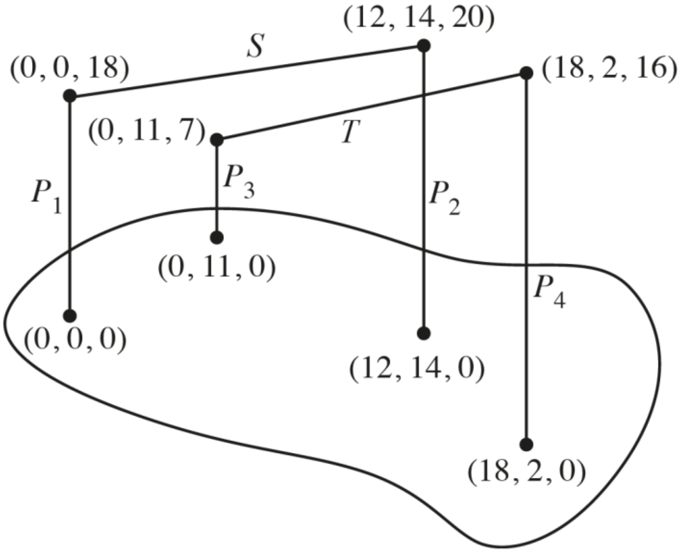
$t = \frac{1}{2}\cos^{-1}\left(\frac{-\sqrt{10}}{10}\right) \approx 0.6245$

This exemplar highlights the common error of starting with the correct expression for the integrating factor $e^{\int 2\tan 2t\,dt}$ but then not being able to evaluate this expression correctly. This response managed to score four of the method marks available.

Question 11 (a)

11 A 3-D coordinate system, whose units are metres, is set up to model a construction site. The construction site contains four vertical poles P_1, P_2, P_3 and P_4. The floor of the construction site is modelled as lying in the x-y plane and the poles are modelled as vertical line segments. One end of each pole lies on the floor of the construction site, and the other end of each pole is modelled by the points (0, 0, 18), (12, 14, 20), (0, 11, 7) and (18, 2, 16) respectively.

A wire, S, runs from the top of P_1 to the top of P_2. A second wire, T, runs from the top of P_3 to the top of P_4. The wires are modelled by straight lines segments. The layout of the construction site is illustrated on the diagram below which is **not** drawn to scale.



A vector equation of the line **segment** that represents the wire S is given by

$$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 18 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 7 \\ 1 \end{pmatrix}, 0 \leqslant \lambda \leqslant 2.$$

(a) Find, in the same form, a vector equation of the line **segment** that represents the wire T. The components of the direction vector should be integers whose only positive common factor is 1. **[2]**

Many candidates did not read the question carefully, and therefore did not give a vector equation of the line segment that represented T in a similar form to the equation given for S. Therefore, many candidates did not begin their equation with $\mathbf{r} =$ or gave a direction vector which did not have the components in the required form.

Question 11 (b)

For the construction site to be considered safe, it must pass two tests.

Test 1: The wires S and T need to be at least 5 metres apart at all positions on S and T.

(b) By using an appropriate formula, determine whether the construction site passes Test 1. **[2]**

Part (b) was generally answered well with many candidates using an appropriate formula (from page 5 of the Formula Booklet), $D = \frac{|(\mathbf{b}-\mathbf{a})\cdot\mathbf{n}|}{|\mathbf{n}|}$, with correct values for $\mathbf{a}$ and $\mathbf{b}$ and a correct vector product for $\mathbf{n}$. Although many candidates correctly found the required distance of 8.033… many then did not compare this value explicitly to 5 or incorrectly said that the site did not pass Test 1.

Question 11 (c)

A security camera is placed at a point Q on wire S.

Test 2: To ensure sufficient visibility of the construction site, the distance between the security camera and the top of P_3 must be at least 19 m.

(c) Determine whether it is possible to find point Q on S such that the construction site passes Test 2. **[3]**

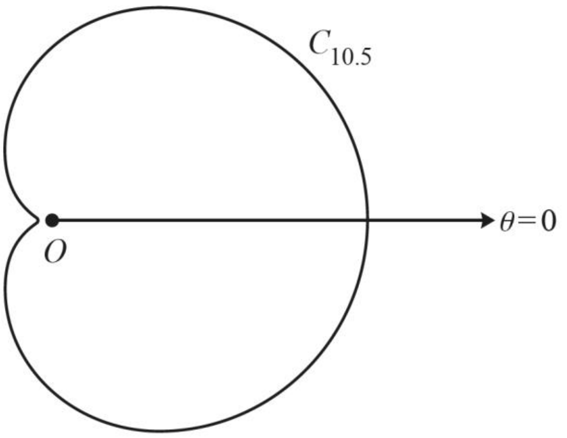
The response to part (c) was more mixed, with quite a few candidates leaving this part blank. Of those that made a correct start, most attempted to find an expression for $|\overrightarrow{PQ}|$ in terms of a parameter λ, and then attempted to solve the corresponding quadratic equation/inequality to show that the range of values of λ did not fall in the interval $0 \le \lambda \le 2$. Apart from sign errors, a number of candidates who correctly showed that $\lambda \ge 2.17 \ldots$ or $\lambda \le -0.637 \ldots$ stated that the construction site did not pass Test 2 but gave no indication of why this was the case. Finally, it was pleasing to note an alternative method where some candidates simply considered the distance between (0, 11, 7) and the two points (0, 0, 18) and (12, 14, 20) and said that as these two distances (15.55… and 17.94…) were both less than 19 the site would fail the second test. However, to score full marks in this second approach candidates had to explicitly state why they were considering these two points (with many not explicitly stating that these two points were the two points on S that would be the furthest away from (0, 11, 7)).

Question 12

12 For any positive parameter k, the curve C_k is defined by the polar equation

$$r = k(\cos\theta + 1) + \frac{10}{k}, 0 \leqslant \theta \leqslant 2\pi.$$

For each value of k the curve is a single, closed loop with no self-intersections. The diagram shows $C_{10.5}$ for the purpose of illustration.



Each curve, C_k, encloses a certain area, A_k.
You are given that there is a single minimum value of A_k.

Determine, in an exact form, the value of k for which C_k encloses this minimum area. **[7]**

Examiners noted, possibly due to timing issues at the end of the examination, that many candidates left this question blank or made a number of routine errors when expanding r^2 in their integral expression for the area enclosed by the polar curve. The most common error being those who stated that $r^2 = k^2(\cos\theta + 1)^2 + \frac{100}{k^2}$. Of those candidates that did expand r^2 correctly, most applied the correct identity $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ and integrated their expression correctly. While most candidates applied the correct limits very few candidates (due to earlier errors) found the correct expression for A_k. Only the strongest candidates seemed to understand that to find the required value of k the expression for A_k needed to be differentiated with respect to k, and this resulting expression then needed to be set equal to zero. Examiners noted that many candidates who had the correct expression for $A_k = \pi\left(\frac{3k^2}{2} + 20 + \frac{100}{k^2}\right)$, incorrectly multiplied through by k^2 therefore not appreciating that they were in fact differentiating an incorrect expression for the area. Finally, some candidates did not give their value for k in an exact form as requested.

Exemplar 3

12

$$A = \frac{1}{2}\int_0^{2\pi} \theta^2 \theta \quad \frac{1}{2}\int_0^{2\pi} r^2\, d\theta = \frac{1}{2}\int_0^{2\pi}\left(k(\cos\theta+1)+\frac{10}{k}\right)^2 d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} k^2\cos^2\theta + 2k^2\cos\theta + k^2 + 20(\cos\theta+1) + \frac{100}{k^2}\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \frac{k^2}{2}(1+\cos 2\theta) + 2k^2\cos\theta + k^2 + 20\cos\theta + 20 + \frac{100}{k^2}\, d\theta$$

$$= \frac{1}{2}\left[\frac{k^2}{2}\theta + \frac{k^2}{4}\sin 2\theta + 2k^2\sin\theta + k^2\theta + 20\sin\theta + 20\theta + \frac{100}{k^2}\theta\right]_0^{2\pi}$$

$$= \frac{1}{2}\left[\left(\frac{3k^2}{2} + 20 + \frac{100}{k^2}\right)\theta + (2k^2+20)\sin\theta + \frac{k^2}{4}\sin 2\theta\right]_0^{2\pi}$$

$$= \frac{1}{2}\left(\frac{3k^2}{2} + 20 + \frac{100}{k^2}\right)2\pi = \left(\frac{3k^2}{2} + 20 + \frac{100}{k^2}\right)\pi$$

$$\frac{dA}{dk} = \pi\left(3k - \frac{200}{k^3}\right) \stackrel{\text{set}}{=} 0$$

$$3k - \frac{200}{k^3} = 0$$

$$\Rightarrow 3k^4 - 200 = 0 \quad , k \neq 0$$

$$k^4 = \frac{200}{3}$$

$$k = \sqrt[4]{\frac{200}{3}} \quad \text{since } k > 0.$$

This exemplar highlights that even at the final stages of the examination, some candidates still managed to show an excellent understanding of the mathematics involved in these unstructured/problem-solving type problems. Furthermore, they managed to structure their response(s) so that it was easy to follow each step of their working. This particular response scored all 7 marks available.

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