Oxford Cambridge and RSA

Wednesday 22 May 2024 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes

1420877967

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

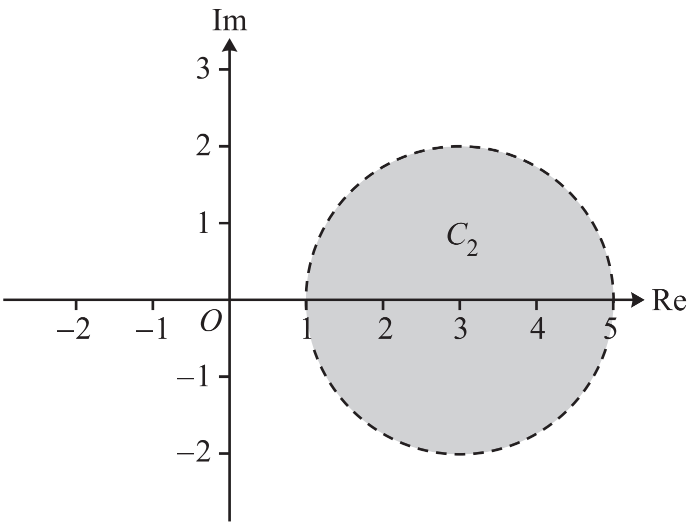
1 Given that $y = \sin^{-1}(x^2)$, find $\frac{dy}{dx}$. **[3]**

2 The locus C_1 is defined by $C_1 = \left\{z : 0 \leqslant \arg(z+i) \leqslant \frac{1}{4}\pi\right\}$.

(a) Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_1. **[2]**

(b) Determine whether the complex number $1.2 + 0.8i$ is in C_1. **[2]**

The locus C_2 is the set of complex numbers represented by the interior of the circle with radius 2 and centre 3. The locus C_2 is illustrated on the Argand diagram below.



(c) Use set notation to define C_2. **[2]**

(d) Determine whether the complex number $1.2 + 0.8i$ is in C_2. **[2]**

3 A transformation T is represented by the matrix $\mathbf{N} = \begin{pmatrix} a & 4 & 2 \\ 5 & 1 & 0 \\ 3 & 6 & 3 \end{pmatrix}$, where a is a constant.

(a) Find $\mathbf{N}^2$ in terms of a. **[3]**

(b) Find det $\mathbf{N}$ in terms of a. **[2]**

The value of a is 13 to the **nearest integer**.

A shape S_1 has volume 11.6 to 1 decimal place. Shape S_1 is mapped to shape S_2 by the transformation T.

A student claims that the volume of S_2 is less than 400.

(c) Comment on the student's claim. **[3]**

4 **In this question you must show detailed reasoning.**

The equation $2x^3 + 3x^2 + 6x - 3 = 0$ has roots α, β and γ.

Determine a cubic equation with integer coefficients that has roots $\alpha^2\beta\gamma, \alpha\beta^2\gamma$ and $\alpha\beta\gamma^2$. **[3]**

5 Express $\dfrac{12x^3}{(2x+1)(2x^2+1)}$ using partial fractions. **[5]**

6 **In this question you must show detailed reasoning.**

Determine the exact value of $\displaystyle\int_9^\infty \frac{18}{x^2\sqrt{x}}\,\mathrm{d}x$. **[4]**

7 **(a)** By using the definitions of $\cosh u$ and $\sinh u$ in terms of e^u and e^{-u}, show that $\sinh 2u \equiv 2\sinh u \cosh u$. **[2]**

The equation of a curve, C, is $y = 16\cosh x - \sinh 2x$.

(b) Show that there is only one solution to the equation $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0$ **[4]**

You are now given that C has exactly one point of inflection.

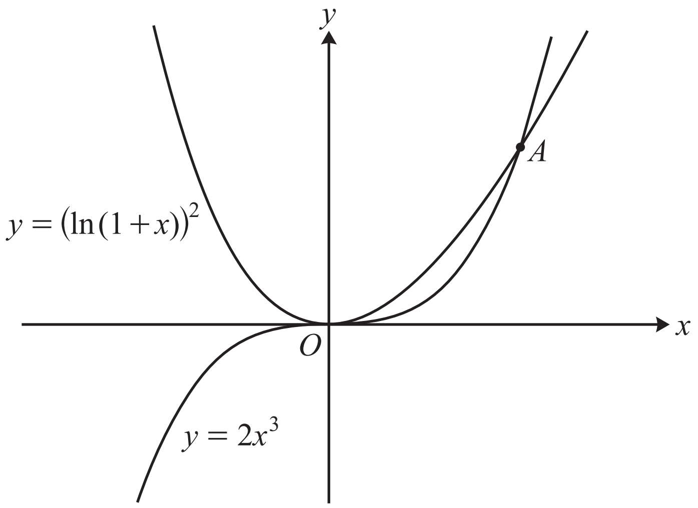
(c) Use your answer to part **(b)** to determine the exact coordinates of this point of inflection. Give your answer in a logarithmic form where appropriate. **[3]**

8 Prove by induction that $11 \times 7^n - 13^n - 1$ is divisible by 3, for all integers $n \geqslant 0$. **[5]**

9 **(a)** Find the Maclaurin series of $(\ln(1+x))^2$ up to and including the term in x^4. **[3]**

The diagram below shows parts of the graphs of the curves with equations $y = (\ln(1+x))^2$ and $y = 2x^3$.

The curves intersect at the origin, O, and at the point A.



(b) **In this question you must show detailed reasoning.**

Use your answer to part **(a)** to determine an approximation for the value of the x-coordinate of A. Give your answer to **2** decimal places. **[3]**

10 A particle B, of mass 3 kg, moves in a straight line and has velocity $v\,\mathrm{m\,s^{-1}}$.

At time t seconds, where $0 \leqslant t < \frac{1}{4}\pi$, a variable force of $-(15\sin 4t + 6v\tan 2t)$ Newtons is applied to B. There are no other forces acting on B. Initially, when $t = 0$, B has velocity $4.5\,\mathrm{m\,s^{-1}}$.

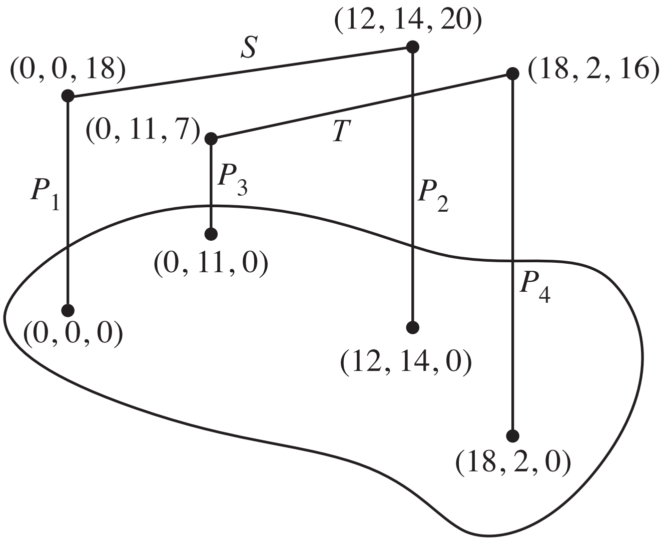
The motion of B can be modelled by the differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} + P(t)v = Q(t)$ where $P(t)$ and $Q(t)$ are functions of t.

(a) Find the functions $P(t)$ and $Q(t)$. **[2]**

(b) Using an integrating factor, determine the first time at which B is stationary according to the model. **[8]**

11 A 3-D coordinate system, whose units are metres, is set up to model a construction site. The construction site contains four vertical poles P_1, P_2, P_3 and P_4. The floor of the construction site is modelled as lying in the *x*-*y* plane and the poles are modelled as vertical line segments. One end of each pole lies on the floor of the construction site, and the other end of each pole is modelled by the points (0, 0, 18), (12, 14, 20), (0, 11, 7) and (18, 2, 16) respectively.

A wire, *S*, runs from the top of P_1 to the top of P_2. A second wire, *T*, runs from the top of P_3 to the top of P_4. The wires are modelled by straight lines segments. The layout of the construction site is illustrated on the diagram below which is **not** drawn to scale.



A vector equation of the line **segment** that represents the wire *S* is given by

$$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 18 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 7 \\ 1 \end{pmatrix}, 0 \leqslant \lambda \leqslant 2.$$

(a) Find, in the same form, a vector equation of the line **segment** that represents the wire *T*. The components of the direction vector should be integers whose only positive common factor is 1. **[2]**

For the construction site to be considered safe, it must pass two tests.

Test 1: The wires *S* and *T* need to be at least 5 metres apart at all positions on *S* and *T*.

(b) By using an appropriate formula, determine whether the construction site passes Test 1. **[2]**

A security camera is placed at a point *Q* on wire *S*.

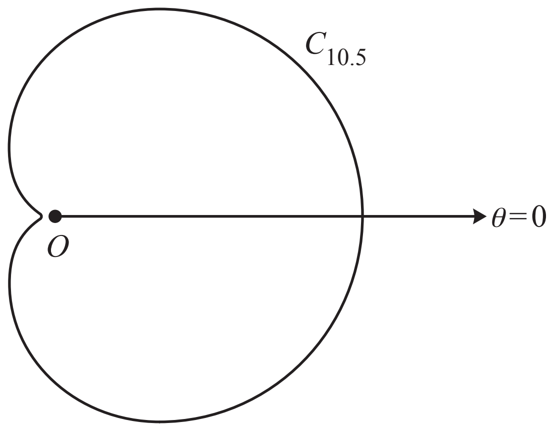
Test 2: To ensure sufficient visibility of the construction site, the distance between the security camera and the top of P_3 must be at least 19 m.

(c) Determine whether it is possible to find point *Q* on *S* such that the construction site passes Test 2. **[3]**

12 For any positive parameter k, the curve C_k is defined by the polar equation

$$r = k(\cos\theta + 1) + \frac{10}{k}, 0 \leqslant \theta \leqslant 2\pi.$$

For each value of k the curve is a single, closed loop with no self-intersections. The diagram shows $C_{10.5}$ for the purpose of illustration.



Each curve, C_k, encloses a certain area, A_k.
You are given that there is a single minimum value of A_k.

Determine, in an exact form, the value of k for which C_k encloses this minimum area. **[7]**

END OF QUESTION PAPER

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