OCR
Oxford Cambridge and RSA

Monday 3 June 2024 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **(a)** Use the method of differences to show that $\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)=1-\frac{1}{n+1}$. **[1]**

(b) Hence determine the following sums.

(i) $\sum_{r=1}^{99}\frac{1}{r}-\frac{1}{r+1}$ **[1]**

(ii) $\sum_{r=100}^{\infty}\frac{1}{r}-\frac{1}{r+1}$ **[3]**

2 **In this question you must show detailed reasoning.**

(a) Solve the equation $x^2-6x+58=0$. Give your solutions in the form $a+b\mathrm{i}$ where a and b are real numbers. **[3]**

(b) Determine, in exact form, $\arg(-10+(5\sqrt{12})\mathrm{i})^5$. **[3]**

3 Matrices **A** and **B** are given by $\mathbf{A}=\begin{pmatrix}4 & -3\\ -2 & 2\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}3 & -5\\ 0 & 1\end{pmatrix}$.

(a) Find $2\mathbf{A}-4\mathbf{B}$. **[2]**

(b) Write down the matrix **C** such that $\mathbf{AC}=2\mathbf{A}$. **[1]**

(c) Find the value of det **A**. **[1]**

(d) **In this question you must show detailed reasoning.**

Use $\mathbf{A}^{-1}$ to solve the equations $4x-3y=7$ and $-2x+2y=9$. **[3]**

4 **In this question you must show detailed reasoning.**

The series S is defined as being the sum of the squares of all positive **odd** integers from 1^2 to 779^2.

Determine the value of S. **[5]**

5 Vectors, **a**, **b** and **c**, are given by $\mathbf{a}=\mathbf{i}+(1-p)\mathbf{j}+(p+2)\mathbf{k}$, $\mathbf{b}=2\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=\mathbf{i}+14\mathbf{j}+(p-3)\mathbf{k}$ where p is a constant.

You are given that $\mathbf{a}\times\mathbf{b}$ is perpendicular to **c**.

Determine the possible values of p. **[6]**

6 In polar coordinates, the equation of a curve, C, is $r = 6\sin(2\theta)\sinh\left(\frac{1}{3}\theta\right)$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

The pole of the polar coordinate system corresponds to the origin of the cartesian system and the initial line corresponds to the positive x-axis.

(a) Explain how you can tell that C comprises a single loop in the first quadrant, passing through the pole. **[3]**

The incomplete table below shows values of r for various values of θ.

θ	0	$\frac{1}{12}\pi$	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{5}{12}\pi$	$\frac{1}{2}\pi$
r	0	0.262			1.851		

(b) Use the copy of the table and the polar coordinate system diagram given in the Printed Answer Booklet to complete the table and sketch C. **[3]**

The point on C which is furthest away from the pole is denoted by A and the value of θ at A is denoted by ϕ.

(c) Show that ϕ satisfies the equation $\phi = \frac{3}{2}\ln\left(\frac{6-\tan 2\phi}{6+\tan 2\phi}\right)$ **[4]**

(d) You are given that the relevant solution of the equation given in part **(c)** is $\phi = 1.0207$ correct to 5 significant figures.

Find the distance from A to the pole. Give your answer correct to **3** significant figures. **[1]**

7 **(a)** Express $17\cosh x - 15\sinh x$ in the form $\mathrm{e}^{-x}(a\mathrm{e}^{bx} + c)$ where a, b and c are integers to be determined. **[3]**

A function is defined by $\mathrm{f}(x) = \frac{1}{\sqrt{17\cosh x - 15\sinh x}}$. The region bounded by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \ln 3$ is rotated by 2π radians about the x-axis to form a solid of revolution S.

(b) **In this question you must show detailed reasoning.**

Use a suitable substitution, together with known results from the formula book, to show that the volume of S is given by $k\pi\tan^{-1}q$ where k and q are rational numbers to be determined. **[7]**

8 A children's play centre has two rooms, a room full of bouncy castles and a room full of ball pits. At any given instant, each child in the centre is playing either on the bouncy castles or in the ball pits. Each child can see one room from the other room and can decide to change freely between the two rooms. It is assumed that such changes happen instantaneously.

The number of children playing on the bouncy castles at time t hours, is denoted by C and the corresponding number of children playing in the ball pits is P. Because the number of children is large for most of the time, C and P are modelled as being continuous.

When there is a different number of children in each room, some children will move from the room with more children to the room with fewer children. A researcher therefore decides to model C and P with the following coupled differential equations.

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \alpha(P-C)+\gamma t$$
$$\frac{\mathrm{d}C}{\mathrm{d}t} = \alpha(C-P)$$

(a) Explain why α must be negative. **[1]**

After examining data, the researcher chooses $\alpha = -2$ and $\gamma = 32$.

(b) Show that P satisfies the second order differential equation $\frac{\mathrm{d}^2P}{\mathrm{d}t^2}+4\frac{\mathrm{d}P}{\mathrm{d}t} = 64t+32$. **[2]**

(c) **(i)** Find the complementary function for the differential equation from part **(b)**. **[1]**

(ii) Explain why a particular integral of the form $P = at+b$ will not work in this situation. **[1]**

(iii) Using a particular integral of the form $P = at^2+bt$, find the general solution of the differential equation from part **(b)**. **[3]**

At a certain time there are 55 children playing in the ball pits and 24 children per hour are arriving at the ball pits.

(d) Use the model, starting from this time, to estimate the number of children in the ball pits 30 minutes later. **[4]**

(e) Explain why the model becomes unreliable as t gets very large. **[1]**

9 In this question, the argument of a complex number is defined as being in the range $[0, 2\pi)$.

You are given that ω_k, where $k = 0, 1, 2, ..., n-1$, are the n n^{th} roots of unity for some integer n, $n \geqslant 3$, and that these are given in order of increasing argument (so that $\omega_0 = 1$).

(a) With the help of a diagram explain why $\omega_k = (\omega_1)^k$ for $k = 2, ..., n-1$. **[3]**

(b) Using the identity given in part **(a)**, show that $\sum_{k=0}^{n-1} \omega_k = 0$. **[2]**

(c) Show that if z is a complex number then $z + z^* = 2\text{Re}(z)$. **[1]**

(d) Using the results from parts **(b)** and **(c)** show that $\sum_{k=0}^{n-1} \text{Re}(\omega_k) = 0$. **[1]**

(e) With the help of a diagram explain why $\text{Re}(\omega_k) = \text{Re}(\omega_{n-k})$ for $k = 1, 2, ..., n-1$. **[1]**

You should now consider the case where $n = 5$.

(f) **(i)** Use parts **(d)** and **(e)** to deduce that $\cos\frac{4\pi}{5} = a + b\cos\frac{2\pi}{5}$, for some rational constants a and b. **[2]**

(ii) Hence determine the exact value of $\cos\frac{2\pi}{5}$. **[2]**

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE

