OCR
Oxford Cambridge and RSA

Friday 7 June 2024 – Afternoon

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Time allowed: 2 hours 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Section A (26 marks)

1 A car A of mass 1200 kg is about to tow another car B of mass 800 kg in a straight line along a horizontal road by means of a tow-rope attached between A and B. The tow-rope is modelled as being light and inextensible. Just before the tow-rope tightens, A is travelling at a speed of $1.5\,\mathrm{m\,s^{-1}}$ and B is at rest. Just after the tow-rope tightens, both cars have a speed of $v\,\mathrm{m\,s^{-1}}$.

(a) Find the value of v. **[2]**

(b) Calculate the magnitude of the impulse on A when the tow-rope tightens. **[2]**

2 One end of a light spring is attached to a fixed point. A mass of 2 kg is attached to the other end of the spring.

The spring hangs vertically in equilibrium. The extension of the spring is 0.05 m.

(a) Find the stiffness of the spring. **[2]**

(b) Find the energy stored in the spring. **[2]**

(c) Find the dimensions of stiffness of a spring. **[1]**

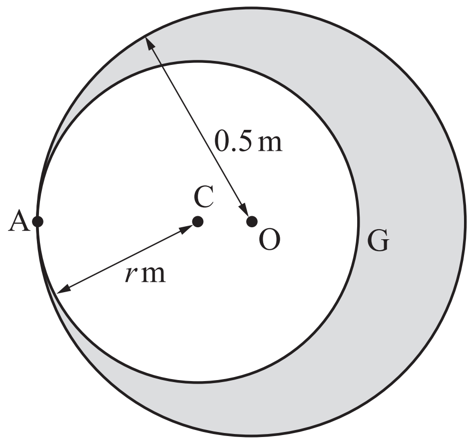
A particle P of mass m is performing complete oscillations with amplitude a on the end of a light spring with stiffness k. The spring hangs vertically and the maximum speed v of P is given by the formula

$$v = Cm^{\alpha}a^{\beta}k^{\gamma},$$

where C is a dimensionless constant.

(d) Use dimensional analysis to determine α, β, and γ. **[4]**

3

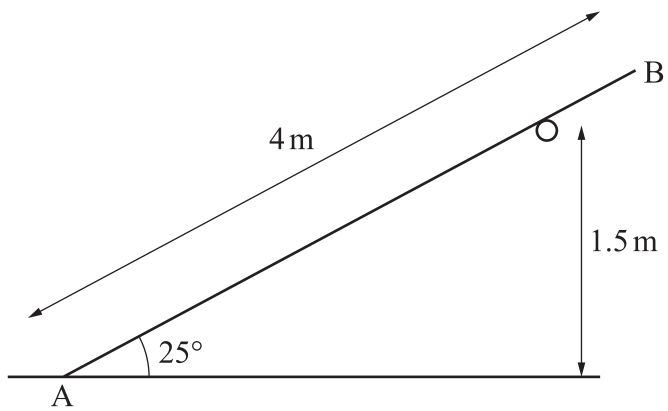


A circular hole with centre C and radius r m, where $r < 0.5$, is cut in a uniform circular disc with centre O and radius 0.5 m. The hole touches the rim of the disc at A (see diagram).

The centre of mass, G, of the remainder of the disc is on the rim of the hole.

Determine the value of r. **[5]**

4



A uniform rod AB has mass 3 kg and length 4 m. The end A of the rod is in contact with rough horizontal ground.

The rod rests in equilibrium on a smooth horizontal peg 1.5 m above the ground, such that the rod is inclined at an angle of 25° to the ground (see diagram).

The rod is in a vertical plane perpendicular to the peg.

(a) Determine the magnitude of the normal contact force between the peg and the rod. **[3]**

(b) Determine the range of possible values of the coefficient of friction between the rod and the ground. **[5]**

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Section B (94 marks)

5 A car of mass 850 kg is travelling along a straight horizontal road. The power developed by the car is constant and is equal to 18 kW. There is a constant resistance to motion of magnitude 600 N.

(a) Find the greatest steady speed at which the car can travel. **[2]**

Later in the journey, while travelling at a speed of $15\,\mathrm{m\,s^{-1}}$, the car comes to the bottom of a straight hill which is inclined at an angle of $\sin^{-1}\left(\frac{1}{40}\right)$ to the horizontal.

The power developed by the car remains constant at 18 kW. The magnitude of the resistance force is **no longer** constant but changes such that the total work done against the resistance force in ascending the hill is 103 000 J. The car takes 10 seconds to ascend the hill and at the top of the hill the car is travelling at $18\,\mathrm{m\,s^{-1}}$.

(b) Determine the distance the car travels from the bottom to the top of the hill. **[5]**

6 **In this question you must show detailed reasoning.**

In this question, positions are given relative to a fixed origin, O. The unit vectors **i** and **j** are in the *x*- and *y*-directions respectively in a horizontal plane. Distances are measured in centimetres and the time, *t*, is measured in seconds, where $0 \leqslant t \leqslant 5$.

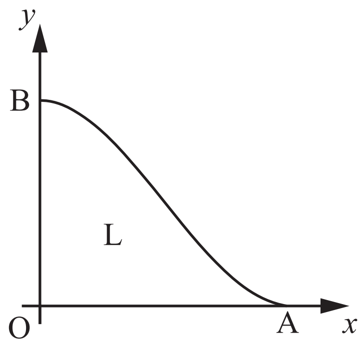
A small radio-controlled toy car C moves on a horizontal surface which contains O.

The acceleration of C is given by $2\mathbf{i} + t\mathbf{j}\,\mathrm{cm\,s^{-2}}$.

When $t = 4$, the displacement of C from O is $16\mathbf{i} + \frac{32}{3}\mathbf{j}\,\mathrm{cm}$, and the velocity of C is $8\mathbf{i}\,\mathrm{cm\,s^{-1}}$.

Determine a cartesian equation for the path of C for $0 \leqslant t \leqslant 5$. You are **not** required to simplify your answer. **[6]**

7 The region bounded by the curve $y = x^3 - 3x^2 + 4$, the positive x-axis and the positive y-axis is occupied by a uniform lamina L. The vertices of L are O, A and B, where O is the origin, A is a point on the positive x-axis and B is a point on the positive y-axis (see diagram).



(a) Determine the coordinates of the centre of mass of L. **[5]**

The lamina L is the cross-section through the centre of mass of a uniform solid prism M.

The prism M is placed on an inclined plane, which makes an angle of 30° with the horizontal, so that OA lies along a line of greatest slope of the plane with O lower down the plane than A.

It is given that M does **not** slip on the plane.

(b) Determine whether M will topple in this case. Give a reason to support your answer. **[2]**

The prism M is now placed on the same inclined plane so that OB lies along a line of greatest slope of the plane with O lower down the plane than B.

It is given that M still does **not** slip on the plane.

(c) Determine whether M will topple in this case. Give a reason to support your answer. **[2]**

8 A particle P of mass $3m$ kg is attached to one end of a light elastic string of modulus of elasticity $4mg$ N and natural length 0.4 m. The other end of the string is attached to a fixed point O. The particle P rests in equilibrium at a point A with the string vertical.

(a) Find the distance OA. **[2]**

At time $t = 0$ seconds, P is given a speed of $2.5\,\text{m s}^{-1}$ vertically downwards from A.

(b) Show that P initially performs simple harmonic motion with amplitude a m, where a is to be determined correct to **3** significant figures. **[5]**

(c) Determine the smallest distance between P and O in the subsequent motion. **[3]**

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9 A particle P of mass 5 kg is released from rest at a point O and falls vertically. A resistance of magnitude $0.05v^2$ N acts vertically upwards on P, where $v\,\text{m s}^{-1}$ is the velocity of P when it has fallen a distance x m.

(a) Show that $\left(\frac{100v}{980-v^2}\right)\frac{dv}{dx} = 1$. **[2]**

(b) Verify that $v^2 = 980\left(1-e^{-0.02x}\right)$. **[4]**

(c) Determine the work done against the resistance while P is falling from O to the point where P's acceleration is $8.36\,\text{m s}^{-2}$. **[5]**

10 A particle P of mass 2 kg is projected vertically upwards from horizontal ground with an initial speed of $14\,\text{m s}^{-1}$. At the same instant a particle Q of mass 8 kg is released from rest 5 m vertically above P. During the subsequent motion P and Q collide. The coefficient of restitution between P and Q is $\frac{11}{14}$.

Determine the time between this collision and P subsequently hitting the ground. **[10]**

11 A particle P of mass 1 kg is fixed to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point O, which is 1.75 m above a horizontal plane. P is held with the string horizontal and taut. P is then projected vertically downwards with a speed of $3.2\,\text{m s}^{-1}$.

(a) Find the tangential acceleration of P when OP makes an angle of 20° with the horizontal. **[2]**

The string breaks when the tension in it is 32 N. At this point the angle between OP and the horizontal is θ.

(b) Show that $\theta = 23.1°$, correct to **1** decimal place. **[5]**

Particle P subsequently hits the plane at a point A.

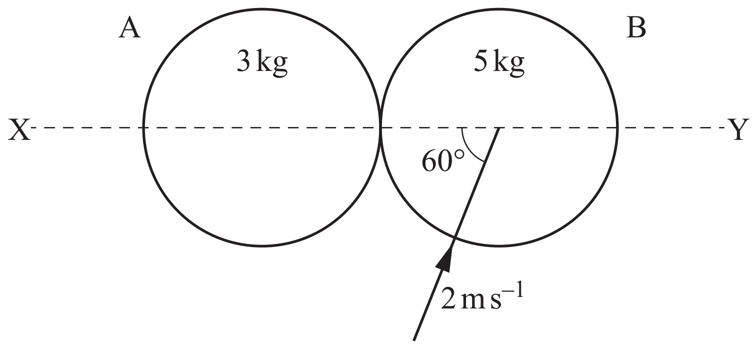
(c) Determine the speed of P when it arrives at A. **[4]**

(d) Show that A is almost vertically below O. **[5]**

12 Two small uniform discs A and B, of equal radius, have masses 3 kg and 5 kg respectively. The discs are sliding on a smooth horizontal surface and collide obliquely.

The contact between the discs is smooth and A is stationary after the collision.

Immediately before the collision B is moving with speed $2\,\mathrm{m\,s^{-1}}$ in a direction making an angle of 60° with the line of centres, XY (see diagram below).



(a) Explain how you can tell that A must have been moving along XY before the collision. **[1]**

The coefficient of restitution between A and B is 0.8.

(b)
- Determine the speed of A immediately before the collision.
- Determine the speed of B immediately after the collision. **[7]**

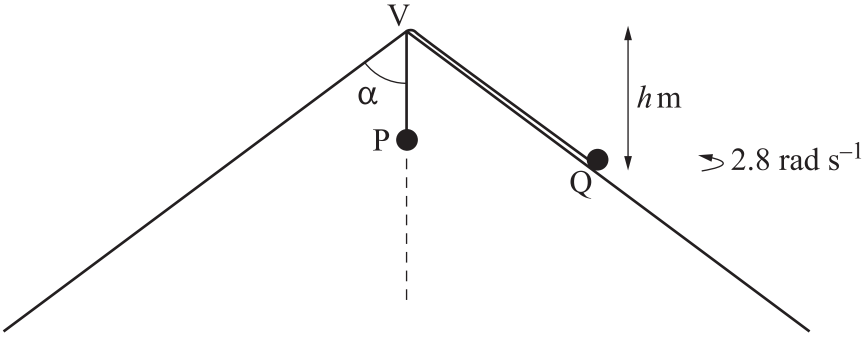
(c) Determine the angle turned through by the direction of B in the collision. **[3]**

Disc B subsequently collides with a smooth wall, which is **parallel** to XY. The kinetic energy of B after the collision with the wall is 95% of the kinetic energy of B before the collision with the wall.

(d) Determine the coefficient of restitution between B and the wall. **[4]**

Turn over for question 13

13



A conical shell, of semi-vertical angle α, is fixed with its axis vertical and its vertex V upwards. A light inextensible string passes through a small smooth hole at V and a particle P of mass 4 kg hangs in equilibrium at one end of the string. The other end of the string is attached to a particle Q of mass 2 kg which moves in a horizontal circle at a constant angular speed 2.8 rad s^{-1} on the smooth outer surface of the shell at a vertical depth h m below V (see diagram).

(a) Show that $k_1 h \sin^2\alpha + k_2 \cos^2\alpha = k_3 \cos\alpha$, where k_1, k_2 and k_3 are integers to be determined. **[7]**

(b) Determine the greatest value of h for which Q remains in contact with the shell. **[3]**

END OF QUESTION PAPER

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