OCR

Oxford Cambridge and RSA

Monday 3 June 2024 – Afternoon

A Level Further Mathematics B (MEI)

Y433/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

QP

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The values below represent the weights of eleven suitcases, in kg.

23 15 17 10 35 18 12 21 22 11 8

(a) Show the result of applying the first fit algorithm to pack the weights into containers that can hold a maximum of 50 kg. **[2]**

(b) Show the result of applying the first fit decreasing algorithm to pack the weights into containers that can hold a maximum of 50 kg. **[2]**

(c) Give a reason why the number of containers used for the packing of the suitcases in part **(b)** is optimal. **[1]**

2 The table shows the distance, in kilometres, along the direct roads between eight towns, A to H. A dash (-) indicates that there is no direct road linking the towns.

	A	B	C	D	E	F	G	H
A	-	31	21	-	-	-	-	-
B	31	-	8	12	-	-	10	20
C	21	8	-	24	-	35	20	36
D	-	12	24	-	30	-	-	-
E	-	-	-	30	-	12	40	10
F	-	-	35	-	12	-	-	25
G	-	10	20	-	40	-	-	15
H	-	20	36	-	10	25	15	-

(a) Use Dijkstra's algorithm to find the shortest path from A to E for the network shown in the table. **[5]**

(b) (i) Apply the **tabular** form of Prim's algorithm on the diagram in the Printed Answer Booklet, **starting at vertex A**, to find the total length of the arcs in the minimum spanning tree for the network shown in the table. **[3]**

(ii) On the vertices provided in the Printed Answer Booklet draw the minimum spanning tree for this network. **[1]**

(c) Write down the length of the shortest path from A to F which passes through all the towns **except** D. **[1]**

(d) A computer takes 0.016 secs to solve the minimum connector problem on the network shown in the table using Prim's algorithm.

Approximately how long will it take the computer to solve the minimum connector problem on a network with 1000 edges using Prim's algorithm? **[2]**

3 A directed network consists of eight nodes, S, A, B, C, D, E, F and T, and seventeen arcs. The arcs represent a system of pipes through which a fluid flows continuously from the source S to a sink T. The weight of each arc is the corresponding capacity of that pipe in litres per minute.

The following LP formulation can be used to find the maximum flow from S to T in this network.

Maximise SA + SB + SC + SD

subject to
SA + BA − AE − AT = 0
SB + CB − BA − BE − BF − BT = 0
SC + DC − CB − CF = 0
SD − DC − DF = 0
AE + BE − EF − ET = 0
BF + CF + DF + EF − FT = 0
SA ⩽ 25, SB ⩽ 61, SC ⩽ 37, SD ⩽ 35, AE ⩽ 25, AT ⩽ 38,
BA ⩽ 39, BE ⩽ 40, BF ⩽ 17, BT ⩽ 49, CB ⩽ 12, CF ⩽ 26,
DC ⩽ 28, DF ⩽ 19, EF ⩽ 25, ET ⩽ 27, FT ⩽ 32

(a) Complete the diagram in the Printed Answer Booklet to show the directed network represented in the above LP formulation. **[2]**

(b) A cut α partitions the vertices into the sets {S, A, B, C, D}, {E, F, T}.

Calculate the capacity of cut α. **[1]**

(c) By considering the objective function of the LP and the answer to part **(b)**, explain why the maximum flow through the network cannot be equal to the capacity of cut α. **[1]**

The LP formulation was run in an online solver and some of the output is shown below.

VARIABLE	VALUE
SA	25.00000
SB	61.00000
AE	25.00000
AT	38.00000
BT	35.00000
CB	12.00000
DC	0.00000
DF	19.00000
EF	0.00000
FT	32.00000

(d) By completing the diagram in the Printed Answer Booklet, determine the maximum value of the flow through the network. **[3]**

(e) Use a suitable cut to prove that this is the maximum flow. **[2]**

It is decided to reverse the flow in pipe CB so that fluid can flow directly from B to C, instead of C to B. The capacity of the pipe is unchanged. No other pipes are changed.

(f) Explain what changes would be required to the LP formulation due to this modified constraint on the flow in pipe BC. **[2]**

4 A student is trying to draw a triangle *XYZ* subject to the following constraints.

- Angle *X* must be at most 10° smaller than angle *Z*.
- Angle *Y* must be at least twice the size of angle *Z*.

Furthermore, the student wishes to maximise the size of angle *Z*.

Let *x*, *y*, and *z* represent the size (in degrees) of angle *X*, *Y* and *Z* respectively.

Complete the initial tableau in the Printed Answer Booklet so that the two-stage simplex method may be used to solve this problem in *x*, *y*, and *z*.

Your tableau should only contain **one** surplus variable and **one** artificial variable.

You must show all your working and you are **not** required to solve the problem. **[8]**

5 The table in **Fig. 5.1** lists the immediate predecessors for each of the eleven activities in a project.

Fig. 5.1

Activity	Immediate predecessors
A	-
B	-
C	-
D	A
E	A
F	B, E
G	B, C, E
H	D, F
I	D, F, G
J	B, C, E
K	B, C, E

(a) Draw an activity network, using activity on arc, to represent the project. Your network should contain exactly **3** dummy activities. **[3]**

Each of the eleven activities in the table requires exactly one worker.

The diagram in **Fig. 5.2** shows how three people can complete the project in the minimum time. Each column in the diagram represent 1 hour. If a box is blank the person is resting for that 1 hour.

Fig. 5.2

	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18	18–19
WORKER 1	A	A	A	E	E	E	E	F	F	F	F	F	F	H	H	H	H	H	H
WORKER 2	B	B		D	D	D	D	G	G	G	G			I	I	I	I		
WORKER 3	C	C	C	C	C			J	J	J	J	K	K	K	K	K			

(b) **(i)** State the critical activities of the project. **[1]**

(ii) State the minimum project completion time. **[1]**

For the remainder of this question, it may be assumed that there is no restriction on the number of workers available to complete the project.

(c) Complete the table in the Printed Answer Booklet to show the total float for each activity. **[4]**

A twelfth activity L is added to the project. Activity L can only begin once activity G is completed, and no activity depends on its completion.

(d) It is given that the duration of activity L is 9 hours.

(i) State the new minimum project completion time. **[1]**

(ii) State the new critical activities. **[1]**

6 A maximisation LP problem in x, y and z is to be solved.

The tableau after the 1st iteration of the simplex method is shown below.

P	x	y	z	s_1	s_2	s_3	RHS
1	-2	0	-1	0	0	1	20
0	1	0	$\frac{11}{3}$	1	0	$-\frac{1}{3}$	$\frac{130}{3}$
0	2	0	-2	0	1	1	40
0	0	1	$-\frac{2}{3}$	0	0	$\frac{1}{3}$	$\frac{20}{3}$

(a) State the value of the objective function after this 1st iteration. **[1]**

(b) Explain how the tableau shows that the pivot value for the 1st iteration came from a value from the y column. **[1]**

(c) By considering the equations in the tableau, formulate the **original** LP problem. You should state the objective and list the constraints as simplified inequalities with integer coefficients. **[5]**

(d) Perform the 2nd iteration of the simplex method, using the tableau in the Printed Answer Booklet, choosing the pivot from the x column and giving each entry as an exact value. **[3]**

After three iterations of the simplex method an optimal solution to the LP problem is obtained and the following three equations are found.

$$x + \frac{3}{14}s_1 + \frac{11}{28}s_2 + \frac{9}{28}s_3 = 25$$

$$y + \frac{1}{7}s_1 - \frac{1}{14}s_2 + \frac{3}{14}s_3 = 10$$

$$z + \frac{3}{14}s_1 - \frac{3}{28}s_2 - \frac{5}{28}s_3 = 5$$

(e) (i) State the optimal values of x, y and z. **[1]**

(ii) Determine the corresponding value of the objective function. **[2]**

END OF QUESTION PAPER