OCR
Oxford Cambridge and RSA

Wednesday 19 June 2024 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

QP

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The table shows some values of x, together with the associated values of a function, $f(x)$.

x	1.9	2	2.1
$f(x)$	0.5842	0.6309	0.6753

(a) Use the information in the table to calculate the most accurate estimate of $f'(2)$ possible. **[2]**

(b) Calculate an estimate of the error when $f(2)$ is used as an estimate of $f(2.05)$. **[2]**

2 You are given that $a = \tanh(1)$ and $b = \tanh(2)$.

A is the approximation to a formed by rounding $\tanh(1)$ to 1 decimal place.

B is the approximation to b formed by rounding $\tanh(2)$ to 1 decimal place.

(a) Calculate the following.

- The relative error R_A when A is used to approximate a.
- The relative error R_B when B is used to approximate b. **[3]**

(b) Calculate the relative error R_C when $C = \frac{A}{B}$ is used to approximate $c = \frac{a}{b}$. **[2]**

(c) Comment on the relationship between R_A , R_B and R_C . **[1]**

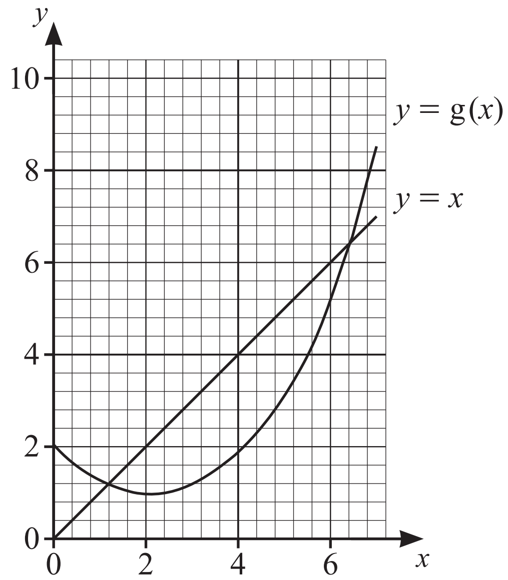
3 The equation $x^2 - \cosh(x-2) = 0$ has two roots, α and β, such that $\alpha < \beta$.

(a) Use the iterative formula

$$x_{n+1} = \mathrm{g}(x_n) \text{ where } \mathrm{g}(x_n) = \sqrt{\cosh(x_n - 2)},$$

starting with $x_0 = 1$, to find α correct to **3** decimal places. **[2]**

The diagram shows the part of the graphs of $y = x$ and $y = \mathrm{g}(x)$ for $0 \leqslant x \leqslant 7$.



(b) Explain why the iterative formula used to find α **cannot** successfully be used to find β, even if x_0 is very close to β. **[1]**

(c) Use the relaxed iteration

$$x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n),$$

with $\lambda = -0.21$ and $x_0 = 6.4$, to find β correct to **3** decimal places. **[2]**

In part **(c)** the method of relaxation was used to convert a divergent sequence of approximations into a convergent sequence.

(d) State **one** other application of the method of relaxation. **[1]**

4 Between 1946 and 2012 the mean monthly maximum temperature of the water surface of a lake in northern England has been recorded by environmental scientists. Some of the data are shown in **Table 4.1**.

Table 4.1

Month	May	June	July	August	September
t = Time in months	0	1	2	3	4
T = Mean temperature in °C	8.8	13.2	15.4	15.4	13.3

Table 4.2 shows a difference table for the data.

Table 4.2

t	T	ΔT	ΔT^2
0	8.8		
1	13.2		
2	15.4		
3	15.4		
4	13.3		

(a) Complete the copy of the difference table in the **Printed Answer Booklet**. **[2]**

(b) Explain why a quadratic model may be appropriate for these data. **[1]**

(c) Use Newton's forward difference interpolation formula to construct an interpolating polynomial of degree 2 for these data. **[4]**

This polynomial is used to model the relationship between T and t. Between 1946 and 2012 the mean monthly maximum temperature of the water surface of the lake was recorded as 8.9 °C for October and 7.5 °C for November.

(d) Determine whether the model is a good fit for the temperatures recorded in October and November. **[2]**

A scientist recorded the mean monthly maximum temperature of the water surface of the lake in 2022. Some of the data are shown in **Table 4.3**.

Table 4.3

Month	May	June	July	August	September
t = Time in months	0	1	2	3	4
T = Mean temperature in °C	10.3	14.7	16.9	16.9	14.8

(e) Adapt the polynomial found in part **(c)** so that it can be used to model the relationship between T and t for the data in **Table 4.3**. **[1]**

5 The root of the equation $\mathrm{f}(x) = 0$ is being found using the method of interval bisection. Some of the associated spreadsheet output is shown in the table below.

	A	B	C	D	E	F
1	a	$\mathrm{f}(a)$	b	$\mathrm{f}(b)$	c	$\mathrm{f}(c)$
2	2	−0.6109	3	6.08554	2.5	1.43249
3	2	−0.6109	2.5	1.43249	2.25	0.17524
4	2	−0.6109	2.25	0.17524	2.125	−0.2677
5	2.125	−0.2677	2.25	0.17524	2.1875	−0.0598
6						

The formula in cell B2 is `=EXP(A2)-A2^2-A2-2` .

(a) Write down the equation whose root is being found. **[2]**

(b) Write down a suitable formula for cell E2. **[1]**

The formula in cell A3 is `=IF(F2<0,E2,A2)` .

(c) Write down a similar formula for cell C3. **[1]**

(d) Complete row 6 of the table on the copy in the **Printed Answer Booklet**. **[2]**

(e) **Without** doing any calculations, write down the value of the root correct to the number of decimal places which seems justified. You must explain the precision quoted. **[1]**

(f) Determine how many more applications of the bisection method are needed such that the interval which contains the root is less than 0.0005. **[3]**

6 **Table 6.1** shows some values of x and the associated values of a function, $y = f(x)$.

Table 6.1

x	1.5	1	2
$f(x)$	0.840 89	1	1.189 21

(a) Explain why it is **not** possible to use the central difference method to calculate an estimate of $\frac{dy}{dx}$ when $x = 1$. **[1]**

(b) Use the forward difference method to calculate an estimate of $\frac{dy}{dx}$ when $x = 1$. **[2]**

A student uses the forward difference method to calculate a series of approximations to $\frac{dy}{dx}$ when $x = 2$ with different values of the step length, h.

These approximations are shown in **Table 6.2**, together with some further analysis.

Table 6.2

h	0.8	0.4	0.2	0.1	0.05	0.025	0.0125	0.006 25
approximation	0.130 452	0.138 647	0.143 381	0.145 942	0.147 277	0.147 959	0.148 304	0.148 477
difference		0.008 195	0.004 734	0.002 561	0.001 335	0.000 682	0.000 345	0.000 173
ratio			0.577 633	0.541 099	0.521 186	0.510 762	0.505 424	0.502 723

(c) **(i)** Explain what the ratios of differences tell you about the order of the method in this case. **[2]**

(ii) Comment on whether this is unusual. **[1]**

(d) Determine the value of $\frac{dy}{dx}$ when $x = 2$ as accurately as possible. You must justify the precision quoted. **[4]**

7 A student is using a spreadsheet to find approximations to $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$ using the midpoint rule, the trapezium rule and Simpson's rule. Some of the associated spreadsheet output with $n = 1$ and $n = 2$, is shown in **Table 7.1**.

Table 7.1

n	M_n	T_n	S_{2n}
1	0.612547	1	
2	0.639735		

(a) Complete the copy of **Table 7.1** in the **Printed Answer Booklet**. Give your answers correct to **5** decimal places. **[3]**

(b) State the value of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$ as accurately as possible. You must justify the precision quoted. **[1]**

The student calculates some more approximations using Simpson's rule. These approximations are shown in the associated spreadsheet output, together with some further analysis, in **Table 7.2**. The values of S_2 and S_4 have been blacked out, together with the associated difference and ratio.

Table 7.2

n	S_{2n}	difference	ratio
1	[redacted]		
2	[redacted]	[redacted]	
4	0.674353	−0.0209	[redacted]
8	0.665199	−0.00915	0.438059
16	0.661297	−0.0039	0.426286
32	0.659675	−0.00162	0.415762
64	0.659015	−0.00066	0.406785

(c) The student checks some of her values with a calculator. She does **not** obtain 0.406785 when she calculates $-0.00066 \div (-0.00162)$. Explain whether the value in the spreadsheet, or her value, is a more precise approximation to the ratio of differences in this case. **[2]**

(d) (i) State the order of convergence of the values in the ratio column. You must justify your answer. **[1]**

(ii) Explain what the values in the ratio column tell you about the order of the method in this case. **[2]**

(iii) Comment on whether this is unusual. **[1]**

(e) Determine the value of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$ as accurately as you can. You must justify the precision quoted. **[4]**

END OF QUESTION PAPER