OCR
Oxford Cambridge and RSA

Tuesday 11 June 2024 – Afternoon

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Time allowed: 2 hours

1335930028

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Differentiation from first principles

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Integration

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$

Integration by parts $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\overline{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \overline{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\overline{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \overline{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A
Pure Mathematics

1 Differentiate the following with respect to x.

(a) $3x^4 - \dfrac{2}{x^2}$ **[2]**

(b) $4\sqrt{x} - 9$ **[2]**

2 The vector $\begin{pmatrix} a \\ b \end{pmatrix}$ has magnitude 6 and direction 60° above the positive x-axis.

Determine the exact values of a and b. **[4]**

3 The function f is defined by $f(x) = x^3 - x^2 - 5x - 3$.

(a) Show that $(x-3)$ is a factor of $f(x)$. **[1]**

(b) Factorise $f(x)$ completely. **[2]**

Three students attempted to draw the graph of $y = (x-a)(x-1)(x+1)$, each using a different value of the constant a. Not all of their graphs were correct. Their graphs are given in the diagrams below. Copies of the diagrams are provided in the Printed Answer Booklet.

(c) Underneath each diagram in the Printed Answer Booklet,

- **either** give the value of a for which this is the correct graph of $y = f(x)$,
- **or**, if there is no value of a for which this graph is correct, write "No value of a". **[3]**

Fig. 1.1

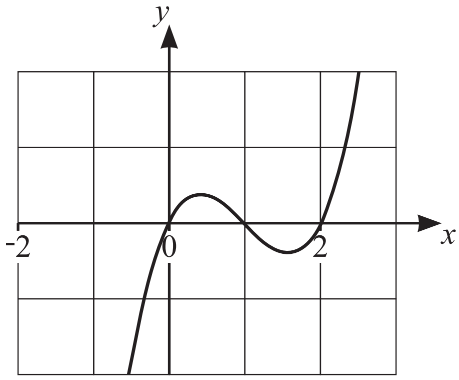


Fig. 1.2

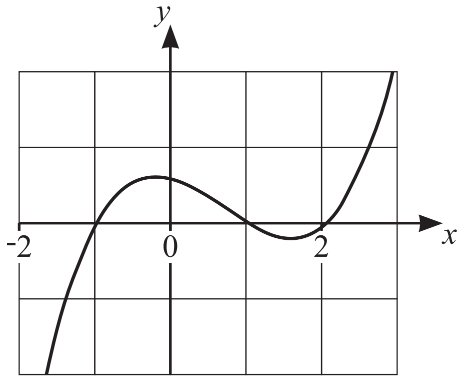
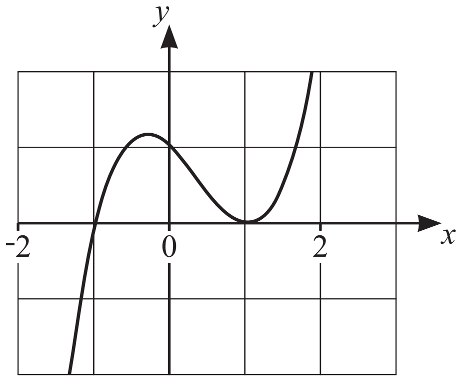
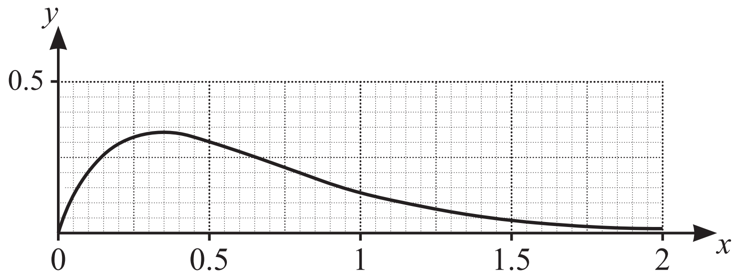


Fig. 1.3



4 The diagram shows part of the graph of $y = xe^{1-3x}$.



(a) Use the sign change method to determine, correct to **2** decimal places, the root of the equation $xe^{1-3x} - 0.2 = 0$, that lies between $x = 0.5$ and $x = 1$. **[3]**

(b) Determine the exact x-coordinate of the maximum point of the curve $y = xe^{1-3x}$. **[3]**

(c) **In this question you must show detailed reasoning.**

Determine the exact area of the region enclosed by the curve $y = xe^{1-3x}$, the x-axis and the line $x = 1$. **[4]**

5 A scientist is monitoring the decline in the population of a certain endangered species of animal in an area where their natural habitat has been damaged.

As a model, the scientist proposes that the rate of decline per year of the population is given by $\frac{1}{80}P^2$, where P is the size of the population t years after the start of the modelling.

(a) Explain how this model gives rise to the differential equation

$$\frac{dP}{dt} = -\frac{1}{80}P^2.$$ **[1]**

The scientist notes that at the start of the monitoring the population is 120.

(b) Use the model to determine an expression for P in terms of t. **[4]**

(c) Use the model to determine the time it takes for the population to reach 10. **[2]**

The model predicts that the population will never reach zero.

(d) By considering the case when $t \geqslant 160$, or otherwise, comment on the validity of the model for large values of t. **[1]**

6 In this question you must show detailed reasoning.

(a) (i) Use the formula for $\cos(A+B)$, and the double angle formulae, to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **[2]**

(ii) Use this result to solve the equation $4\cos^3\theta - 3\cos\theta - \frac{\sqrt{2}}{2} = 0$ for $0° \leqslant \theta \leqslant 180°$. **[3]**

(b) (i) Show that $\left(x + \frac{\sqrt{2}}{2}\right)\left(4x^2 - 2\sqrt{2}x - 1\right) = 4x^3 - 3x - \frac{\sqrt{2}}{2}$. **[1]**

(ii) Hence find the exact roots of the equation $4x^3 - 3x - \frac{\sqrt{2}}{2} = 0$. **[2]**

(c) Use the results from parts **(a)(ii)** and **(b)(ii)** to show that $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$. **[2]**

7 Two arithmetic progressions, A and B, each have 100 terms denoted by a_i and b_i respectively, where $i = 1, 2, 3, \ldots 100$.

The common difference of A is d, where d is a positive integer.

The two progressions have the following properties.

- $a_1 = b_{100} = 4$
- $b_1 = a_{100}$

(a) You are given that there is at least one value of i for which $b_i = 10 + a_i$.

Show that, in this case,

$i = \frac{101}{2} - \frac{5}{d}$. **[6]**

(b) Hence show that it is impossible for the equation $b_i = 10 + a_i$ to hold unless d takes certain values, which should be stated. **[2]**

Section B
Statistics

8 Sweets from a certain manufacturer are sold in packets. Thirty per cent of the sweets are orange, and these are randomly distributed amongst the packets. Each packet contains 15 sweets.

The number of orange sweets in a randomly chosen packet is denoted by X.

(a) Find the following probabilities.

(i) $P(X = 4)$ **[1]**

(ii) $P(X \geqslant 4)$ **[2]**

(b) **(i)** Write down an expression for $P(X = r)$. **[1]**

(ii) Explain the connection between the expression in part **(b)(i)** and the binomial expansion of $(0.7 + 0.3)^n$, for a specific value of n which should be stated. **[2]**

9 **(a)** The masses, M grams, of bags of flour are modelled by the distribution N(1002, 2.25).

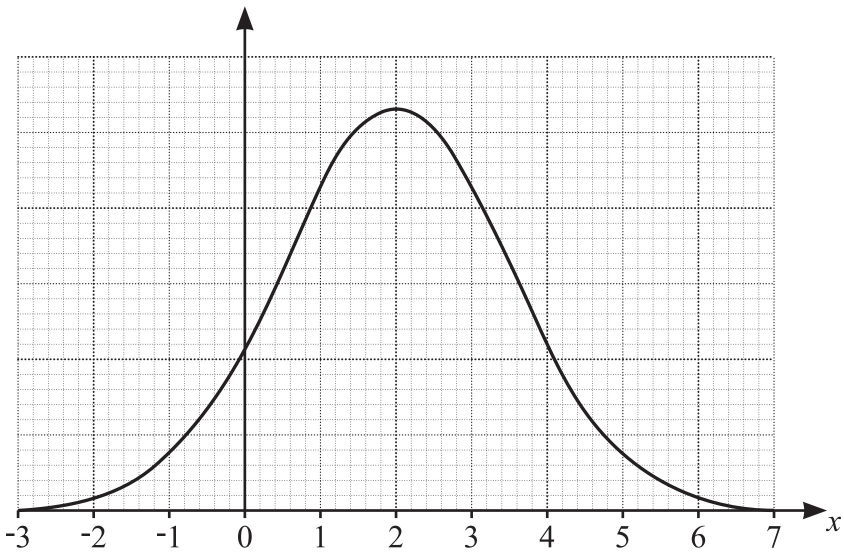
Find $P(1000 < M < 1005)$. **[1]**

(b) The masses, in grams, of bags of sugar are modelled by the distribution $N(\mu, \sigma^2)$.

You are given that 20% of bags have masses greater than 502 g and 30% of bags have masses less than 499 g.

Determine the values of μ and σ. Give your answers correct to **2** decimal places. **[5]**

(c) The diagram shows the probability distribution of a normal variable, X.



(i) Write down estimates of the x-coordinates of the points of inflection on the graph. **[1]**

(ii) Hence write down an estimate of the standard deviation of X, explaining your method. **[1]**

10 Each month, the manager of a large store records the number, c, of customers who visit the store, and the amount, £h, spent on heating during that month. The manager wants to test whether there is linear correlation between c and h.

For a randomly chosen year the value of Pearson's product-moment correlation coefficient, r, between c and h was -0.798, correct to 3 significant figures.

(a) Using the table below, carry out the test at the 1% significance level. **[5]**

(b) Describe briefly two main features of a scatter diagram that could be drawn to illustrate the values of c and h for this year. There is no need to draw a diagram. **[2]**

(c) The manager makes the following statement.

"The value of r shows that when we spend more on heating, fewer customers visit the store. So we should spend less on heating."

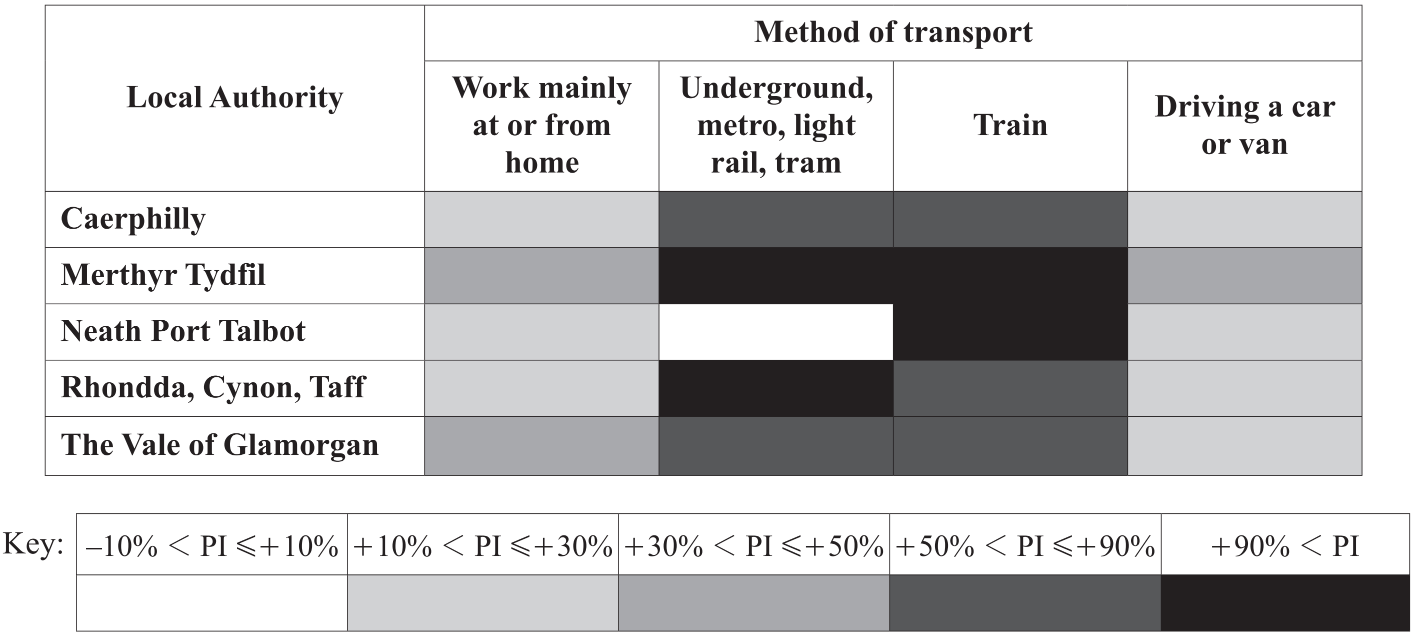
Comment briefly on this statement, making reference to the context. **[2]**

(d) Give a statement about a probability to explain the meaning of the value 0.7155 in the table below. **[2]**

Critical values of Pearson's product-moment correlation coefficient

	1-tail test	5%	2.5%	1%	0.5%
	2-tail test	10%	5%	2%	1%
n	10	0.5494	0.6319	0.7155	0.7646
	11	0.5214	0.6021	0.6851	0.7348
	12	0.4973	0.5760	0.6581	0.7079
	13	0.4762	0.5529	0.6339	0.6835

11 The chart below represents the percentage increases (PI) in the numbers of employees using four different methods of travel to work from 2001 and 2011, in five different Local Authorities (LAs) in Wales.



(a) **(i)** State, with a reason, which of the four methods of transport probably had the greatest overall percentage growth in these LAs between 2001 and 2011. **[1]**

(ii) Explain why your answer to part **(a)(i)** is **not** definite. **[1]**

(b) A student suggests that the chart can be used to estimate the total percentage change for these methods of transport in each individual LA.

Give **two** reasons why the student is likely to be wrong. **[2]**

(c) A student wants to investigate the trend from 2001 to 2011 in numbers using underground, metro, light rail or tram. The actual numbers of people using these methods in these LAs in 2001 were all less than 50 (and in one case was 4).

Explain why this means that the chart does **not** provide very helpful information for the student. **[1]**

(d) Let D denote the number of people in the Vale of Glamorgan whose usual method of travel to work is "Driving a car or van", and let H denote the number of people in the Vale of Glamorgan who "Work mainly at or from home".

Between 2001 and 2011 the increase in D was approximately 3.5 times the increase in H.

Use this fact and the information in the chart to estimate the ratio $D : H$ in 2001. **[3]**

12 Ryan has to choose one student at random from a group of 11 students. Ryan makes the choice using a single throw of two fair, six-sided dice, together with the following table.

Total score on the two dice	2	3	4	5	6	7	8	9	10	11	12
Student chosen	A	B	C	D	E	F	G	H	I	J	K

(a) Show that this sampling method is **not** random. **[2]**

Sasha suggests making the choice using a single throw of two fair, six-sided dice, together with the following table.

Scores on the two dice	1, 1	1, 2	2, 1	1, 3	3, 1	1, 4	4, 1	1, 5	5, 1	1, 6	6, 1
Student chosen	A	B	C	D	E	F	G	H	I	J	K

Ryan says that a further instruction is needed to complete the method.

(b) (i) Write a suitable further instruction. **[1]**

(ii) Using Sasha's method, state the probability of choosing student E. **[1]**

13 At an election last year, 20% of voters in Aytown voted for the Now Party. A researcher plans to test whether the proportion of voters in Aytown who support the Now Party this year is greater than 0.2. The hypotheses for the test will be $H_0: p = 0.2$ and $H_1: p > 0.2$.

The researcher surveys a random sample of 200 voters in Aytown and notes the number X who say they support the Now Party.

The random variable Y has a normal distribution which is a good approximation to the distribution of X when the null hypothesis is correct.

The significance level of the test is 5%.

(a) Find the value of y such that $P(Y > y) = 0.05$. **[2]**

(b) Use your answer to part **(a)** to determine the smallest value of X that would result in rejecting the null hypothesis. **[3]**

14 For a certain value of the constant p, the random variable X has the probability distribution given in the table.

x	1	2	3	4
$P(X = x)$	p	$\frac{1}{6}p$	p^2	$\frac{1}{2}$

Two independent values, X_1 and X_2, of X are found.

Determine $P(X_2 = 2X_1 \mid X_2 > X_1)$. **[8]**

END OF QUESTION PAPER

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