OCR
Oxford Cambridge and RSA

Thursday 20 June 2024 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

1336236120

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Integration

$\displaystyle\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\displaystyle\int f'(x)\big(f(x)\big)^n\,dx = \frac{1}{n+1}\big(f(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

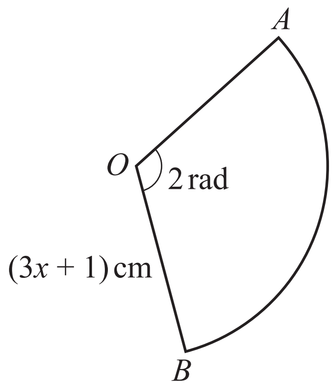
Section A
Pure Mathematics

1 Simplify each of the following.

(a) $(2a^2)^3 \times \frac{3}{4}a^{-1}$ **[2]**

(b) $\frac{4x^2 - 9}{(2x^2 + 5x - 12)(2x + 3)}$ **[2]**

2 **In this question you must show detailed reasoning.**



The diagram shows a sector AOB of a circle with centre O and radius $(3x + 1)$ cm. The angle AOB is 2 radians. The area of sector AOB is less than $(44x - 7)\,\text{cm}^2$.

Find the set of possible values of x. Give your answer in set notation. **[5]**

3 **(a)** Expand $(3 - 2x)^{-2}$ in ascending powers of x up to and including the term in x^2. **[4]**

(b) State the set of values of x for which this expansion is valid. **[1]**

(c) When $\frac{a + x}{(3 - 2x)^2}$ is expanded in ascending powers of x, the coefficient of x is zero.

Determine the value of the constant a. **[2]**

4 **(a)** Show that the equation $2\cot^2 x - 9\operatorname{cosec} x - 3 = 0$ can be expressed in the form

$5\sin^2 x + 9\sin x - 2 = 0.$ **[3]**

(b) **(i)** **In this question you must show detailed reasoning.**

Hence solve, for $0 < \theta < \pi$,

$2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3 = 0.$

Give your answers correct to **3** decimal places. **[4]**

The small angle approximation for $\sin 2\theta$ is used to find an approximation for the smallest positive solution of the equation $2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3 = 0$.

(ii) Show that this approximate solution is accurate to **2** decimal places. **[2]**

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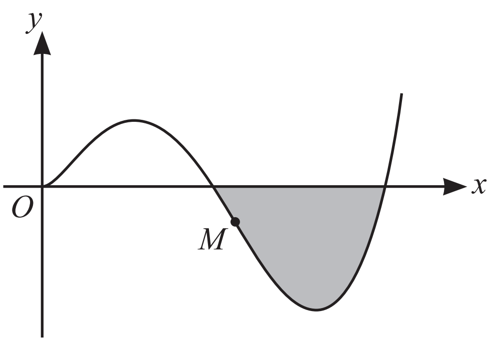

The diagram shows the curve with equation $y = (x^3 - 2x^2)\ln x$. The curve has a point of inflection at the point M.

(a) **(i)** Show that the x-coordinate of M satisfies the equation

$$x = \frac{6 + (4 - 6x)\ln x}{5}.$$ **[5]**

(ii) Use an iterative formula, based on the equation in part **(a)(i)**, to determine the x-coordinate of M correct to **2** decimal places. Use an initial value of 1.1 and show the result of each step of the iterative process. **[2]**

(b) Determine the exact area of the shaded region, giving your answer in the form $p\ln q - r$, where p and r are positive rational numbers and q is a positive integer. **[6]**

6 The curve C is defined, for $0 \leqslant t < 2\pi$, by the parametric equations

$x = 4k + k\sin t, \quad y = 2 + 4\cos t,$

where k is a constant.

(a) Find a cartesian equation for C. You do **not** need to simplify your answer. **[2]**

You are given that C is a circle.

(b) **(i)** Determine the radius of C. **[2]**

(ii) Find the possible coordinates for the centre of C. **[2]**

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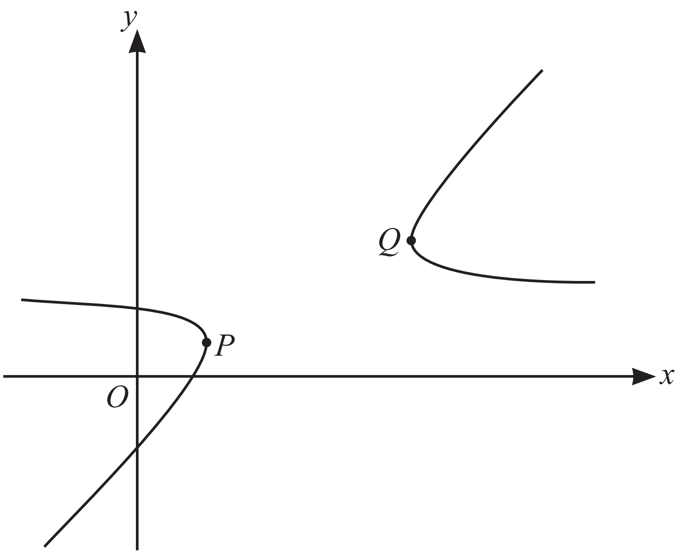


The diagram shows the curve $5x - 2xy + 2y^2 - k = 0$, where k is a positive integer.

At the points P and Q on the curve, the tangents to the curve are parallel to the y-axis.

Given that the difference in the y-coordinates of P and Q is 3, determine the x-coordinates of P and Q. **[7]**

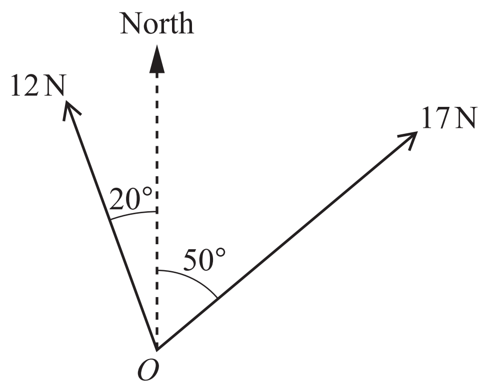
Section B
Mechanics

8 A particle P is moving with constant acceleration $(-5\mathbf{i}+2\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t=0$ seconds, P is at the origin and has velocity $(\mathbf{i}+3\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the displacement of P at time $t=2$ seconds. **[2]**

(b) Determine the speed of P at time $t=2$ seconds. **[4]**

9



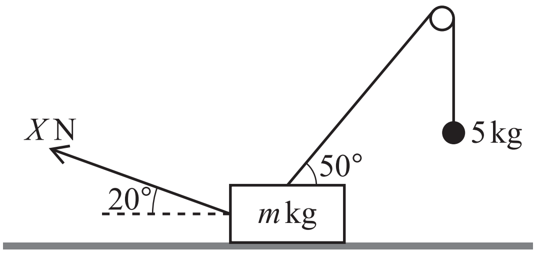
Two horizontal forces of magnitudes 17 N and 12 N act at a point O along bearings of 050° and 340° respectively (see diagram).

(a) Determine the magnitude and bearing of the resultant force. **[6]**

A third horizontal force $\mathbf{F}$ is now applied at O. The three forces are in equilibrium.

(b) State the magnitude of $\mathbf{F}$ and give the bearing along which it acts. **[2]**

10



A block of mass m kg is on smooth horizontal ground with one end of a light inextensible rope attached to its upper surface. The other end of the rope is attached to an object of mass 5 kg. The rope passes over a small smooth pulley, and the object hangs vertically below the pulley. The part of the rope between the block and the pulley makes an angle of 50° with the horizontal. A force of magnitude X N acts on the block at an angle of 20° above the horizontal in the vertical plane containing the rope (see diagram).

You are given that the block is in equilibrium.

(a) Determine the value of X. **[3]**

You are also given that the magnitude of the contact force exerted by the ground on the block is 147 N.

(b) Determine the value of m. **[3]**

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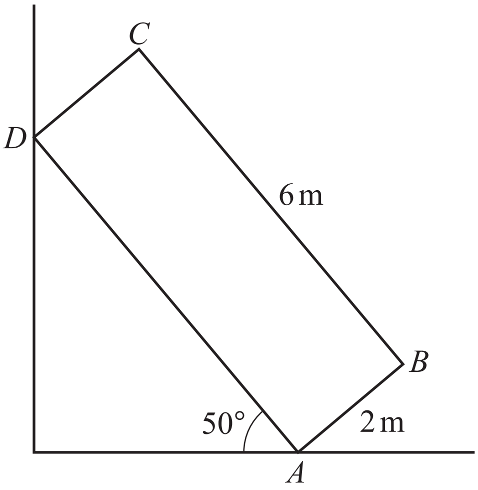


A uniform rectangular lamina $ABCD$ has a mass of 0.5 kg. The length of AB is 2 m, and the length of BC is 6 m. The lamina is in limiting equilibrium with corner A in contact with rough horizontal ground and corner D in contact with a smooth vertical wall. The lamina rests in a vertical plane that is perpendicular to the wall, with AD inclined at 50° to the horizontal (see diagram).

(a) By taking moments, show that the magnitude of the normal contact force between the lamina and the wall is 1.24 N, correct to **3** significant figures. **[4]**

(b) Determine the coefficient of friction between the lamina and the ground. **[3]**

12 A particle P moves in a straight line. The velocity $v\,\text{m s}^{-1}$ of P at time t seconds is given by

$v = \frac{1}{12}kt(t-3)$ for $0 \leqslant t \leqslant 6$,

$v = \dfrac{54k}{t^2}$ for $6 \leqslant t \leqslant 9$,

where k is a positive constant.

(a) Sketch, on the axes in the Printed Answer Booklet, the velocity-time graph for P for values of t from 0 to 9. **[3]**

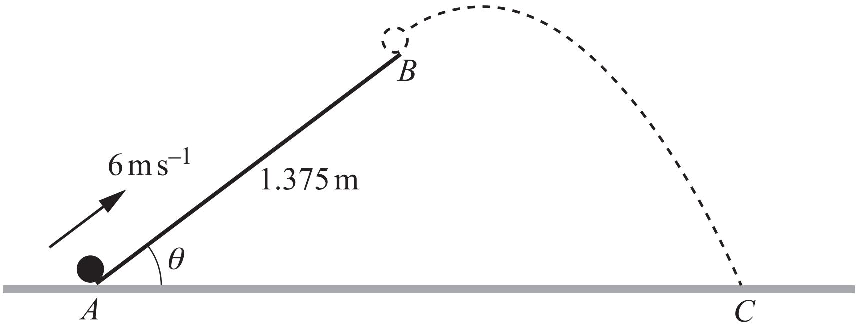
(b) State the value of t in the interval $0 \leqslant t \leqslant 9$ when the acceleration of P is zero. **[1]**

(c) **In this question you must show detailed reasoning.**

You are given that the total distance travelled by P in the interval $0 \leqslant t \leqslant 9$ is 84 m.

Find the value of k. **[6]**

13



The points A and B are the lower and upper ends, respectively, of a line of greatest slope on a plane inclined at an angle θ to the horizontal, where $\sin\theta = 0.6$ and $AB = 1.375$ m (see diagram).

A particle P is projected up the plane with speed $6\,\mathrm{m\,s^{-1}}$ from A towards B.

The plane at A is fixed to the ground which is horizontal.

The surface of the plane is rough and the coefficient of friction between P and the plane is 0.25.

(a) Show that the speed of P at B is $3.8\,\mathrm{m\,s^{-1}}$. **[6]**

The particle leaves the slope at B and moves freely under gravity.

The particle first lands at a point C on the horizontal ground. The time taken for P to travel from A to C is T seconds.

(b) Determine the value of T. **[6]**

END OF QUESTION PAPER

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