GCE

Further Mathematics A

Y541/01: Pure Core 2

A Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some

method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

   Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$\left(\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)=\right)\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\left(\frac{1}{3}+\frac{1}{3}-\frac{1}{4}\right)+\ldots$ $\left(\ldots+\frac{1}{n-1}\right)-\frac{1}{n}+\frac{1}{n}-\frac{1}{n+1}=1-\frac{1}{n+1}$	B1 [1]	1.1	AG. Writing the sum out so that the cancellation pattern is clear. If dots missing then some indication of existence of intermediate terms must be seen. Condone absence of + signs if listed by r value.	Must see $r=1$ and $r=n$ and at least two justified cancellations. Or algebraic cancellation: $=\sum_{r=1}^{n}\frac{1}{r}-\sum_{r=1}^{n}\frac{1}{r+1}=\sum_{r=1}^{n}\frac{1}{r}-\sum_{r=2}^{n+1}\frac{1}{r}$ $=\frac{1}{1}+\sum_{r=2}^{n}\frac{1}{r}-\left(\sum_{r=2}^{n}\frac{1}{r}+\frac{1}{n+1}\right)=1-\frac{1}{n+1}$
	(b)	(i)	$\left(1-\frac{1}{1+99}=1-\frac{1}{100}\right)=\frac{99}{100}$ or 0.99 oe	B1 [1]	1.1		
		(ii)	eg $\lim_{n\to\infty}\frac{1}{n+1}=0$ or $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)=1$	B1	1.1	For correct consideration of limit as n tends to infinity. Could be embedded but limit must be explicitly evaluated.	**B0** for use of ∞ in fraction (unless correct limit work seen) or incorrect limit notation eg $\lim_{n\to\infty}\frac{1}{n+1}\to 0$
			$\left(\sum_{r=100}^{\infty}\left(\frac{1}{r}-\frac{1}{r+1}\right)\right)$ $=\sum_{r=1}^{\infty}\left(\frac{1}{r}-\frac{1}{r+1}\right)-\sum_{r=1}^{99}\left(\frac{1}{r}-\frac{1}{r+1}\right)$	M1	1.1	Rewriting sum as difference of two sums with correct limits. Condone missing summation content but correct relationship between summation limits must be clear.	
			$=1-\frac{99}{100}=\frac{1}{100}$ www	A1 [3]	1.1		If **B0M0** then **SCB1** for 0.01 from 1 – 0.99 = 0.01.

Question			Answer	Marks	AO	Guidance	
2	(a)		$x=\frac{--6\pm\sqrt{(-6)^2-4\times1\times58}}{2\times1}$	**M1**	1.1	**DR**. Correctly using formula or completing the square. Condone "–6 ±√Δ" for **M1**.	Condone missing brackets under root if –6 squares to 36 (ie Δ = –196 rather than –268).
			$\sqrt{-196}=14\text{i}$	**B1FT**	1.1	FT their negative discriminant. Writing square root of negative number as the correct multiple of i.	"–196" (or "–49") must be seen.
			$x=\frac{6\pm14\text{i}}{2}=3\pm7\text{i}$	**A1**	2.5	Must be $a + b$i.	
	(a)		**Alternative method:**				
			The roots are $\alpha = a + b\text{i}$ and $\beta = a - b\text{i}$ (where a and b are real)	**B1**		Using the fact that the roots of a real quadratic form a complex conjugate pair.	May be embedded.
			$\alpha + \beta = -(-6)/1 = 6$ and $\alpha\beta = 58/1 = 58$ So $2a = 6$ (=>$a = 3$) and $a^2 + b^2 = 58$	**M1**		Finding the numerical value of the sum and product of the roots.	Could also be found by expanding $(x - (a + b\text{i}))(x - (a - b\text{i}))$ and comparing with equation. Could also substitute (a + bi) into the equation to derive $(2ab - 6b) = 0$ and $a^2 - b^2 - 6a + 58 = 0$.
			So $b^2 = 58 - 9 = 49$ so roots are $3 \pm 7\text{i}$	**A1**		Must be $a + b$i.	a real =>$b \neq 0$ =>$a = 3$ =>$b = \pm7$
				[3]			

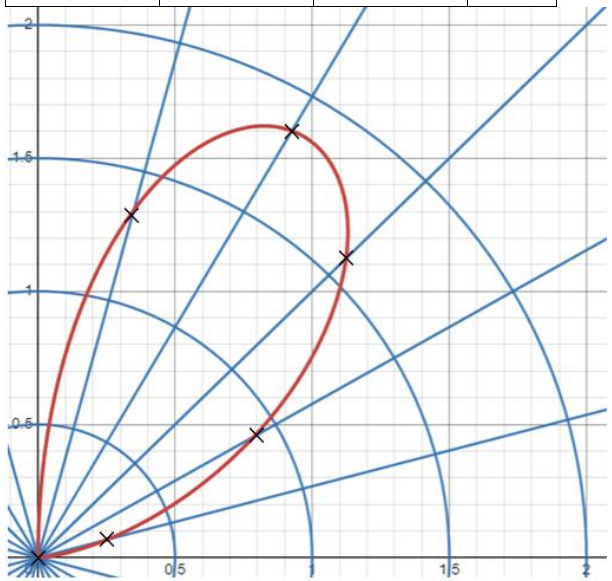
Question			Answer	Marks	AO	Guidance	
	(b)		$\arg\left(-10+5\sqrt{12}\mathrm{i}\right)=\tan^{-1}\frac{5\sqrt{12}}{-10}\left(=\frac{2}{3}\pi\right)$	**M1**	1.1	**DR**. Using correct formula for argument of complex number with non-zero real and imaginary parts.	Condone $\tan\alpha=\frac{5\sqrt{12}}{-10}\Rightarrow\alpha=-\frac{\pi}{3}$ or $\tan^{-1}\frac{5\sqrt{12}}{10}\Rightarrow\alpha=\frac{2\pi}{3}$ or $-\frac{\pi}{3}$ for **M1**.
			$\left(\arg\left(-10+5\sqrt{12}\mathrm{i}\right)^5=5\arg\left(-10+5\sqrt{12}\mathrm{i}\right)=\right)$ $5\times\frac{2}{3}\pi\left(=\frac{10}{3}\pi\right)$	**M1**	1.1	Using De Moivre's Theorem for their angle. Condone error in modulus if shown.	or using a valid method for finding z^5 explicitly (eg by expansion or by writing $-10+5\sqrt{12}\mathrm{i}=20\mathrm{e}^{\frac{2}{3}\pi\mathrm{i}}$) $z^5=-1600000-1600000\sqrt{3}\,\mathrm{i}$ or $\left(20^5\right)\mathrm{e}^{5\times\frac{2}{3}\pi\mathrm{i}}$
			so required angle is $-\frac{2}{3}\pi$ $\left(\text{or } \frac{4}{3}\pi\right)$	**A1**	1.1	cao	
				[3]			

Question			Answer	Marks	AO	Guidance	
3	(a)		$\mathbf{C}=2\begin{pmatrix}4 & -3\\ -2 & 2\end{pmatrix}-4\begin{pmatrix}3 & -5\\ 0 & 1\end{pmatrix}$ $=\begin{pmatrix}8 & -6\\ -4 & 4\end{pmatrix}-\begin{pmatrix}12 & -20\\ 0 & 4\end{pmatrix}$ $=\begin{pmatrix}8-12 & -6--20\\ -4-0 & 4-4\end{pmatrix}$	M1	1.1	Sufficient working to demonstrate knowledge of scalar multiplication of a matrix and subtraction of matrices. Can be implied by 3 out of 4 entries correct.	
			$=\begin{pmatrix}-4 & 14\\ -4 & 0\end{pmatrix}$	A1	1.1		Or BC
				[2]			
	(b)		$(\mathbf{C}=)\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$ or $2\mathbf{I}$	B1 **[1]**	2.2a		
	(c)		$(\det\mathbf{A} = 4\times2 - (-3)(-2) = 8 - 6 =)\ 2$	B1 **[1]**	1.1		
	(d)		$\begin{pmatrix}4 & -3\\ -2 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}7\\ 9\end{pmatrix}$	M1	1.1	**DR** Expressing the system in matrix form. Can be implied by the next line.	Matrix method must be used. Any other method 0/3.
			$\frac{1}{2}\begin{pmatrix}2 & 3\\ 2 & 4\end{pmatrix}\begin{pmatrix}7\\ 9\end{pmatrix}$ or $\mathbf{A}^{-1}\begin{pmatrix}7\\ 9\end{pmatrix}$ if $\mathbf{A}^{-1}$ defined	M1	1.1	Forming correct solution as matrix/vector product with inverse matrix.	FT their det**A** from **(c)**. Except for this, inverse must be correct.
			so $x=\frac{41}{2},\ y=25$	A1	1.1	20½ or 20.5	Condone $\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}\frac{41}{2}\\ 25\end{pmatrix}$ but not $\begin{pmatrix}x\\ y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}41\\ 50\end{pmatrix}$
				[3]			

Question			Answer	Marks	AO	Guidance	
4			$S=\sum_{r=1}^{390}(2r-1)^2$ or $S=\sum_{r=0}^{389}(2r+1)^2$	**M1**	3.1a	**DR**. Correctly setting up S as a sum of odd squares (condone incorrect limit combination: allow 0 or 1 for lower and 389 or 390 or 399 or 400 for upper). Treat 399 or 400 as MR (799 for 779).	
			$=\sum_{r=1}^{390}\left(4r^2-4r+1\right)=4\sum_{r=1}^{390}r^2-4\sum_{r=1}^{390}r+\sum_{r=1}^{390}1$ or $=\sum_{r=0}^{389}\left(4r^2+4r+1\right)=4\sum_{r=0}^{389}r^2+4\sum_{r=0}^{389}r+\sum_{r=0}^{389}1$	**M1**	2.2a	Expanding and separating (with their limits). Term "1" must be dealt with correctly although condone poor notation if their 390 appropriately seen later)	**M0M1** is possible
			$\sum_{r=1}^{390}r^2=\frac{1}{6}\times390(390+1)(2\times390+1)$ or 19 849 115 or 79 396 460	**B1FT**	1.1	FT only 389 or 390. Correctly using the formula for the sum of squares up to and including 390 (or 389)	or 19 697 015 from 0 or 1 up to 389
			$\sum_{r=1}^{390}r=\frac{390(390+1)}{2}$ or 76 245 or 304 980 and $\sum_{r=1}^{390}1=390$	**B1FT**	1.1	FT only 389 or 390. Correctly using the formula for the sum of integers up to and including 390 (or 389) <u>and</u> "sigma 1 = n" leading to their n.	or 75 855 from 0 or 1 up to 389 NB **B1B1** can only be attained with consistent upper limits
			So S = 4×19 849 115 – 4×76 245 + 390 = 79 091 870	**A1**	1.1		

			Alternative method: $S = 1^2 + 3^2 + 5^2 + \ldots + 779^2$ $= \sum_{r=1}^{779} r^2 - (2^2 + 4^2 + 6^2 + \ldots + 778^2)$	**M1**		Correctly setting up *S* and writing it as the difference between two sums of squares.	or $\sum_{r=1}^{780} r^2 - (2^2 + 4^2 + 6^2 + \ldots + 780^2)$
			$\sum_{r=1}^{779} r^2 = \frac{1}{6} \times 779(779+1)(2\times779+1)$ $(=157\ 879\ 930)$	**B1**		Correctly using the formula for the sum of squares up to and including 779 (or 780)	or 158 488 330 up to 780
			$S = \sum_{r=1}^{779} r^2 - (2^2 + 4^2 + 6^2 + \ldots + 778^2)$ $= \sum_{r=1}^{779} r^2 - 4(1^2 + 2^2 + 3^2 + \ldots + 389^2)$	**M1**		Bringing out a factor of 2^2 (or 4) to convert the sum of even squares to a sum of squares. Could be separate.	
			$\sum_{r=1}^{389} r^2 = \frac{1}{6} \times 389(389+1)(2\times389+1)$ $(= 19\ 697\ 015)$	**B1**		Correctly using the formula for the sum of squares up to and including 389 (or 390)	or 19 849 115 up to 390 NB **B1B1** can only be attained for 779 and 389 or 780 and 390
			So *S* = 157 879 930 – 4×19697015 = 79 091 870	**A1**			158 488 330 – 4×19 849 115
				[5]			

Question			Answer	Marks	AO	Guidance	
5			$\mathbf{a}\times\mathbf{b}=\begin{pmatrix}1\\1-p\\p+2\end{pmatrix}\times\begin{pmatrix}2\\1\\1\end{pmatrix}$ $=\begin{pmatrix}(1-p)\times1-(p+2)\times1\\(p+2)\times2-1\times1\\1\times1-(1-p)\times2\end{pmatrix}$	**M1***	3.1a	Forming the cross-product. Two components correct (possibly unsimplified) or all correct but for wrong sign globally.	or expanding $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&1-p&p+2\\2&1&1\end{vmatrix}$
			$=\begin{pmatrix}-1-2p\\2p+3\\2p-1\end{pmatrix}$	**A1**	1.1		
			$\begin{pmatrix}1\\14\\p-3\end{pmatrix}.\begin{pmatrix}-1-2p\\2p+3\\2p-1\end{pmatrix}$ $=1(-1-2p)+14(2p+3)+(p-3)(2p-1)$	**M1**	1.1	Correctly forming the scalar product of **c** and their **a×b** (even if one or both incorrect)	If **M0M0** then **SCB1** for correctly forming scalar product of any two vectors containing *p*.
			c perpendicular to **a×b** =>**c.(a×b) =** 0 soi	**B1**	3.1a	Rule for two vectors being perpendicular soi.	
			$\Rightarrow -1-2p+28p+42+2p^2-7p+3=0$ $\Rightarrow 2p^2+19p+44=0$	**depM1***	1.1	Rearranging to 3-term quadratic equation in *p*.	"= 0" can be implied by correct solution.
			$\therefore p=-4$ or $p=-\frac{11}{2}$ cao	**A1**	1.1	$(p + 4)(2p + 11) = 0$	
				[6]			

Question			Answer	Marks	AO	Guidance	
6	**(a)**		$r = 0$ when $\theta = 0$ and when $\theta = \pi/2$ (and the function is continuous so there is a loop).	**B1**	2.4		Condone use of "origin" rather than "pole" in explanations.
			Must be in **1st quadrant** since θ only takes values between **0 and ½**π.	**B1**	2.1	Must link domain of θ with quadrant.	
			There are **only** two values of θ for which $r = 0$, therefore there is **only one** loop (two loops require three values of θ/tangents at the pole).	**B1** **[3]**	2.2a	Convincingly explaining why it is a single loop. θ values/$r = 0$ must be linked to loop.	or "There are no other solutions to $r = 0$ (in the domain)".
	(b)		$\frac{1}{6}\pi$ | $\frac{1}{4}\pi$ | $\frac{5}{12}\pi$ | $\frac{1}{2}\pi$ 0.912 | 1.589 | 1.351 | 0	**B1***	1.1	All 4 entries correct and non-zero ones correct to at least 3 sf.ie awrt 0.912, 1.59, 1.35.	
				**M1**	1.1	All points, theirs and given, plotted approximately correctly (ie on the θ = constant lines and in the correct half between the appropriate r = constant lines)	Points can be inferred by curve
				A1dep*	2.2a	Smooth curve plotted between correctly plotted points (see criteria above) joining at the pole. Single maximum r at some angle between $\pi/4$ and $5\pi/12$ (it might be on the wrong side of $\theta = \pi/3$.	Curve needs to be: smooth (except at pole) curved (not line segments) closed single loop cusp at pole

				[3]			
	(c)		$\frac{\mathrm{d}r}{\mathrm{d}\theta}=6\sinh\frac{1}{3}\theta\times\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin 2\theta\right)+6\sin 2\theta\times\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sinh\frac{1}{3}\theta\right)$	**M1**	3.1a	Attempting to differentiate $r(\theta)$ using the product rule.	$a\sinh\frac{1}{3}\theta\ \cos 2\theta+b\sin 2\theta\cosh\frac{1}{3}\theta$ where a, b are non-zero constants
			$\frac{\mathrm{d}r}{\mathrm{d}\theta}=12\cos 2\theta\sinh\frac{1}{3}\theta+2\sin 2\theta\cosh\frac{1}{3}\theta$	**A1**	1.1	All correct (any form)	
			$\therefore$ at A, $12\cos 2\phi\sinh\frac{1}{3}\phi+2\sin 2\phi\cosh\frac{1}{3}\phi=0$ $\therefore 12\cos 2\phi\sinh\frac{1}{3}\phi=-2\sin 2\phi\cosh\frac{1}{3}\phi$ $\therefore 6\tanh\frac{1}{3}\phi=-\tan 2\phi$	**M1**	1.1	Setting their derivative to 0 and rearranging to a form containing tan and tanh only.	If using exponential form, must reach tan and reduce to one exponential term. eg $e^{\frac{2}{3}\phi}(6+\tan 2\phi)=6-\tan 2\phi$
			$\therefore\frac{1}{3}\phi=\tanh^{-1}(-\frac{1}{6}\tan 2\phi)$ $=\frac{1}{2}\ln\left(\frac{1+(-\frac{1}{6}\tan 2\phi)}{1-(-\frac{1}{6}\tan 2\phi)}\right)$ $\therefore\phi=\frac{3}{2}\ln\left(\frac{6-\tan 2\phi}{6+\tan 2\phi}\right)$	**A1** **[4]**	2.2a	AG so intermediate step must be shown. Condone use of θ rather than ϕ but final answer must be in terms of ϕ (or explicitly stating "$\theta=\phi$").	
	(d)		r(= 6sin(2×1.0207)×sinh(1.0207/3)) = 1.85 (3 sf) cao	**B1** **[1]**	1.1		0.0741 comes from using degrees.

Question			Answer	Marks	AO	Guidance	
7	**(a)**		$17\cosh x - 15\sinh x = 17\frac{e^x + e^{-x}}{2} - 15\frac{e^x - e^{-x}}{2}$	**M1**	3.1a	Substituting correct exponential definitions of hyperbolic functions into expression	
			$= \frac{17-15}{2}e^x + \frac{17+15}{2}e^{-x}\left(= e^x + 16e^{-x}\right)$	**M1**	1.1	Collecting terms	eg $s\,e^x + t\,e^{-x}$ where s, t are non-zero constants, possibly unsimplified.
			$= e^{-x}(e^{2x} + 16)$ (so a = 1, b = 2 and c = 16) www	**A1**	2.2a	Answer can be embedded. ISW.	
				[3]			

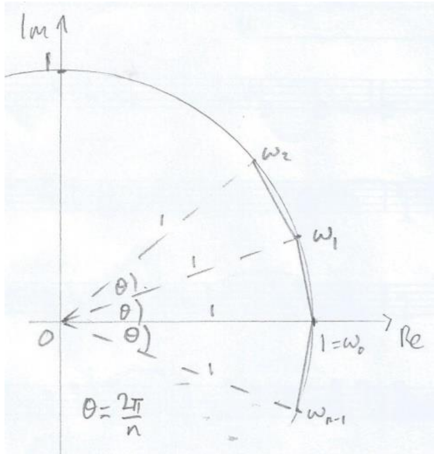
	(b)		$V=\pi\int_0^{\ln 3}\left(\frac{1}{\sqrt{17\cosh x-15\sinh x}}\right)^2 \mathrm{d}x$	**M1**	1.1	**DR**. Formula for VoR correctly used in solution (limits and/or π may come later) soi	dx must be seen here but can be implied later.
			$=\pi\int_0^{\ln 3}\frac{1}{\mathrm{e}^{-x}(\mathrm{e}^{2x}+16)}\mathrm{d}x$ or $\pi\int_0^{\ln 3}\frac{\mathrm{e}^x}{\mathrm{e}^{2x}+16}\mathrm{d}x$	**M1**	1.1	Squaring out and using their part **(a)** formula	
			$u=\mathrm{e}^x$	**M1**	3.1a	Useful substitution stated or used	eg $u=\mathrm{e}^{-x}$ or $\mathrm{e}^x=4\tan u$ etc
			$\mathrm{d}u=\mathrm{e}^x\mathrm{d}x \Rightarrow \therefore V=\pi\int_{\ldots}^{\ldots}\frac{\mathrm{e}^x}{\mathrm{e}^{2x}+16}\mathrm{d}x=\pi\int_{\ldots}^{\ldots}\frac{1}{u^2+16}\mathrm{d}u$	**M1***	1.1	Making the substitution (including dx) to reduce to integrable form. Ignore limits here.	$\mathrm{e}^x\mathrm{d}x=4\sec^2 u\,\mathrm{d}u$ and $V=\frac{\pi}{4}\int \mathrm{d}u$ if using $\mathrm{e}^x=4\tan u$
			$\int_{\ldots}^{\ldots}\frac{1}{u^2+16}\mathrm{d}u=\left[\frac{1}{4}\tan^{-1}\frac{u}{4}\right]_{\ldots}^{\ldots}$	**B1FT**	1.1	Correct arctan integration (ignore limits etc). FT their 16 and √16	Could be implicit in the substitution. (ie $\int \mathrm{d}u=[u]$ if using $\mathrm{e}^x=4\tan u$)
			$x=0\Rightarrow u=\mathrm{e}^0=1, x=\ln 3\Rightarrow u=\mathrm{e}^{\ln 3}=3$ $\therefore V=\frac{\pi}{4}\left[\tan^{-1}\frac{u}{4}\right]_1^3=\frac{\pi}{4}\left(\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{1}{4}\right)$	**depM1***	1.1	Dealing with limits correctly (either converting to u-space or substituting back to x-space – if the latter ignore missing "x =" in the limits if correct at resubstitution)	Could see $\frac{\pi}{4}\left[\tan^{-1}\frac{\mathrm{e}^x}{4}\right]_0^{\ln 3}$
			$\tan\left(\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{1}{4}\right)=\frac{\frac{3}{4}-\frac{1}{4}}{1+\frac{3}{4}\times\frac{1}{4}}=\frac{12-4}{16+3}=\frac{8}{19}$ $\therefore \tan^{-1}\frac{3}{4}-\tan^{-1}\frac{1}{4}=\tan^{-1}\frac{8}{19}$ $\therefore V=\frac{1}{4}\pi\tan^{-1}\frac{8}{19}\quad\left(\text{so } k=\frac{1}{4}\text{ and } q=\frac{8}{19}\right)$	**A1**	3.1a	Formula for tan(A – B) must be used. (NB It is possible to find 8/19 using tan and $\tan^{-1}$ on a calculator)	0.3129987969... alone or used to derive k and/or q gets **A0**

				[7]			

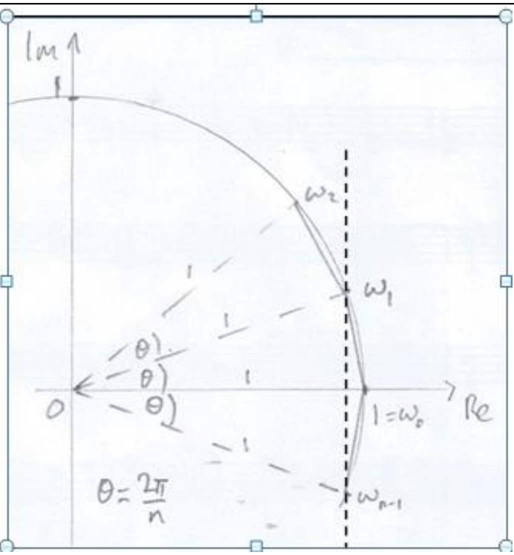
Question			Answer	Marks	AO	Guidance	
8	(a)		**If $C - P > 0$**(or more children are playing in the bouncy castle than in the ball pits so**) children will leave** the bouncy castle and hence **d*C*/d*t* will be negative**(and so α must be negative).	B1 [1]	2.4	Properly reasoned argument. Could be related to $C - P < 0$. ie • some conditional • consequence in terms of movement of children • consequence in terms of sign of derivative	Ignore reference to d*P*/d*t*.
	(b)		$\frac{dP}{dt} = 2C - 2P + 32t$ $\therefore \frac{d^2P}{dt^2} = 2\frac{dC}{dt} - 2\frac{dP}{dt} + 32$ $= 2\times -2(C-P) - 2\frac{dP}{dt} + 32$ $= 4P - 4C - 2\frac{dP}{dt} + 32$ $\frac{dP}{dt} = 2C - 2P + 32t$ $\therefore 2C = \frac{dP}{dt} + 2P - 32t$ $\therefore \frac{d^2P}{dt^2} = 4P - 2\left(\frac{dP}{dt} + 2P - 32t\right) - 2\frac{dP}{dt} + 32$ $= 4P - 2\frac{dP}{dt} - 4P + 64t - 2\frac{dP}{dt} + 32$ $= -4\frac{dP}{dt} + 64t + 32$ $\therefore \frac{d^2P}{dt^2} + 4\frac{dP}{dt} = 64t + 32$ www	M1 A1	3.4 1.1	Correctly differentiating the d*P*/d*t* equation and then substituting from the d*C*/d*t* equation. Condone sign/coefficient errors when substituting. AG. Substitution for *C* (or eg elimination of *C* – *P*) must be seen. Must reach AG. Could see eg P' or $\dot{P}$ etc. Allow in terms of α and γ if values of α and γ quoted at end.	$\frac{dP}{dt} = \alpha(P-C) + \gamma t$ $\therefore \frac{d^2P}{dt^2} = \alpha\left(\frac{dP}{dt} - \frac{dC}{dt}\right) + \gamma$ $\therefore \frac{d^2P}{dt^2} = \alpha\frac{dP}{dt} - \alpha\frac{dC}{dt} + \gamma$ $= \alpha\frac{dP}{dt} - \alpha(\alpha C - \alpha P) + \gamma$ $\frac{dP}{dt} = \alpha(P-C) + \gamma t$ $\therefore \alpha C = \alpha P + \gamma t - \frac{dP}{dt}$ $\therefore \frac{d^2P}{dt^2} = \alpha\frac{dP}{dt} - \alpha^2 C + \alpha^2 P + \gamma$ $= \alpha\frac{dP}{dt} - \alpha\left(\alpha P + \gamma t - \frac{dP}{dt}\right) + \alpha^2 P + \gamma$ $= \alpha\frac{dP}{dt} - \alpha\gamma t + \alpha\frac{dP}{dt} + \gamma$ $\therefore \frac{d^2P}{dt^2} - 2\alpha\frac{dP}{dt} = -\alpha\gamma t + \gamma$

				[2]			
	(c)	**(i)**	Auxiliary equation is $m^2 + 4m = 0$ so $m = 0$ or $m = -4$ so CF is $(P =)A + Be^{-4t}$	**B1** **[1]**	1.1	Condone missing "$P =$" and/or x instead of t / y instead of P here. Condone Ae^{0t}	$m^2 - 2\alpha m = 0$ so $m = 0$ or $m = 2\alpha$ so $(P =)A + Be^{2\alpha t}$
		(ii)	Because the function $P = b$ is one of the functions that form the CF (so will come to 0 when plugged in to the LHS).	**B1** **[1]**	2.4	or other correct reason eg this trial function doesn't produce a term in t to compare with $64t$. If considering the CF, must address the fact that there is a constant term in the CF and so there cannot be a constant term in the trial function.	"The complementary function already has a constant in it". "Part of the trial function is included in the complementary function". Not "There is no P term in the differential equation". Not "It matches the form of the complementary function".
		(iii)	$P = at^2 + bt \Rightarrow P' = 2at + b$ and $P'' = 2a$ $\therefore 2a + 4(2at + b) = ...$	**M1**	3.4	Correctly differentiating to find the first and second derivatives and substituting into the LHS of the DE in **(b)**.	
			$\therefore 8at + 4b + 2a = 64t + 32$ $\therefore 8a = 64 \Rightarrow a = 8,$ $4b + 2a = 32 \Rightarrow 4b = 32 - 2(8) = 16 \Rightarrow b = 4$	**A1**	3.4	Finding a and b correctly.	$\therefore 2a - 2\alpha(2at + b) = -\alpha\gamma t + \gamma$ $\therefore -4a\alpha = -\alpha\gamma \Rightarrow a = \frac{\gamma}{4}$ $2a - 2\alpha b = \gamma \Rightarrow 2\alpha b = \frac{\gamma}{2} - \gamma = -\frac{\gamma}{2}$ $\therefore b = \frac{-\gamma}{4\alpha}$
			$P = A + Be^{-4t} + 8t^2 + 4t$	**A1FT** **[3]**	1.1	FT their CF + their numerical PI. Must be "$P =$". Must be $P(t)$.	$P = A + Be^{2\alpha t} + \frac{\gamma}{4}t^2 - \frac{\gamma}{4\alpha}t$

	(d)		$t = 0$, $P = 55 \Rightarrow A + B = 55$ $P' = -4Be^{-4t} + 16t + 4$ $t = 0$, $P' = 24 \Rightarrow -4B + 4 = 24 \Rightarrow B = -5$ $\Rightarrow A = 60$ $t = \frac{1}{2} \Rightarrow P = 60 - 5e^{-4\times\frac{1}{2}} + 8\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)$ $= 60 - 5e^{-2} + 2 + 2 = 63.323...$ so 63 children cao www	**M1** **M1** **M1** **A1** **[4]**	3.3 3.4 3.3 3.4	Using first initial condition. Correctly differentiating their *P* to find *P′* (can be implied by next line) Substituting t=0 correctly in *P′* to obtain the second initial condition and using the latter to solve for *A* and *B*. Must be from correct *P*.	
	(e)		The number of children cannot increase indefinitely.	**B1** **[1]**	3.5b	or other reasonable limitation of the model in the long-term which could be environmental. eg the room has a finite size the centre will close. $P \sim 8t^2$ as t gets large **which is not realistic**.	Not just "Children are being added constantly". Not "The number of children grows exponentially".

Question			Answer	Marks	AO	Guidance	
9	**(a)**		Diagram showing $\boldsymbol{\omega_1}$ as the 'first' non-real vertex of a regular n-gon **with 1** as the 0th vertex **and at least one other vertex** shown with the correct relationship (ie on unit circle with same angular distance).	**B1**	2.1		Diagram should clearly show equal angular distance between the roots and an equal distance (of 1) from O to each root. At least 3 points including ω_0 and ω_1 shown. For this **B1** allow an n-gon with a specific value of n chosen.
			(Since it is a root of unity)the modulus of ω_1 is 1 so multiplying by it leaves the modulus unchanged....	**B1**	2.2a	Dealing with modulus (could be incorporated in the below).	$\lvert\omega_k\rvert = 1$ since it is a root of unity $\lvert\omega_1\rvert = 1 \Rightarrow \lvert\omega_1^k\rvert \left(= \lvert\omega_1\rvert^k\right) = 1$
			...(since the n roots of unity are represented by the n vertices on the unit circle of a regular n-gon then)rotation by the argument of the first (ω_1) (ie adding an angle) takes you to the second and so on.	**B1**	2.4	Dealing with argument. Accept a well-reasoned argument based on multiplication by ω_1 representing a pure rotation (by the required angle). Could be argued by induction if rigorous.	$\arg(\omega_k) = \frac{2\pi k}{n}$ $\arg(\omega_1) = \frac{2\pi}{n}$ $\Rightarrow \arg\left(\omega_1^k\right) = k\arg(\omega_1) = \frac{2\pi k}{n}$
			Alternative method for last B1B1: $\omega_1 = e^{\frac{2\pi}{n}i}$ and $\omega_k = e^{\frac{2\pi k}{n}i}$	**B1**		ω_1 can be implied if appearing in the equation below	

			$\therefore \omega_1^k = \left(e^{\frac{2\pi}{n}i}\right)^k = e^{\frac{2\pi k}{n}i} = \omega_k$	**B1**			
				[3]			
	(b)		$(\omega_1^0 = 1 = \omega_0$ and so) $\sum_{k=0}^{n-1}\omega_k = \sum_{k=0}^{n-1}\omega_1^k$ which is a GP with $(a = 1)$, $r = \omega_1$ $(\neq 1)$ and n terms. $= \frac{1\times(\omega_1^n - 1)}{\omega_1 - 1} = \frac{1-1}{\omega_1 - 1} = \frac{0}{\omega_1 - 1} = 0$ (since ω_1 is an n^{th} root of unity so $\omega_1^n = 1$).	**M1** **A1** **[2]**	3.1a 2.2a	Using the identity from **(a)** and recognising the GP (can be implied by the formula). AG so reasoning must be shown,	$\sum_{k=0}^{n-1}\omega_k = \sum_{k=0}^{n-1}\omega_1^k$ and recognition of $\omega_1^n - 1 = (\omega_1 - 1)(\omega_1^{n-1} + \omega_1^{n-2} + \ldots + \omega_1 + 1)$ If GP not recognised then justification for $\omega_1 - 1 \neq 0$ must also be given.
	(c)		$z = a + bi$ $z^* = a - bi$ $\therefore z + z^* = 2a = 2\text{Re}(z)$	**B1** **[1]**	2.1	AG	
	(d)		$\left(\sum_{k=0}^{n-1}\omega_k = 0\right)$ $\therefore \sum_{k=0}^{n-1}\omega_k^* = 0$ $\therefore \sum_{k=0}^{n-1}\omega_k + \sum_{k=0}^{n-1}\omega_k^* = 0$ or $\sum_{k=0}^{n-1}\left(\omega_k + \omega_k^*\right) = 0$ $\therefore \sum_{k=0}^{n-1} 2\text{Re}(\omega_k) = 0$ $\therefore \sum_{k=0}^{n-1}\text{Re}(\omega_k) = 0$	**B1** **[1]**	3.1a		

	(e)		 The roots of unity form a regular *n*-gon which is symmetrical in the real axis.	**B1** **[1]**	2.1	Or the (non-real) roots of unity come in complex conjugate pairs (since they are roots of the real polynomial $z^n = 1$).	Could use symmetry of cos function in geometric context (eg $\cos\frac{2}{5}\pi = \cos\left(2\pi - \frac{2}{5}\pi\right)$ etc)
	(f)	**(i)**	$\arg\omega_1 = \frac{2\pi}{5}$ soi So from **(d)** and **(e)**, $1 + 2\cos\frac{2\pi}{5} + 2\cos\frac{4\pi}{5} = 0$ $\left(\therefore 2\cos\frac{4\pi}{5} = -1 - 2\cos\frac{2\pi}{5}\right)$ $\therefore \cos\frac{4\pi}{5} = -\frac{1}{2} - \cos\frac{2\pi}{5}$ $a = -\frac{1}{2},\ b = -1$	**M1** **A1** **[2]**	3.1a 2.2a	Finding the first argument. Could be embedded. *a* and *b* can be embedded.	Could see sum of all 5 real parts.

	(f)	**(ii)**	$\therefore 2\cos^2\frac{2\pi}{5}-1=-\frac{1}{2}-\cos\frac{2\pi}{5}$ $\therefore 4c^2+2c-1=0,\ c=\cos\frac{2\pi}{5}$ $c=\frac{-1\pm\sqrt{5}}{4}$	**M1**	3.1a	Using correct double angle formula to derive and solve a quadratic equation in $\cos\frac{2\pi}{5}$ (must have real solutions). Could be BC.	
			$\frac{2\pi}{5}$ is an acute angle so $\cos\frac{2\pi}{5}>0$. So $\frac{-1-\sqrt{5}}{4}$ is rejected so $\cos\frac{2\pi}{5}=\frac{-1+\sqrt{5}}{4}$.	**A1** **[2]**	2.3	Clear rejection after valid argument leading to correct answer.	

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