

GCE

Further Mathematics A

Y544/01: Discrete Mathematics

A Level

Mark Scheme for June 2024

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It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.
2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal <http://www.rm.com/support/ca>
3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **number of required** standardisation responses.

YOU MUST MARK THE REQUIRED NUMBER OF PRACTICE AND STANDARDISATION RESPONSES BEFORE YOU CAN BE APPROVED TO MARK LIVE SCRIPTS.

MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.
4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.
5. The RM Assessor **comments box** is used by your team leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**
If you have any questions or comments for your team leader, use the phone, the RM Assessor messaging system, or e-mail.
6. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

7. Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

8. Subject Specific Marking Instructions

- a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
 - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

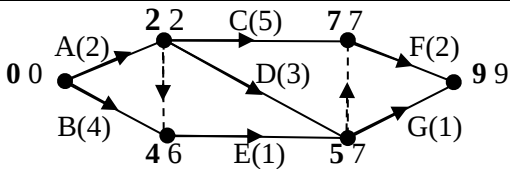
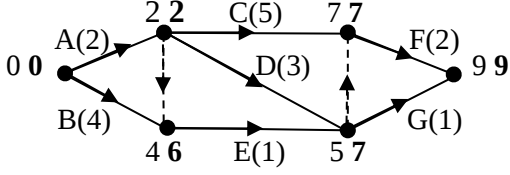
- g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

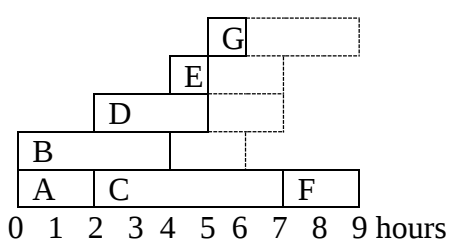
Question		Answer	Marks	AO	Guidance																																								
1	(a)	<table><tr><td>A chooses</td><td>B</td><td>B</td><td>B</td><td>C</td><td>C</td><td>C</td><td>D</td><td>D</td><td>D</td></tr><tr><td>B chooses</td><td>A</td><td>C</td><td>D</td><td>A</td><td>D</td><td>D</td><td>A</td><td>C</td><td>C</td></tr><tr><td>C chooses</td><td>D</td><td>D</td><td>A</td><td>D</td><td>A</td><td>B</td><td>B</td><td>A</td><td>B</td></tr><tr><td>D chooses</td><td>C</td><td>A</td><td>C</td><td>B</td><td>B</td><td>A</td><td>C</td><td>B</td><td>A</td></tr></table>	A chooses	B	B	B	C	C	C	D	D	D	B chooses	A	C	D	A	D	D	A	C	C	C chooses	D	D	A	D	A	B	B	A	B	D chooses	C	A	C	B	B	A	C	B	A	M1	1.1	Any three correct derangements (not including the one given)
		A chooses	B	B	B	C	C	C	D	D	D																																		
		B chooses	A	C	D	A	D	D	A	C	C																																		
		C chooses	D	D	A	D	A	B	B	A	B																																		
D chooses	C	A	C	B	B	A	C	B	A																																				
	A1	1.1	All correct and no repeats or extras (except allow given derangement repeated)																																										
	[2]																																												
1	(b)	Existence	B1	1.2																																									
			[1]																																										
1	(c)	e.g. A might choose B but B not choose A	B1	2.4	An example to explain why graph must be directed Or an appropriate description involving who is choosing (gifting) and who was chosen (receiving) e.g. The person chosen by a worker need not choose that worker A giving a gift to B is not the same as B giving a gift to A																																								
			[1]																																										
1	(d)	8	B1	1.1	8 cao 2×4 (= 8), 4+4 (= 8), 4 arcs for each of two years (so 8)																																								
			[1]																																										
1	(e)	V + R = E + 2 ⇒ 5 + R = 8 + 2 R = 5	M1 A1 FT [2]	1.1 1.1	Attempt to use Euler’s formula, with 5, 2 and ‘their 8’, allow other notation (e.g. A for R) 5 (regions) or ‘their 8’ – 3 FT their 8 from part (d)																																								

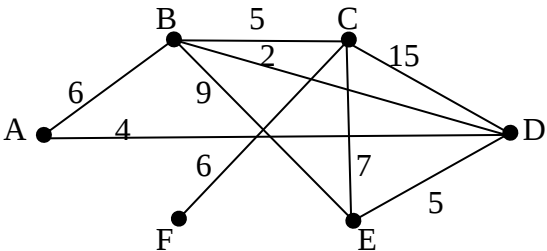
Question		Answer	Marks	AO	Guidance																																						
2	(a)	$P = 2x - y + z \Rightarrow P - 2x + y - z = 0$ $3x - 4y - z \leq 30 \Rightarrow 3x - 4y - z + s = 30$ $x - y \leq 6 \Rightarrow x - y + t = 6$ $x - 3y + 2z \geq -2 \Rightarrow -x + 3y - 2z \leq 2$ $\Rightarrow -x + 3y - 2z + u = 2$	M1	1.1	Dealing with objective and using slack variables to remove inequalities May be implied from tableau																																						
		<table border="1"><tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>u</td><td>RHS</td></tr><tr><td>1</td><td>-2</td><td>1</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>3</td><td>-4</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>30</td></tr><tr><td>0</td><td>1</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>6</td></tr><tr><td>0</td><td>-1</td><td>3</td><td>-2</td><td>0</td><td>0</td><td>1</td><td>2</td></tr></table>	P	x	y	z	s	t	u	RHS	1	-2	1	-1	0	0	0	0	0	3	-4	-1	1	0	0	30	0	1	-1	0	0	1	0	6	0	-1	3	-2	0	0	1	2	M1
P	x	y	z	s	t	u	RHS																																				
1	-2	1	-1	0	0	0	0																																				
0	3	-4	-1	1	0	0	30																																				
0	1	-1	0	0	1	0	6																																				
0	-1	3	-2	0	0	1	2																																				
			A1	1.1	Correct tableau, or equivalent																																						
			[3]																																								
2	(b)	Pivot on 1 in x column	M1	1.1	Pivot row and column correct for a valid pivot choice for their initial tableau, pivot must be positive and from minimum positive ratio ($\text{RHS} \div \text{pivot col}$), but condone any non-zero value in P row																																						
		<table border="1"><tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>u</td><td>RHS</td></tr><tr><td>1</td><td>0</td><td>-1</td><td>-1</td><td>0</td><td>2</td><td>0</td><td>12</td></tr><tr><td>0</td><td>0</td><td>-1</td><td>-1</td><td>1</td><td>-3</td><td>0</td><td>12</td></tr><tr><td>0</td><td>1</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>6</td></tr><tr><td>0</td><td>0</td><td>2</td><td>-2</td><td>0</td><td>1</td><td>1</td><td>8</td></tr></table>	P	x	y	z	s	t	u	RHS	1	0	-1	-1	0	2	0	12	0	0	-1	-1	1	-3	0	12	0	1	-1	0	0	1	0	6	0	0	2	-2	0	1	1	8	M1
P	x	y	z	s	t	u	RHS																																				
1	0	-1	-1	0	2	0	12																																				
0	0	-1	-1	1	-3	0	12																																				
0	1	-1	0	0	1	0	6																																				
0	0	2	-2	0	1	1	8																																				
			A1	1.1	Correct tableau, or equivalent																																						
			[3]																																								
2	(c)	$x = 6, y = 0, z = 0$	B1FT	2.2a	ft their iterated tableau from part (b)																																						
			[1]																																								
2	(d)	Move along edge $z = 0, t = 0$	B1FT	3.1b	(i.e. both are = 0 in each row of table below)																																						
		From $x = 6, y = 0, z = 0, s = 12, t = 0, u = 8$ to $x = 10, y = 4, z = 0, s = 16, t = 0, u = 0$	B1FT	2.2a	Sufficient to have the (exactly) 3 zero values correct in each row ft their iterated tableau from part (b)																																						
			[2]																																								

Question		Answer	Marks	AO	Guidance																				
3	(a)	Beth <table><tr><td></td><td>X</td><td>Y</td><td>Z</td><td>row min</td></tr><tr><td>P</td><td>2</td><td>-3</td><td>c</td><td>-3</td></tr><tr><td>Amir Q</td><td>-3</td><td>b</td><td>4</td><td>-3</td></tr><tr><td>R</td><td>a</td><td>-1</td><td>-2</td><td>min(a, -2)</td></tr></table> min(a, -2) > -3 -3 < a < 0		X	Y	Z	row min	P	2	-3	c	-3	Amir Q	-3	b	4	-3	R	a	-1	-2	min(a, -2)	B1 M1 A1 [3]	2.1 1.1 2.2a	Row minima calculated (at least as far as showing that rows P and Q each have row min = -3) Ignore any other working Appropriate comparison, may be implied from a > -3 or a ≥ -3 seen cao
	X	Y	Z	row min																					
P	2	-3	c	-3																					
Amir Q	-3	b	4	-3																					
R	a	-1	-2	min(a, -2)																					
3	(b)	Beth <table><tr><td></td><td>X</td><td>Y</td><td>Z</td></tr><tr><td>P</td><td>2</td><td>-3</td><td>c</td></tr><tr><td>Amir Q</td><td>-3</td><td>b</td><td>4</td></tr><tr><td>R</td><td>a</td><td>-1</td><td>-2</td></tr></table> col max 2 b max(c, 4) b < 2 and b < max(c, 4) 0 < b < 2		X	Y	Z	P	2	-3	c	Amir Q	-3	b	4	R	a	-1	-2	B1 M1 A1 [3]	2.1 1.1 2.2a	Col maxima calculated (or negatives of these values) (at least as far as showing that col X has col max = 2) Ignore any other working Either of these seen, may be implied from b < 2 or b ≤ 2 seen Accept 0 < b < min(2, c)				
	X	Y	Z																						
P	2	-3	c																						
Amir Q	-3	b	4																						
R	a	-1	-2																						
3	(c)	Stable ⇒ min for row R = max for col Y so min(a, -2) = b But min(a, -2) < 0 and b > 0 so cannot be equal Hence unstable	M1 M1d A1	3.4 1.1 2.2a	Interpret stability in terms of min for row R and max for col Y Using (a, -2) and b Explain why condition fails, accept a < 0 < b Written descriptions must be unambiguous Hence not stable, from correct explanation seen																				

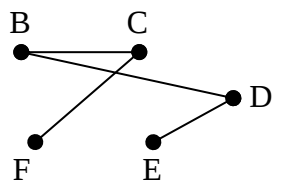
Question		Answer	Marks	AO	Guidance																
3	(c)	Alternative method 1 Stable $\Rightarrow -1 = \min(a, -2)$ But $\min(a, -2) \leq -2$, so $\neq -1$ Hence unstable	M1 M1d A1		Interpret stability in terms of entry in cell (R, Y) = -1 Or Stable $\Rightarrow -1 = b$ Or But $b > 0$, so $\neq -1$ Explain why condition fails Hence not stable, from correct explanation seen																
		Alternative method 2 <table border="1"><tr><td></td><td>X</td><td>Y</td><td>Z</td></tr><tr><td>P</td><td>2, -2</td><td>-3, 3</td><td>c, -c</td></tr><tr><td>Q</td><td>-3, 3</td><td>b, -b</td><td>4, -4</td></tr><tr><td>R</td><td>a, -a</td><td>-1, 1</td><td>-2, 2</td></tr></table> Hence unstable		X	Y	Z	P	2, -2	-3, 3	c, -c	Q	-3, 3	b, -b	4, -4	R	a, -a	-1, 1	-2, 2	M1 M1d A1		Finding Nash equilibrium cells Need not explicitly show pay-offs for Beth Amir: (P, X), (Q, Y), (P, Z), (Q, Z) depending on value of c Beth: (P, Y), (Q, X), (R, X), (R, Z) depending on value of a (R, Y) is not a Nash equilibrium Hence not stable, from correct explanation seen
			X	Y	Z																
		P	2, -2	-3, 3	c, -c																
		Q	-3, 3	b, -b	4, -4																
R	a, -a	-1, 1	-2, 2																		
Alternative method 3 e.g. If Beth plays safe and chooses Y then Amir's best choice is Q But Q is not Amir's play-safe strategy Hence unstable	M1 M1d A1		'Chasing' best options May use an informal approach with at least first step $Y \rightarrow Q$ or $R \rightarrow$ 'X or Z (depending on value of a)' condone $R \rightarrow Z$ Written descriptions must be unambiguous Or equivalent, or similarly for Beth, or (R, Y) is not a Nash equilibrium Hence not stable, from correct explanation seen																		
Alternative method 4 Minimax for columns is $\min(2, b) > 0$ Maximin for rows is $\max(-3, \min(a, -2)) < 0$ So minimax $\neq$ maximin, hence unstable	M1 M1 A1		Using minimax and maximin for whole table Minimax (2, b) or b and > 0 o.e. Maximin (-3, a , -2) or (a , -2) < 0 o.e. Hence not stable, from correct explanation seen																		
			[3]																		

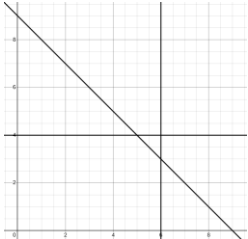
Question		Answer	Marks	AO	Guidance																
4	(a)	 Minimum project completion time = 9 hours	B1	1.1	Forward pass unambiguously placed at vertices (ignore ends)																
			B1	1.1	9 cao																
			[2]																		
4	(b)	 $Float_{ij} = LET_j - EET_i - D_{ij}$ <table><tr><th>Activity</th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th><th>G</th></tr><tr><th>Float (hours)</th><td>0</td><td>2</td><td>0</td><td>2</td><td>2</td><td>0</td><td>3</td></tr></table>	Activity	A	B	C	D	E	F	G	Float (hours)	0	2	0	2	2	0	3	B1	1.1	Backward pass unambiguously placed at vertices (ignore ends)
Activity	A	B	C	D	E	F	G														
Float (hours)	0	2	0	2	2	0	3														
			B1	1.1	Float 0 for A, C and F																
			M1	1.1	Float correct for any two of B, D, E, G Or from their EET and LET																
			A1	1.1	Float correct for B, D, E and G																
			[4]																		
4	(c)	Independent $float_{ij} = \max(EET_j - LET_i - D_{ij}, 0)$ Interfering float = float – independent float <table><tr><th>Activity</th><th>B</th><th>D</th><th>E</th><th>G</th></tr><tr><th>Independent float (hours)</th><td>0</td><td>0</td><td>0</td><td>1</td></tr><tr><th>Interfering float (hours)</th><td>2</td><td>2</td><td>2</td><td>2</td></tr></table>	Activity	B	D	E	G	Independent float (hours)	0	0	0	1	Interfering float (hours)	2	2	2	2	M1	1.1	At least 3 correct or ft their early event times and late event times	
Activity	B	D	E	G																	
Independent float (hours)	0	0	0	1																	
Interfering float (hours)	2	2	2	2																	
			M1	1.1	At least 3 correct or ft their float and independent float times																
			A1	2.2a	All correct cao Ignore critical activities if given (with both types of float = 0)																
			[3]																		

Question		Answer	Marks	AO	Guidance																						
4	(d)	 0 1 2 3 4 5 6 7 8 9 hours	B1	1.1	Critical activities A, C, F labelled and correct early event times and durations (on a single row or accept them on separate rows) May use other representations																						
			M1	1.1	Non-critical activities B, D, E, G labelled and correct early event times and durations (i.e. the solid line boxes), may use other representations																						
			A1	1.1	Float correct, lasting until late event times (i.e. dashed boxes), may use other representations																						
			[3]																								
4	(e)	e.g. <table border="1" data-bbox="369 818 983 895"><tr><td>Worker 1</td><td>A</td><td>A</td><td>C</td><td>C</td><td>C</td><td>C</td><td>C</td><td></td><td>F</td><td>F</td></tr><tr><td>Worker 2</td><td>B</td><td>B</td><td>B</td><td>B</td><td>D</td><td>D</td><td>D</td><td>E</td><td>G</td><td></td></tr></table>	Worker 1	A	A	C	C	C	C	C		F	F	Worker 2	B	B	B	B	D	D	D	E	G		M1	1.1	A, B, C, D and E positioned appropriately, with correct durations and precedences
			Worker 1	A	A	C	C	C	C	C		F	F														
			Worker 2	B	B	B	B	D	D	D	E	G															
A1	2.2a	F, G positioned appropriately, with correct durations and precedences (F must follow the completion of both D and E) <u>and</u> project is completed in 10 hours (ends at 9-10)																									
[2]																											
4	(f)	e.g. <table border="1" data-bbox="369 1126 983 1204"><tr><td>Worker 1</td><td>A</td><td>A</td><td>C</td><td>C</td><td>C</td><td>C</td><td>C</td><td></td><td>F</td><td>F</td></tr><tr><td>Worker 2</td><td>B</td><td>B</td><td>B</td><td>B</td><td>E</td><td>D</td><td>D</td><td>D</td><td>G</td><td></td></tr></table>	Worker 1	A	A	C	C	C	C	C		F	F	Worker 2	B	B	B	B	E	D	D	D	G		M1	3.1b	D starts at time 5 (i.e. 5-6)
			Worker 1	A	A	C	C	C	C	C		F	F														
			Worker 2	B	B	B	B	E	D	D	D	G															
A1	2.2a	All positioned appropriately, with correct durations and precedences, project is completed in 10 hours (ends at 9-10)																									
[2]																											

Question		Answer	Marks	AO	Guidance
5	(a)	Nearest neighbour can stall	B1 [1]	2.5	e.g. may not find a cycle or may not find least weight cycle BOD descriptions that do not close the cycle Accept valid descriptions concerning the structure of the graph but not descriptions using hypothetical weights
5	(b)	Ad hoc method is likely to find an acceptable solution and is quicker than checking every possible cycle	B1 [1]	2.4	Solution is good enough and method is quick Accept 'quicker' or 'easier' or 'more efficient', o.e. Or any of the following o.e. Algorithmic approach is likely to involve an exhaustive check Algorithmic approach finds (upper and lower) bounds only (not necessarily optimal solution) No known algorithm (currently) (for TSP)
5	(c)		M1 A1 [2]	1.1 1.1	Correct arcs (i.e graph correct) At most one missing or extra arc Ignore any working for part (f) Correct network, i.e. correct graph weighted correctly, with no errors. Allow ambiguous placing if plausibly correct
5	(d)	A – D – B – C – F Stalls before visiting E	M1 A1 [2]	1.1 1.1	A – D – B – C – F No E or 'stall' or 'stop' or other explicit evidence that n.n. does not reach every vertex

Question		Answer	Marks	AO	Guidance																								
5	(e)	e.g. A – B – C – F – C – E – D – A Length = 39(km)	M1 A1 [2]	3.4 3.4	Any closed route through all six vertices, using only arcs from QP Correct length of their route, provided it is less than 50 (km)																								
5	(f)	Dijkstra's algorithm starting from F A <table border="1"><tr><td>6</td><td>17</td></tr><tr><td colspan="2">17</td></tr></table> B <table border="1"><tr><td>3</td><td>11</td></tr><tr><td colspan="2">11</td></tr></table> C <table border="1"><tr><td>2</td><td>6</td></tr><tr><td colspan="2">6</td></tr></table> D <table border="1"><tr><td>4</td><td>13</td></tr><tr><td colspan="2">21 13</td></tr></table> E <table border="1"><tr><td>5</td><td>13</td></tr><tr><td colspan="2">13</td></tr></table> F <table border="1"><tr><td>1</td><td>0</td></tr><tr><td colspan="2"></td></tr></table> F to B = 11 (km) and F to C = 6 (km) F to A = 17, F to D = 13, F to E = 13 (km)	6	17	17		3	11	11		2	6	6		4	13	21 13		5	13	13		1	0			M1* A1 M1d* A1 A1 [5]	3.1a 1.1 1.1 1.1 1.1	Evidence of using Dijkstra's algorithm May be seen on diagram from part (c) Need not use box format Order of labelling not necessary Using F as the starting vertex Evidence of updating (at D if starting from F but e.g. at C if starting from A) Shortest distance F to B = 11, F to C = 6 Correct in a list or table or seen as permanent labels, o.e. Shortest distances F to A = 17, F to D = 13, F to E = 13 Correct in a list or table or seen as permanent labels, o.e. SC If M0 for dependent method mark (i.e. at best M1 A1 M0) then SCB1 for all distances correct F to A = 17, F to B = 11, F to C = 6, F to D = 13, F to E = 13
6	17																												
17																													
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11																													
2	6																												
6																													
4	13																												
21 13																													
5	13																												
13																													
1	0																												

Question		Answer	Marks	AO	Guidance																																										
5	(g)	A B C D E F <table border="1" style="border-collapse: collapse; text-align: center;"> <thead> <tr> <th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr> </thead> <tbody> <tr> <td></td><td>6</td><td>11</td><td>4</td><td>9</td><td>17</td></tr> <tr> <td>6</td><td></td><td>5</td><td>2</td><td>7</td><td>11</td></tr> <tr> <td>11</td><td>5</td><td></td><td>7</td><td>7</td><td>6</td></tr> <tr> <td>4</td><td>2</td><td>7</td><td></td><td>5</td><td>13</td></tr> <tr> <td>9</td><td>7</td><td>7</td><td>5</td><td></td><td>13</td></tr> <tr> <td>17</td><td>11</td><td>6</td><td>13</td><td>13</td><td></td></tr> </tbody> </table>	A	B	C	D	E	F		6	11	4	9	17	6		5	2	7	11	11	5		7	7	6	4	2	7		5	13	9	7	7	5		13	17	11	6	13	13		B1 B1 [2]	 2.1 3.1b	Ignore errors or omissions in row F and column F Lead diagonal may be blank or '0' or '-' or similar, but not contain any non-zero values Bold values correct (unshaded values excluding row F and col F) All shaded values correct
A	B	C	D	E	F																																										
	6	11	4	9	17																																										
6		5	2	7	11																																										
11	5		7	7	6																																										
4	2	7		5	13																																										
9	7	7	5		13																																										
17	11	6	13	13																																											
5	(h)	e.g. BC = 5 BD = 2 DE = 5 CF = 6  AD = 4 and AB = 6 (lower bound) $18 + 10$ = 28	M1 M1 A1 [3]	 3.4 2.2a 2.2a	Any spanning tree for {B, C, D, E, F} or {A, B, C, D, E, F} (need not be MST) Arcs may be listed (in any order) or tree shown as a diagram Weights are not necessary here Use two (different) least weight arcs from A, for example see AB and AD (and no others) or see 4+6 or see +10 or implied from final answer 28 28 cao from correct MST for {B, C, D, E, F} or {A, B, C, D, E, F}																																										

Question	Answer	Marks	AO	Guidance
6	(a) $12 - a - b$ red beads (in each bracelet) Max $P = 3\{6a + 2b + 3(12 - a - b)\}$ pence $= 9a - 3b$ (+ constant) subject to $a \leq 20/3 \Rightarrow a \leq 6, b \leq 12/3 = 4$ $12 - a - b \leq 10/3 \Rightarrow a + b \geq 9$ a, b integers	B1 M1 A1 B1 B1 [5]	3.1b 3.4 1.1 3.3 3.3	$12 - a - b$ red beads per bracelet, seen or implied from P expression Reasonable attempt at objective in terms of these 2 variables only Or a positive multiple with like terms collected Correct integer-valued upper bounds for a and b Or correct with fractions or decimals ($3a \leq 20, 3b \leq 12$, o.e.) <u>and</u> statement 'a, b integers' o.e. seen Correct integer-valued lower bound for $a + b$ Or correct with fractions or decimals ($3a + 3b \geq 26$, o.e.) <u>and</u> statement '$a+b$ integer valued' or 'a, b integers' o.e. seen Ignore inequality $a + b \leq 12$ (from red ≥ 0, but is redundant) Ignore $a \geq 0, b \geq 0$ if seen (redundant)
6	(b) $(a, b) = (5, 4)$ or $(6, 3)$ or $(6, 4)$  6 amber beads, 3 brown beads, 3 red beads	M1 A1 A1 [3]	1.1 1.1 3.2a	List (at least one) feasible or near feasible (a, b) pair (within ± 1 for each) or sketch constraints May be implied from final answer Calculating number of red beads ($= 12 - a - b$) for their feasible or near feasible (a, b) pair (within ± 1 for each) with a, b and $12 - a - b$ non-negative integers Interpret this as optimal solution for all three colours, using colour names or initial letters, or variable for red defined, cao. Not statement $12 - a - b = 3$ without 'red' or r, o.e.

Question		Answer	Marks	AO	Guidance
6	(c)	$(a, b, r) = (5, 4, 3)$ or $(6, 4, 2)$ or $(6, 3, 3)$ Optimal solution only uses 3 brown beads in each bracelet	M1 A1 [2]	1.1 2.2a	Giving all three (and only these) feasible solutions, in any way Identifying any (known) difference between optimal solution $(6, 3, 3)$ and both of the others, apart from the profit
6	(d)	$(6, 3, 3)$ gives profit 153 pence and leaves 2 amber beads (and 3 brown and 1 red) So total profit is $153 + 2k$ (pence) $(6, 4, 2)$ leaves same number of amber beads as $(6, 3, 3)$ and profit from bracelets is less so total profit is less $(5, 4, 3)$ leaves 5 amber beads (and 1 red) and gives total profit $141 + 5k$ (pence) $141 + 5k > 153 + 2k \Rightarrow k > 4$, min $k = 5$ When $k = 5$, total profit is 166 (pence) Least possible value of Sasha's maximum total profit is 166 pence	M1 A1 A1 M1 A1 [5]	3.4 3.4 3.4 3.1a 2.2a	Calculating number of amber beads remaining for any feasible solution (i.e. 2 amber beads for either $a = 6$ solution or 5 amber beads for $a = 5$ solution, may be implied by sight of $2k$ or $5k$ Explaining why $(6, 4, 2)$ cannot be optimal $(6, 4, 2)$ leaves 2 amber beads (and 4 red) and gives total profit $150 + 2k < 153 + 2k$ May be implied from $153 + 2k$ as the only total profit for $a = 6$ Calculating total profit $141 + 5k$ Comparing 'their' total profits to find a min for k Or implied from sight of $k > 4$ or $k = 4$ or (min) $k = 5$ 166 (pence may be implied) or £1.66
6	(e)	e.g. May not sell all the items made	B1 [1]	3.5b	Any valid reason that does not involve the numerical values of the profits or the number of beads used or the costs e.g. May not have enough time to make all the items People may not want to purchase beads separately

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