GCE

Further Mathematics B MEI

Y436/01: Further pure with technology

A Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking.*

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

   a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

   For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

   It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   Award NR (No Response)
   -       if there is nothing written at all in the answer space and no attempt elsewhere in the script
   -       OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
   -       OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.
   Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

   If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

   b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

   If you are in any doubt whatsoever you should contact your Team Leader.

   c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
*   When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
*   When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
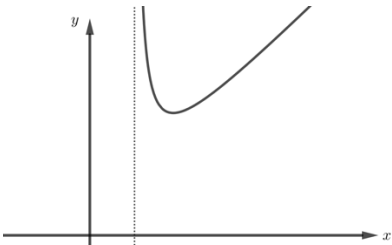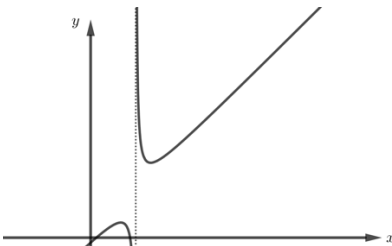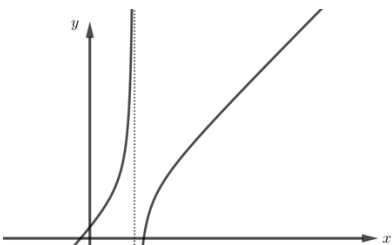   NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.
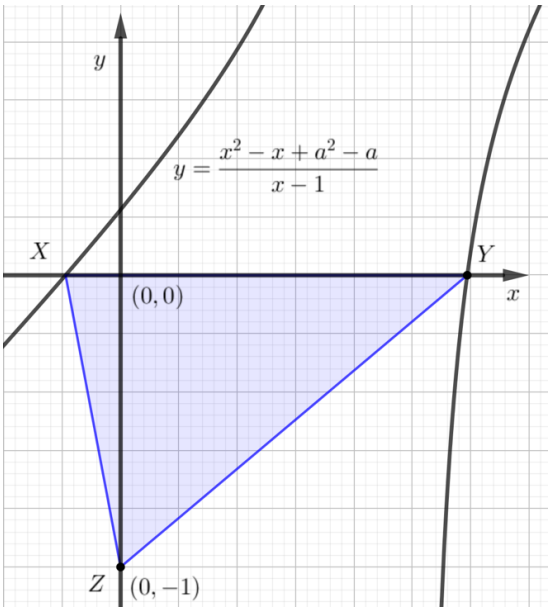
g. Rules for replaced work and multiple attempts:
*   If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
*   If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.

- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)	(i)	$a = -0.5$	B1	1.1	Maximum and minimum in correct quadrants. Two parts to curve separated by vertical asymptote. Condone not labelling axis intersection and not labelling asymptotes.
			$a = -0.1$	B1	1.1	Maximum and minimum in correct quadrants. Two parts to curve separated by vertical asymptote. Condone not labelling axis intersection and not labelling asymptotes.
			$a = 0.5$	B1	1.1	Two parts to curve separated by vertical asymptote. No maximum or minimum. Condone not labelling axis intersection and not labelling asymptotes. Condone not including oblique asymptotes. If included, sketch should be compatible with its inclusion.

Question			Answer	Marks	AO	Guidance
				[3]		
1	**(a)**	**(ii)**	The curves for the cases $a = -0.5$ and $a = -0.1$ have turning points.	**B1**	1.1	Allow stationary point rather than turning point. Allow the curves for the cases $a = -0.5$ and $a = -0.1$ approach the oblique asymptote $y = x$ from above, whereas the curve for the case $a = 0.5$ approaches from below. Allow the curves for the cases $a = -0.5$ and $a = -0.1$ are in the positive quadrant (or equivalent) to the right of the vertical asymptote, whereas the curve for the case $a = 0.5$ does not.
				[1]		
1	**(a)**	**(iii)**	$a = 1$	**B1**	1.1	
				[1]		
1	**(a)**	**(iv)**	$x = 1$	**B1**	1.1	Must be an equation.
				[1]		
1	**(a)**	**(v)**	$y = x + A + \dfrac{a^2 - a}{x - 1}$ $= \dfrac{(x + A)(x - 1) + a^2 - a}{x - 1}$ Equating coefficients gives	**M1**	1.1	Any valid method including CAS.
			$A - 1 = -1$. So $A = 0$.	**A1**	1.1	Alternative working may lead to $A + a^2 - a = a^2 - a$. or $x^2 + (A - 1)x - A = x^2 - x$ and comparing coefficients.
				[2]		
1	**(a)**	**(vi)**	By **(a)(v)**	**B1**	1.1	Value may be found using CAS so may be implied by correct final answer.
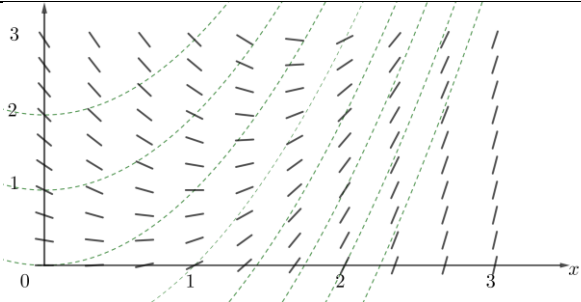
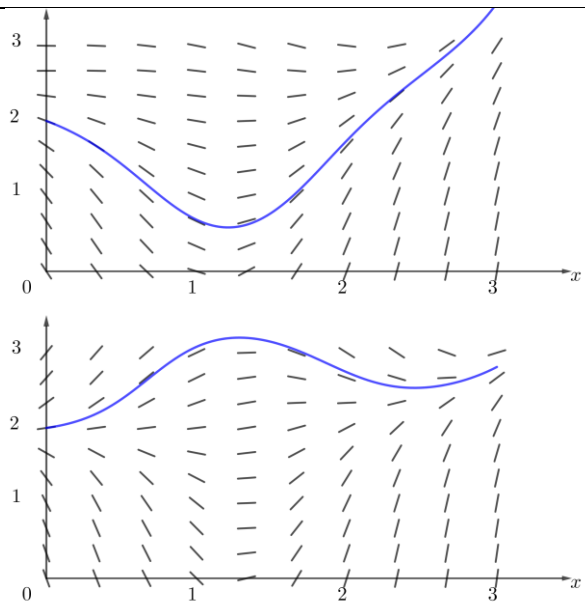| Question			Answer	Marks	AO	Guidance
			$\lim_{x \to \infty} \left(\dfrac{x^2 - x + a^2 - a}{x - 1} - x \right)$ $= \lim_{x \to \infty} \left(x + \dfrac{a^2 - a}{x - 1} - x \right)$ $= \lim_{x \to \infty} \left(\dfrac{a^2 - a}{x - 1} \right)$ $= 0.$	**B1** **[2]**	**1.1**	
1	**(a)**	**(vii)**	$y = x$ is an oblique asymptote [as x approaches infinity].	**B1** **[1]**	**1.2**	Allow the curve tends towards the line $y = x$ as x approaches infinity. Condone not considering behaviour as x approaches minus infinity explicitly.

Question			Answer	Marks	AO	Guidance
1	**(b)**	**(i)**	A diagram is			Diagram not required.
			Intersection with x-axis given by $$\frac{x^2 - x + a^2 - a}{x - 1} = 0$$	**M1(soi)**	**3.1a**	Maybe implied by use of CAS or by correct statements of both X and Y coordinates.
			X has coordinates $$\left(\frac{1}{2} - \frac{\sqrt{1 + 4a - 4a^2}}{2}, 0\right)$$	**A1**	**1.1**	Statement in terms of coordinates is not required.
			Y has coordinates $$\left(\frac{1}{2} + \frac{\sqrt{1 + 4a - 4a^2}}{2}, 0\right)$$	**A1**	**1.1**	Statement in terms of coordinates is not required. May see both points stated as "one" equation or implied by base calculation. Condone this approach.
			Area is $$\frac{1}{2} \times \sqrt{1 + 4a - 4a^2} \times 1.$$	**A1**	**2.2a**	Correct use of base and height of triangle to give area. May see $\sqrt{\frac{1}{4} + a - a^2}$ as a final answer.

Question			Answer	Marks	AO	Guidance
				[4]		
1	**(b)**	**(ii)**	Differentiate $$\frac{1-2a}{\sqrt{1+4a-4a^2}}.$$ So $$a = \frac{1}{2}$$	**B1FT**	**3.1a**	Use of CAS is expected to differentiate or maximise the area. May be implied by correct answer. Alternative. $\frac{1}{2} \times \sqrt{1+4a-4a^2} = \sqrt{\frac{1}{2} - \left(a - \frac{1}{2}\right)^2}$. Since $\left(a - \frac{1}{2}\right)^2 \geqslant 0$ for all a. $\sqrt{\frac{1}{2} - \left(a - \frac{1}{2}\right)^2} \leqslant \sqrt{\frac{1}{2}}$ for all a. Maximum occurs when $a = \frac{1}{2}$
			Maximum area $\frac{\sqrt{2}}{2}$.	**B1** **[2]**	**1.1**	Or equivalent. Allow follow through from **1(b)(i).**

Question			Answer	Marks	AO	Guidance
2	(a)	(i)	136	B1 [1]	1.1	May see math.factorial(136!) % 137
2	(a)	(ii)	137 is prime by part **2(a)(i)** and Wilson's theorem.	B1 [1]	1.1	Must include clear statement regarding primality of 137.
2	(b)	(i)	Appropriate structured program	M1	3.3	Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors based on correct answer to 2**(b)(ii).** Program must include code to implement Wilson's theorem. This may include use of math.factorial or direct computation of factorial.
			Loop for j up to $n + 1$. Check for solution with if statement and print result.	A1	2.1	

			Fully correct programme.	A1 [3]	2.5	Example Python code
						def wlistprimes(n):
						  for j in range (2, n +1):
						    if (math.factorial(j - 1) + 1) % j == 0:
						      print(j)
						Must not produce 1 as a result.

2	(b)	(ii)	263, 269, 271, 277, 281, 283 and 293.	B1 [1]	1.1	
2	(c)	(i)	2 is the only even prime number.	B1 [1]	2.2a	
2	(c)	(ii)	A number congruent to 0 (mod 4) is divisible by 4 and hence not prime.	B1 [1]	2.2a	

2	(c)	(iii)	263, 271 and 283.	B1 [1]	1.1	
2	(c)	(iv)	N leaves a remainder 3 when divided by c_1, c_2 or c_3.	B1 [1]	2.2a	
2	(c)	(v)	3	B1 [1]	1.1	
2	(d)	(i)	3	B1 [1]	2.2a	Accept -1.
2	(d)	(ii)	M is a positive integer so by the fundamental theorem of arithmetic M can be written uniquely as the product of powers of prime numbers.	B1	1.2	
			Expressing each power of a prime as a product gives the required form.	B1 [2]	2.4	It is sufficient to state that the q_is are not necessarily distinct.
2	(d)	(iii)	For a contradiction suppose $q_i \equiv 1 \pmod 4$ for all integers i where $1 \le i \le r$.	M1	3.1a	Discussion of $M > 2$ not required. May also see consideration of $q_i = 2$. Since M is odd, the case $q_i \equiv 2 \pmod 4$ is not possible. Condone no discussion of this point.
			Then $M \equiv q_1 \cdots q_r \pmod 4$ $\equiv 1 \times \cdots \times 1 \pmod 4$ $\equiv 1 \pmod 4$.	A1	2.1	
			But $M \pmod 4 = 3$.	A1	1.1	
			This is a contradiction so $q_i \not\equiv 1 \pmod 4$ for all integers i where $1 \le i \le r$.	A1 [4]	2.2a	Or equivalently, this is a contradiction, so $q_i \equiv 3 \pmod 4$ for some integer i where $1 \le i \le r$.

Question			Answer	Marks	AO	Guidance
2	(d)	(iv)	By (d)(iii) at least one q_i satisfies $q_i \equiv 3(\text{mod } 4)$ and (d)(ii) implies $M \equiv 0 \ (\text{mod } q_i)$.	B1	3.1a	
			By assumption $p_1, \dots, p_n$ is a complete list of primes congruent to 3 (mod 4).	B1	1.1	Seen or implied use of this assumption from the question stem.
			[Since M is not divisible by 3,] this q_i must be one of the $p_2, \dots, p_n$.	B1	2.2a	May see an equivalent argument that none of the q_is is in the list of the p_js.
			So $M \equiv 4p_2 \dots p_n + 3 \ (\text{mod } q_i)$ $\equiv 3(\text{mod } q_i)$.	B1	2.1	
			This is a contradiction to $M \ (\text{mod } q_i) \equiv 0$ so there are infinitely many primes congruent to $3(\text{mod } 4)$	B1	2.1	If making an argument using the fact that none of q_is is in the list of the p_js then will be able to assert the existence of a new unlisted prime congruent to $3(\text{mod } 4)$.
				[5]		

Question			Answer	Marks	AO	Guidance
3	(a)	(i)	$y = x^2 - 2x + 2 - e^{-x}$.	B1 [1]	1.1	Accept solutions by CAS.
3	(a)	(ii)	$\frac{dy}{dx} = m \Leftrightarrow x^2 - y = m$. Which is the equation of a parabola.	B1 [1]	2.1	
3	(a)	(iii)		M1 A1 [2]	1.1 1.1	Sketch of isoclines (dotted lines) not required. Correct slopes close to both axes and gradients approximately constant along parabolic curves. Fully correct sketch allowing reasonable BOD for transcription.

Question			Answer	Marks	AO	Guidance
3	(b)	(i)		B1	1.1	General shape with a minimum. Curve should be tangent to the tangent field. Do not award mark if curve crosses a tangent field line in a manner that place the curve in conflict with the tangent field. That is the presented curve is qualitatively impossible.
				B1 [2]	1.1	General shape with a maximum. Curve should be tangent to the tangent field. Do not award mark if curve crosses a tangent field line in a manner that place the curve in conflict with the tangent field. That is the presented curve is qualitatively impossible.
3	(b)	(ii)	e.g. Local minimum close to $x = 1$	B1 [1]	1.1	Alternatives. **Fig 3.1.** has a single turning point close to $x = 1$, while **Fig. 3.2** has two turning points. In **Fig. 3.2** has a maximum close to $x = 1$ and **Fig. 3.1** does not. In **Fig. 3.1.** the function decreases from $y = 2$ [before increasing]. **Fig. 3.1.** has a point of inflection.
3	(b)	(iii)	$a = -5.5$	B1 [1]	2.2b	Accept $-6 \leq a \leq -4.5$.

Question			Answer	Marks	AO	Guidance
3	(c)	(i)				Give reasonable BOD for possible transcription errors and consider correct answer to **3(c)(ii)** as evidence of correct formulae in the spreadsheet.
			A2 contains 0	**B1**	**3.1a**	Allow for a and h to be varied.
			B2 contains 1			Must reference a cell, a slider or equivalent formulation.
			I1 contains 0.1 (value of h)			
			K1 contains 0 (value of a)			
			C2 =A2^2-B2+$K$1*COS(A2)*COS(B2)	**B1**	**2.4**	Formulae for x_{n+1} and y_{n+1}.
			D2 =$I$1*C2			Allow the specification of "C2" using function notation.
			A3 =A2+$I$1			
			B3=B2+D2			
				[2]		
3	(c)	(ii)	Spreadsheet gives 0.618559	**B1**	**1.1**	Answer must be given to at least 4 decimal places 0.6186 given the context of spreadsheet working.
				[1]		
3	(c)	(iii)	Actual solution is 0.64346934.	**B1**	**3.2b**	Must see comparison to actual solution value in part **(a)(i).**
			So accurate to 1 decimal place.			Accept relative accuracy of approximately 4%.
						Accept absolute value of difference of approximately 0.0249 (4 decimal places).
				[1]		
3	(c)	(iv)	Use a smaller value of h.	**B1** **[1]**	**3.5c**	Also accept a named alternative method such as modified Euler.

Question			Answer	Marks	AO	Guidance
3	(d)	(i)				Give reasonable BOD for possible transcription errors and consider correct answer to **3(d)(ii)** as evidence of correct formulae in the spreadsheet. Allow for transcription of only changes necessary to **3(c)(i)**.
			A2 contains 0 B2 contains 1 I1 contains 0.1 (value of h) K1 contains -0.5 (value of a) C2=A2^2 - B2 + \$K\$1*COS(A2)*COS(B2) D2 =\$I\$1*C2 (value of k_1) E2= A2+\$I\$1 F2=B2+D2 G2=\$I\$1*(E2^2- F2+\$K\$1*COS(E2)*COS(F2)) (value of k_2)	**B1**	3.1a	Columns for k_1 and k_2. May see k_1 and k_2 included explicitly within other formulae.
			A3=A2+\$I\$1 B3=B2+(1/2)*(D2+G2)	**B1**	2.4	Formulae for x_{n+1} and y_{n+1}. Allow the specification of "C2" and "G2" using function notation. Allow follow through from part **(c)(i).**
				[2]		
3	(d)	(ii)	Spreadsheet gives 0.504226904	**B1**	1.1	Answer must be given to at least 4 decimal places 0.5042 given the context of spreadsheet working.
				[1]		
3	(d)	(iii)	With $h = 0.1$ there is a sign change in $\frac{\mathrm{d}y}{\mathrm{d}x}$ between $x = 0.8$ and $x = 0.9$.	**M1**	3.4	Must see evidence that h is being reduced to produce better estimates.
			Reducing to $h = 0.01$ shows that $0.83 < c < 0.84$. So $c = 0.8$ to 1 decimal place.	**A1**	2.2b	**SC B1** for correct value of c with no working.

| 3 | (d) | (iv) | From the spreadsheet with $h = 0.01$
 $0.39218291 < d < 0.392334706$
 So $d = 0.392$ to 3 decimal places. | **[2]**
 M1

 A1
 [2] | **3.4**

 2.2b | Note $h = 0.1$ is insufficient to allow an estimate of d to 2 decimal places.
 SC B1 for correct value of c with no working. |

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