GCE

Mathematics A

H240/02: Pure Mathematics and Statistics

A Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING

RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

   If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

   M
   A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
   A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   B
   Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		$12x^3 + 4x^{-3}$ oe	B2 [2]	1.1 1.1	B1 each term, isw incorrect attempts to simplify
1	(b)		$2x^{-\frac{1}{2}}$ oe	B2 [2]	1.1 1.1	B1 this term correct B1 no second term

Question			Answer	Marks	AO	Guidance
2			$a^2 + b^2 = 36$	M1*	3.1a	Attempt at Pythagoras =36 oe, may see $\sqrt{a^2 + b^2} = 6$
			$\frac{b}{a} = \sqrt{3}$	M1*	1.2	oe or $\frac{b}{a} = \tan 60$ or $\frac{b}{a} = 1.73$, condone $\frac{a}{b}$ May see equivalent statements in sin or cos.
			$a^2 + 3a^2 = 36$ or $\frac{b^2}{3} + b^2 = 36$	M1 dep*	1.1	Substitute both expressions (dependent on **both** previous M marks) – must reach an equation in a or b only.
			$a = 3, b = 3\sqrt{3}$	A1	1.1	A0 for negative answers (if not disregarded) Accept $b = \sqrt{27}$ If no (or insufficient) working then SC **B1B1** (max 2/4) for each correct answer (must be exact).
			Alternative method			
			$(a =) 6\cos 60$	M1		Allow this mark for 6cos or 6sin of 30,60,120° Must see this step oe.
			$a = 3$	A1		
			$(b =) 6\sin 60$	M1		Allow this mark for 6cos or 6sin of 30,60,120° provided it is consistent with their other expression (i.e. not the same). Must see this step oe.
			$b = 3\sqrt{3}$	A1		If no (or insufficient) working then SC **B1B1** (max 2/4) for each correct answer (must be exact).
				[4]		

Question			Answer	Marks	AO	Guidance
3	(a)		$f(3) = 3^3 - 3^2 - 5 \times 3 - 3 = 0$	B1 [1]	1.1	Must see substitution; *or* factorise showing factor $(x - 3)$ No conclusion required beyond =0 (so e.g. accept long division leading to remainder 0).
3	(b)		$(x - 3)(px^2 + qx + r)$ $(x - 3)(x^2 + 2x + 1)$ $(x - 3)(x + 1)(x + 1)$	M1 A1 [2]	3.1a 1.1	Attempt quadratic factor by inspection or division. Look for $(x - 3)$ and a quadratic or two linear factors. NB this step may be seen in (a). or $(x - 3)(x + 1)^2$
3	(c)		Fig 1.1: no value (of a) Fig 1.2: (a =) 2 Fig 1.3: (a =) 1	B1 B1 B1 [3]	2.2a 2.2a 2.2a	 Accept values between 1.9 and 2.1 Accept values between 0.9 and 1.1

Question			Answer	Marks	AO	Guidance
4	(a)		$x\mathrm{e}^{1-3x} - 0.2$ evaluated for any two values of x that give results with opposite signs.	M1	3.1a	Values of iterates not needed, i.e. condone <0 and >0 etc. but signs (and values if given) must be correct to 1sf – see table.
			$x = 0.79$ (2dp)	B1	1.1	cao (Allow this mark even with no/insufficient working).
			($x = 0.79$ to 2 dp) because the change of sign occurs between 0.785 and 0.795.	A1	2.2a	By showing two values between 0.785-0.795 with opposite signs Values of iterates not needed, i.e. condone <0 and >0 etc, but if given must be correct to 1sf e.g. $x = 0.79 \Rightarrow y = 0.0007$ **and** $x = 0.795 \Rightarrow y = -0.0009$
				[3]		(see table below) (Note that exact root is 0.792…)

x	$\boldsymbol{x\mathrm{e}^{1-3x} - 0.2}$
0.1	0.00138
0.5	0.10327
0.7	0.03301
0.75	0.01488
0.78	0.00424
0.785	0.00249
0.786	0.00214
0.787	0.00179
0.788	0.00144
0.789	0.00109
0.79	0.00074
0.791	0.00040
0.792	0.00005
0.793	-0.00030
0.794	-0.00065
0.795	-0.00099
0.796	-0.00134
0.797	-0.00169
0.798	-0.00203
0.799	-0.00238
0.8	-0.00272
1	-0.06466

(Note that exact root is 0.792…)

Question			Answer	Marks	AO	Guidance
4	(b)		$\frac{dy}{dx} = e^{1-3x} + x(-3)e^{1-3x} (= 0)$	M1	3.1a	For an attempt at differentiating using the product rule, with at least one term correct (their derivative must have two terms).
			e^{1-3x} is never 0, hence can divide by it $(e^{1-3x}(1-3x) = 0)$ $(\rightarrow 1-3x = 0)$	B1	2.1	May be implied by not giving a corresponding solution (provided another solution for x is reached). (NB this mark can be gained following M0 provided their derivative has e^{1-3x} as a factor)
			$x = \frac{1}{3}$	A1 [3]	1.1	Not decimal, www (i.e. must come from a correct derivative)
4	(c)		**DR**			
			$\int_0^1 xe^{1-3x}dx = \left[x\frac{e^{1-3x}}{-3}\right]_0^1 - \int_0^1 \frac{e^{1-3x}}{-3}dx$ oe	M1	3.1a	M1 for attempting integration by parts with at least one term correct, must see limits (condone swapped limits, may appear later). Must have two terms.
				A1	2.1	A1 for both terms correct
			$= \left[x\frac{e^{1-3x}}{-3}\right]_0^1 - \left[\frac{e^{1-3x}}{9}\right]_0^1$	A1	2.1	A1 for both terms correct after 2nd integral performed
			$= \frac{1}{9}e - \frac{4}{9}e^{-2}$ or $\frac{e^3-4}{9e^2}$ oe	A1 [4]	1.1	Need not be simplified, isw any incorrect attempts to simplify

Question			Answer	Marks	AO	Guidance
5	(a)		(The rate of change with respect to time is $\frac{dP}{dt}$), which is $-\frac{1}{80}P^2$ because the population is in **decline**.	B1 [1]	3.3	Must see use of **decrease/decline** linked to the negative sign. Condone answers that do not refer to $\frac{dP}{dt}$ Examples: • "it's $-\frac{1}{80}P^2$ because decreasing" B1 • "decreasing" [and nothing further] B0 • "decline implies negative sign" B1

Question			Answer	Marks	AO	Guidance
5	(b)		$\frac{dP}{P^2} = -\frac{dt}{80}$ oe	M1*	3.4	Attempt to separate variables, must see P and dP on the same side May see $\frac{dt}{dP} = \frac{-80}{P^2}$ as a first step. In this case M1 is implied by a subsequent attempt to integrate the RHS w.r.t. P (so do not give M1 for this statement alone).
			$-\frac{1}{P} = -\frac{t}{80}(+c)$ oe	A1	1.1	May see $\frac{1}{P} = \frac{t}{80}(+c)$ etc. Allow without $+c$
			$c = -\frac{1}{120}$ oe $\left(\frac{1}{P} = \frac{3t+2}{240}\right)$	M1 dep*	2.1	Substitute (0, 120) and attempt to find c (which may not be this value). Must reach a value of c for this mark.
			$P = \frac{240}{3t+2}$ oe	A1 [4]	1.1	Must be in terms of P (P=…) but need not be simplified. isw incorrect attempts to simplify following P=…
5	(c)		$10 = \frac{240}{3t+2}$ or $t = \frac{80}{P} - \frac{2}{3}$	M1	3.4	Either substitute P = 10 into their equation from (b) or rearrange to make t the subject.
			$t = 7\frac{1}{3}$ (years) oe	A1 [2]	1.1	Accept 7.33 (3sf), ignore units.
5	(d)		‘for large t, P is small’ or $t \geq 160 \Rightarrow P < 1$ (P is modelled as continuous but in fact the number of animals is discrete), at this time the **actual population** would be 0.	B1 [1]	3.5a	Allow $t = 160 \Rightarrow P = 0.5$ (Use of the value t=160 not required) Any correct statement in context relating to the model predicting non-integer values between 0 and 1. (Note that the statement ‘will never reach zero’ is given in the question so gains no credit). Acceptable examples (for a comment, in combination with the statement that P is small for large t): • “cannot have part of an animal” • “the actual population has reached 0” • “not possible to have a population of 0.49(8)” • “the population will go extinct when less than 1”

Question			Answer	Marks	AO	Guidance
6	(a)	(i)	**DR** $\cos 3\theta = \cos(2\theta + \theta)$ $= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ $= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta$ $= 4\cos^3\theta - 3\cos\theta$ **AG**	 **B1** **B1**	 **1.1** **1.1**	Condone a small slip in early irrelevant working before substituting, but must be fully correct after this. oe in terms of θ, not 2θ May be implied by next line oe in terms of $\cos\theta$ only Must reach AG or give a conclusion (even if just 'QED') www
			Alternative 1 $= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ $= \cos^3\theta - (1 - \cos^2\theta)\cos\theta - 2(1 - \cos^2\theta)\cos\theta$ $= 4\cos^3\theta - 3\cos\theta$ **AG**	 **B1** **B1**		 oe in terms of θ, not 2θ May be implied by next line oe in terms of $\cos\theta$ only Must reach AG or give a conclusion (even if just 'QED') www
			Alternative 2 $= (1 - 2\sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ $= (1 - 2(1 - \cos^2\theta))\cos\theta - 2(1 - \cos^2\theta)\cos\theta$ $= 4\cos^3\theta - 3\cos\theta$ **AG**	 **B1** **B1**		 oe in terms of θ, not 2θ May be implied by next line oe in terms of $\cos\theta$ only Must reach AG or give a conclusion (even if just 'QED') www
				[2]		
6	(a)	(ii)	**DR** $\cos 3\theta = \frac{\sqrt{2}}{2}$ $3\theta = 45^o$ or 315^o or 405^o $\theta = 15^o$ or 105^o or 135^o	 **M1** **A1** **A1** **[3]**	 **1.1** **1.1** **1.1**	 oe Allow A1 for two correct values of 3θ. Ignore other values. This mark is not implied by correct final answers but accept equivalent correct working e.g. a graph of $\cos 3\theta$. Accept radians for this mark only: $3\theta = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$ cao. Ignore values outside of the range 0-180°, but do not accept radians for this mark. This mark may be given following M1A0.

Question			Answer	Marks	AO	Guidance
6	(b)	(i)	**DR** $4x^3 - 2\sqrt{2}x^2 - x + 2\sqrt{2}x^2 - 2x - \frac{\sqrt{2}}{2}$ oe $= 4x^3 - 3x - \frac{\sqrt{2}}{2}$ **AG**	**B1** **[1]**	**1.1**	Must see a correct multiplied out form and AG or conclusion.
6	(b)	(ii)	**DR** $x = -\frac{\sqrt{2}}{2}$ $x = \frac{\sqrt{2}+\sqrt{6}}{4}$ and $x = \frac{\sqrt{2}-\sqrt{6}}{4}$ oe	**B1** **B1** **[2]**	**2.1** **1.1**	oe, must be exact May see $x = \frac{2\sqrt{2}\pm\sqrt{24}}{8}$ etc.
6	(c)		**DR** cos 15° is a root of the equation in **(b)(ii)** **(a)(ii):** cos 105° < 0, cos 135° < 0, cos 15° > 0 **(b)(ii):** $-\frac{\sqrt{2}}{2} < 0$ and $\frac{\sqrt{2}-\sqrt{6}}{4} < 0, \frac{\sqrt{2}+\sqrt{6}}{4} > 0$ $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$ **AG**	**B1** **B1** **[2]**	**2.4** **3.2a**	soi - for 'spotting' the connection. This mark can be gained regardless of their answers to **(a)(ii)** and **(b)(ii)**. $x = \cos\theta$ or 3 correctly paired roots are sufficient for this mark Condone $x = \cos 15$ Justification for selecting this root (may say e.g. "cos 15 is the **only** positive root") This may be implied by matching each pair of answers correctly (but all three must be present or mentioned).

Question			Answer	Marks	AO	Guidance
7	(a)		common difference of B is $-d$	B1	1.1	soi
			$(b_1 = a_{100} =) \ 4 + 99d$	B1	1.1	soi (or $a_1 + 99d$)
			$b_i = 4 + 99d + (i-1)(-d)$ or $4 + 100d - id$	M1	3.1a	For an attempt at the general term b_i using b_1 ($= a_{100}$), i.e. an expression of the form $a_1 + kd + (i-1)(\pm d)$ oe
				A1	2.1	Fully correct expression for b_i
			$4 + 100d - id = 4 + (i-1)d + 10$ $(2id = 101d - 10)$	M1	1.1	Equating their b_i to an expression for $a_i + 10$ May see $a_1 + (i-1)d + 10$
			$\Rightarrow i = \frac{101}{2} - \frac{5}{d}$ **AG**	A1	1.1	www (so this mark is dependent on all previous marks) May see a_1 used throughout instead of 4 Condone other letters used throughout (e.g. n for i and c for $-d$): - Must recover to d in order to solve (A1A1) - Must recover to i to demonstrate the AG (A1)
				[6]		
7	(b)		$\frac{101}{2} - \frac{5}{d}$ not always integer [between 1 & 100]	B1	2.4	soi (accept e.g. "i must be an integer")
			$d = 2$ or 10	B1	3.2b	Ignore negative values (but B0 for any additional positive values)
				[2]		

Question			Answer	Marks	AO	Guidance
8	(a)	(i)	0.219 (3 sf)	B1	1.1	awrt 0.219 (Condone 21.9%)
				[1]		
8	(a)	(ii)	$1 - P(X \leq 3)$ or $1 - 0.297$	M1	3.4	Allow M1 for $1 - P(X \leq 4)$ or $1 - 0.515$ or 0.484 May be implied by correct answer.
			$= 0.703$ (3 sf)	A1	1.1	awrt 0.703 (Condone 70.3%)
				[2]		
8	(b)	(i)	$\binom{15}{r} 0.7^{15-r} 0.3^r$ oe	B1	1.2	Allow $(1 - 0.3)$ for 0.7 Accept $^{15}C_r$ etc. but must be in terms of r
				[1]		
8	(b)	(ii)	$n = 15$	B1	3.3	Must be stated (not implied)
			$P(X = r)$ is the $(r \pm 1)$th **term** of the expansion	B1	2.4	Allow $P(X = r)$ is a **term** of the expansion Condone "appears in the expansion" but not "it is the expansion" and not "the coefficient of a term"
				[2]		Condone $P(X = r)$ is the rth **term** of the expansion

Question			Answer	Marks	AO	Guidance
9	(a)		0.886 (3 sf)	B1 [1]	1.1	awrt 0.886 (Condone 88.6%)
9	(b)		$\Phi(0.8)$ and $\Phi(0.3)$	M1*	3.1a	Attempted, values not required and allow truncated or rounded values e.g. 0.84… etc. throughout. May be implied by ±0.84162 & ±0.52440
			$0.84162 = \frac{502-\mu}{\sigma}$ and $-0.52440 = \frac{499-\mu}{\sigma}$	A1	3.4	Allow $\Phi(0.8)$ and $\Phi(0.3)$ – or use of these expressions throughout (need not see values for Φ but if given they must be correct)
			$\frac{0.84162}{-0.52440} = \frac{502-\mu}{499-\mu}$ oe **or** $(0.84162 + 0.52440)\sigma = 502 - 499$ oe	M1 dep*	1.1	One correct equation in one unknown (FT their Φ values) (may be implied by correct answers).
			μ = 500.15 (2 dp) and σ = 2.20 (2 dp)	A1 A1 [5]	1.1 1.1	Correct answers to 2sf (500 and 2.2) or better Correct answers to exactly 2dp, cao SCB1 for one correct answer to 2dp (max 4/5) or if one/both answers are given with no/insufficient working (max 1/5).
9	(c)	(i)	0.5 and 3.5	B1 [1]	1.1	Allow 0 to 1, 3 to 4
9	(c)	(ii)	1.5			$\frac{\text{Their }'3.5' - '0.5'}{2}$ or $\lvert 2 - '3.5' \rvert$ or $\lvert 2 - '0.5' \rvert$ FT their points of inflection (Note that $\mu \approx 2$)
			Inflection points are one sd from mean	B1FT [1]	1.2	Need value and explanation (calculation alone is not sufficient)

Question			Answer	Marks	AO	Guidance
10	(a)		H_0: $\rho = 0$ H_1: $\rho \neq 0$ where ρ is the **correlation coefficient** for the population **or** where ρ is the **correlation coefficient** between amount spent (h) and no. of customers (c)	**B1** **B1**	**1.1** **2.5**	Subtract B1 for each error: • Undefined ρ: B1B0 • 1-tail: B1B0 • Allow other letters (but not r, c or h: B1B0) • Hypotheses in words (no parameter): B1B0 o H_0: There is no correlation o H_1: There is correlation o Do not allow "negative" or "positive" correlation for H_1: B0B0 Accept "pmcc" for correlation coefficient
			0.798 > 0.7079 oe	**B1FT**	**1.1**	FT their setup/hypotheses • e.g. for H_1: $\rho < 0$, compare 0.798 with 0.6581 Must use $n = 12$ and specify a corresponding value from the table • 0.6581 or 0.7079 only (NB not 0.6**85**1 from $n = 11$) Must compare this with 0.798 with the same sign • Condone $-0.798 < -0.7079$ or $\lvert-0.798\rvert$ • Do not accept $-0.798 < 0.6581$ NB this is the only mark that can be scored with no hypotheses
			Reject H_0	**M1**	**1.1**	This step must be seen, consistent with their hypotheses and their comparison. Condone Accept H_1
			Sufficient evidence for a (linear) **correlation** between amount spent (h) and no. of customers (c) oe	**A1**	**2.2b**	Conclusion must be in context, not definite and consistent with their hypotheses and comparison. • Disregard any mention of "negative" or "positive" • "Relationship" A0 • "Prove(d)" A0 • Condone "there is evidence of a linear correlation between h and c" • Condone "significant" for "sufficient" Must conclude that there is evidence **for** a correlation.
				[5]		

Question			Answer	Marks	AO	Guidance
10	(b)		Points (fairly) close to a (straight) **line** with negative gradient oe	**B1** **B1** **[2]**	**1.2** **1.2**	For a statement about the relative strength of the linear correlation: • Accept "points form a **line**" or • Accept "points lie (relatively) close to the **line**" • Not "points are close together" or "close to each other" For a statement about the appearance of the negative correlation: • Accept "line from 2nd to 4th quadrant" • Accept "two clusters in top left and bottom right" • Accept "line will be downwards sloping" • Not "negatively correlated" (must be a feature of the scatter diagram) This mark only may be implied by a sketch (showing a scatter diagram with negative correlation, with or without a line of best fit).
10	(c)		Correlation does not imply causation A suggested 3rd factor affecting both *c* & *h* e.g. time of year, temperature, weather	**B1** **B1** **[2]**	**2.3** **2.4**	oe, may be implied (but do not allow "independent") Any sensible comment about the statement but must be in context: • Accept "Some people may not visit if the store is too cold" • Accept "the shop being too warm may mean customers don't want to go inside" • Not "There may be a third factor affecting both *c* and *h*" (a possible factor must be specified) • Accept "more people in the store may mean there is less need for heating" (so the implication might be the other way around) or equivalent statements about causation

Question			Answer	Marks	AO	Guidance
10	(d)		If no (linear) correlation in the population, then for (a sample of) 10 (pairs) $P(r > 0.7155) = 0.01$ or $P(\lvert r \rvert > 0.7155) = 0.02$	**B1** **B1** **[2]**	**1.2** **2.5**	For the setup, condone "n=10" but must reference 'no correlation' Allow $\geq$ Accept an equivalent statement in words, but it must be about a probability: • e.g. "The **probability** that the pmcc is greater than 0.7155 is 1%" • e.g. "There is a 1% **chance** that there is actually no correlation when r is greater than 0.7155."

Question			Answer	Marks	AO	Guidance
11	(a)	(i)	**Train**, because all increases over 50%	**B1** **[1]**	**2.2b**	Must specify **Train** and give a reason. Any sensible reason related to chart, e.g.: • "Train has two of highest category and three of the 2nd highest" • May refer to colours (e.g. dark, darker, darkest)
11	(a)	(ii)	The chart does not show where in the range each value is **Or** We do not know the (relative) sizes of each LA	**B1** **[1]**	**2.3**	Any sensible reason relating to either 'exact values' or 'actual/relative sizes of each LA'. Examples: • "we do not know exact values because the data is grouped" B1 • "grouped" (without reference to exact values) B0 • "increases could be much higher than 90%" B1 • "low % of high number may be greater than high % of low number" B1 • "not all workers accounted for" B0 • "may be errors in data" B0 NB throughout this question, do not credit reasons that don't relate to the question or that attempt to criticise the data set (rather than the chart/presentation and reasoning from it).

Question			Answer	Marks	AO	Guidance
11	(b)		Do not know the initial proportions of each mode of transport Do not know the exact % increases	B1 B1 [2]	2.3 2.4	Any 2 distinct correct reasons. Examples: • "% ranges are wide" B1 • "Do not know the upper limit for +90% PI" B1 • "Do not know values within -10 to +10%" B1 • "Do not know baseline values for each mode" B1 • "size of the LAs" (not relevant here) B0 Do not credit criticism of underlying data or grouping of modes of transport (e.g. walking/working from home) B0
11	(c)		(Small) changes in these **small numbers** will lead to **large % increases** (which may give a misleading impression of the overall trend).	B1 [1]	2.4	Must refer to numbers or absolute values, not just %, e.g.: • "actual increases small (so hard to see trend)" B1 • Do not credit references to grouped data / exact values here (not relevant) B0
11	(d)		Increase in D is 20% and in H is 40% $0.2D_{01} \approx 3.5\times0.4H_{01}$ D:H is 7:1	M1 M1 A1 [3]	3.1a 1.1 2.2b	M1 for sight of midpoints: 20% and 40% • Allow 0.2 and 0.4 • Condone 1.2 and 1.4 Alternatively may see comparison of corresponding lower and upper bounds e.g. 10% -> 30% and 30% -> 50% (this leads to an estimate of 8.15) – M1 For setting up two expressions and correctly using 3.5 (must be on the correct side). Do not accept 1.2,1.4 here (NB 3.5×0.4=1.4) Allow = instead of ≈ Must be using two estimates for midpoints or considering both sets of lower/upper bounds Accept 3.5:0.5. If lower and upper bounds used, accept 8(.15):1 Need not be given as a ratio: accept $\frac{D_{01}}{H_{01}} \approx 7$ oe but not $D = 7H$ Note that 7:2 often comes from not using any information from the chart (M0M0A0)

Question			Answer	Marks	AO	Guidance
12	(a)		Two correct, unequal probabilities: e.g. $P(2) = \frac{1}{36}, P(3) = \frac{2}{36}$	M1	2.4	Must be seen. Condone a correct statement e.g. "The probability of obtaining 2 is not the same as the probability of getting 3" – must refer to specific scores, need not compute probabilities (but if given these must be correct).
			Probabilities not equal (and hence not random).	A1	3.5b	A generalised conclusion must be present: • Allow "probabilities are not the same" • May come at the beginning e.g. "the **chance** of each student being chosen is not equal because…"
			Alternative using combinations: Demonstrate that there are more ways of obtaining one answer than another e.g. "there are more ways of obtaining 4 than 2"	M1		Must be seen. Must refer to specific scores but need not compute the number of combinations/ways (but if given these must be correct).
			(Hence student C is more likely to be chosen than student A) and therefore the **probabilities are not equal** (and hence not random).	A1		A generalised conclusion must be present: • Accept "student __ is **more likely** to be chosen than student __, **therefore it is not random**" • Accept "some scores are **more likely** than others"
				[2]		
12	(b)	(i)	Throw repeatedly until one of these pairs is obtained	B1	3.5c	Any valid correction to the method, e.g.: • "If obtain a pair not included, throw again." • "make the two dice distinguishable" (because 1,3 and 3,1 have different students) or "throw in order" Condone extending to the entire sample space (e.g. "assign a further 2 pairs to each student and the remaining 3 to 'throw again'") but the method must remain random.
				[1]		
12	(b)	(ii)	$\frac{1}{11}$	B1	3.4	Accept $\frac{1}{36}$
				[1]		

Question			Answer	Marks	AO	Guidance
13	(a)		$\mu = 40$, $\sigma^2 = 32$ $P(Y > y) = 0.05 \Rightarrow y = 49.3$ (3 sf)	**M1** **A1** **[2]**	**1.2** **1.1**	May see N(40,32) awrt 49.3
13	(b)		$P(X \geq 50) = 0.049\ldots$	**M1**	**3.1**	Attempt $P(X \geq$ '50'$)$ using B $\left(200, \frac{1}{5}\right)$ for their '49'+1 Allow this mark for P(X<50)=0.95… The value must be linked to their correct threshold value of '49'+1(=50) (or '49'), if an inequality is given it must be correct.
			$P(X \geq 49) = 0.069\ldots$	**M1**	**2.1**	Attempt $P(X \geq$ '49'$)$ using B $\left(200, \frac{1}{5}\right)$ for their '49' Allow this mark for P(X<49)=0.93… The value must be linked to their correct threshold value of '49' (or '49'–1=48), if an inequality is given it must be correct.
			Smallest value of X is 50	**A1** **[3]**	**1.1**	Condone $X \geq 50$ (dependent on both method marks) SCB1 for correct answer with no/insufficient working (max. 1/3)

Question			Answer	Marks	AO	Guidance
14			$p+\frac{1}{6}p+p^2+\frac{1}{2}=1$	M1*	3.1a	Forming this equation in p, must be fully correct with =1 soi
			$p^2+\frac{7}{6}p-\frac{1}{2}=0$ or $6p^2+7p-3=0$ oe	M1 dep*	1.1	Rearrange their equation to solvable form $ap^2 + bp + c = 0$ and attempt to solve (may be implied by one or both correct roots)
			$p=\frac{1}{3}$	A1	1.1	
			$p=-\frac{3}{2}$ invalid	B1	2.3	For sight of $p=-\frac{3}{2}$ oe provided this root **not** used in subsequent working. Condone "the other root is negative" or "$p > 0$"
			(Hence probabilities are): x: 1, 2, 3, 4 P($X = x$): $\frac{1}{3}$, $\frac{1}{18}$, $\frac{1}{9}$, $\frac{1}{2}$	A1	1.1	These values are likely to be seen in, and may be implied by, subsequent working. Note that a correct numerical denominator or final answer also implies this mark.
			$\left(\frac{\mathrm{P}(X_2=2X_1\cap X_2>X_1)}{\mathrm{P}(X_2>X_1)}=\frac{\mathrm{P}(X_2=2X_1)}{\mathrm{P}(X_2>X_1)}\right)$ Any of: $p\cdot\frac{p}{6}+\frac{p}{6}\cdot\frac{1}{2}=\frac{1}{3}\times\frac{1}{18}+\frac{1}{18}\times\frac{1}{2}$ or $p\cdot\frac{p}{6}+p\cdot p^2+p\cdot\frac{1}{2}+\frac{p}{6}\cdot p^2+\frac{p}{6}\cdot\frac{1}{2}+p^2\cdot\frac{1}{2}$ $=\frac{1}{3}\times\frac{1}{18}+\frac{1}{3}\times\frac{1}{9}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{18}\times\frac{1}{9}+\frac{1}{18}\times\frac{1}{2}+\frac{1}{9}\times\frac{1}{2}$	M1	2.1	Either numerator or denominator attempted (correct form with 2 terms in the numerator or 6 terms in the denominator). FT their probabilities. May be implied by any of: • a correct expression for one of the numerator or denominator either in p or with their probabilities • a correct final answer
			$\frac{p\cdot\frac{p}{6}+\frac{p}{6}\cdot\frac{1}{2}}{p\cdot\frac{p}{6}+p\cdot p^2+p\cdot\frac{1}{2}+\frac{p}{6}\cdot p^2+\frac{p}{6}\cdot\frac{1}{2}+p^2\cdot\frac{1}{2}}$ $=\frac{\frac{1}{3}\times\frac{1}{18}+\frac{1}{18}\times\frac{1}{2}}{\frac{1}{3}\times\frac{1}{18}+\frac{1}{3}\times\frac{1}{9}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{18}\times\frac{1}{9}+\frac{1}{18}\times\frac{1}{2}+\frac{1}{9}\times\frac{1}{2}}$ oe $\left(=\frac{\frac{5}{108}}{\frac{101}{324}}\ or\ \frac{0.0463}{0.3117}\right)$	M1	1.1	Division attempted with a 2-term numerator and 6-term denominator soi either in p or with their probabilities (may see numerator and denominator computed separately and then an attempt to divide)
			$=\frac{15}{101}$ or 0.149 (3sf)	A1 [8]	1.1	oe, need not be simplified

Appendix – Additional Marking Guidance

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>13</td><td>(b)</td><td></td><td></td><td></td><td></td><td>SCB1 for correct answer with no/insufficient working (max. 1/3)
Further marking guidance:
For the M1 marks, require two distinct probabilities from the correct binomial distribution B $\left(200, \frac{1}{5}\right)$, which correspond to two values centred around 49 ± 1 or their answer from (a) ± 1.
Candidates need not state an inequality, but each probability must be associated with an appropriate value of X and if an inequality is given it must be correct.
<table>
<tr><td>x</td><td>P($X < x$)</td><td>P($X \leq x$)</td><td>P($X > x$)</td><td>P($X \geq x$)</td></tr>
<tr><td>48</td><td>0.9056</td><td>0.93097</td><td>0.06903</td><td>0.0944</td></tr>
<tr><td>49</td><td>0.93097</td><td>0.95065</td><td>0.04935</td><td>0.06903</td></tr>
<tr><td>50</td><td>0.95065</td><td>0.9655</td><td>0.0345</td><td>0.04935</td></tr>
</table>
Additional clarifications:
• The above table shows likely values for x=49 ± 1
• FT their ‘49’ from (a) for M marks only (check values)
• NB for candidates whose ‘49’ is ~65 or greater, their probabilities may appear as ‘1’ and ‘0’ – allow M1M1 if this follows correct setup.
• Condone 2 probabilities for the same x provided inequalities are given correctly
• Probability values may be given to any degree of accuracy, truncated or rounded.
Do not accept any single probabilities i.e. P($X = x$)</td></tr>
</table>

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