Oxford Cambridge and RSA

AS Level Further Mathematics B (MEI) (H635)
A Level Further Mathematics B (MEI) (H645)

Formulae Booklet

INSTRUCTIONS

- Do **not** send this Booklet for marking. Keep it in the centre or recycle it.

INFORMATION

- This document has **16** pages.

AS and A Level Further Mathematics B (MEI)

Contents

A level Mathematics

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}, \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small Angle Approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0 : x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Sample Variance

$s^2 = \frac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \overline{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\overline{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The Binomial Distribution

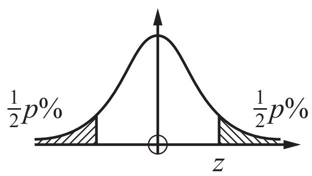
If $X \sim \mathrm{B}(n, p)$ then $P(X=r) = {}^{n}\mathrm{C}_{r} p^{r} q^{n-r}$ where $q = 1-p$
Mean of X is np

Hypothesis test for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two and three dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

-

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Core Pure

Complex Numbers

De Moivre's theorem:
$\{r(\cos\theta + i\sin\theta)\}^n = r^n(\cos n\theta + i\sin n\theta)$

Roots of unity:

The roots of $z^n = 1$ are given by $z = \exp\left(\frac{2\pi k}{n}i\right)$ for $k = 0, 1, 2, \ldots, n-1$

Vectors and 3-D geometry

Cartesian equation of a plane is
$n_1x + n_2y + n_3z + d = 0$

Cartesian equation of a line in 3-D is
$\frac{x-a_1}{d_1} = \frac{y-a_2}{d_2} = \frac{z-a_3}{d_3}$

Vector product $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix}$ $= |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$ where $\mathbf{a}$, $\mathbf{b}$, $\hat{\mathbf{n}}$, in that order, form a right-handed triple.

Distance between skew lines is $\left|\frac{\mathbf{d}_1 \times \mathbf{d}_2}{|\mathbf{d}_1 \times \mathbf{d}_2|}.(\mathbf{a}_1 - \mathbf{a}_2)\right|$ where $\mathbf{a}_1$ is the position vector of a point on the first line and $\mathbf{d}_1$ is parallel to the first line, similarly for the second line.

Distance between point (x_1, y_1) and line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Distance between point (x_1, y_1, z_1) and plane $n_1x + n_2y + n_3z + d = 0$ is $\frac{|n_1x_1 + n_2y_1 + n_3z_1 + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\text{arsinh}\, x = \ln[x + \sqrt{(x^2 + 1)}]$

$\text{arcosh}\, x = \ln[x + \sqrt{(x^2 - 1)}],\ x \geq 1$

$\text{artanh}\, x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),\ -1 < x < 1$

Calculus

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\mathrm{arsinh}\left(\dfrac{x}{a}\right)$ or $\ln(x+\sqrt{x^2+a^2})$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\mathrm{arcosh}\left(\dfrac{x}{a}\right)$ or $\ln(x+\sqrt{x^2-a^2}) \quad (x > a)$

The mean value of $\mathrm{f}(x)$ on the interval $[a,b]$ is $\dfrac{1}{b-a}\int_a^b \mathrm{f}(x)\,\mathrm{d}x$

Area of sector enclosed by polar curve is $\dfrac{1}{2}\int r^2\,\mathrm{d}\theta$

Series

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

$$\mathrm{f}(x) = \mathrm{f}(0) + \mathrm{f}'(0)x + \frac{\mathrm{f}''(0)}{2!}x^2 + \ldots + \frac{\mathrm{f}^{(r)}(0)}{r!}x^r + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \text{ for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots (-1 < x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \text{ for all } x$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (\lvert x\rvert < 1, n \in \mathbb{R})$$

Mechanics

Motion in a circle

For motion in a circle,

tangential velocity is $v = r\dot{\theta}$

radial acceleration is $\frac{v^2}{r}$ or $r\dot{\theta}^2$ towards the centre

tangential acceleration is $r\ddot{\theta}$

Further Pure with Technology

Numerical solution of differential equations

For $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$:

Euler's method: $x_{n+1} = x_n + h \quad y_{n+1} = y_n + h\mathrm{f}(x_n, y_n)$

Modified Euler method (A Runge-Kutta method of order 2):

$k_1 = h\mathrm{f}(x_n, y_n)$

$k_2 = h\mathrm{f}(x_n + h, y_n + k_1)$

$x_{n+1} = x_n + h,\ y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$

Runge-Kutta method of order 4:

$k_1 = h\mathrm{f}(x_n, y_n)$

$k_2 = h\mathrm{f}(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

$k_3 = h\mathrm{f}(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$

$k_4 = h\mathrm{f}(x_n + h, y_n + k_3)$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

Gradient of tangent to a polar curve

For a curve $r = \mathrm{f}(\theta)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta + r\cos\theta}{\frac{\mathrm{d}r}{\mathrm{d}\theta}\cos\theta - r\sin\theta}$

Extra Pure

Multivariable calculus

$\nabla \mathrm{g} = \textbf{grad}\ \mathrm{g} = \begin{pmatrix} \dfrac{\partial \mathrm{g}}{\partial x} \\ \dfrac{\partial \mathrm{g}}{\partial y} \\ \dfrac{\partial \mathrm{g}}{\partial z} \end{pmatrix}$. If $\mathrm{g}(x, y, z)$ can be written as $z = \mathrm{f}(x, y)$ then $\textbf{grad}\ \mathrm{g} = \begin{pmatrix} \dfrac{\partial \mathrm{f}}{\partial x} \\ \dfrac{\partial \mathrm{f}}{\partial y} \\ -1 \end{pmatrix}$

Numerical methods

Solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1}=x_n-\dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

For the iteration $x_{n+1}=\mathrm{g}(x_n)$ the relaxed iteration is $x_{n+1}=(1-\lambda)x_n+\lambda\mathrm{g}(x_n)$.

Numerical integration

To estimate $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$:

The midpoint rule:

$$M_n=h\left(y_{\frac{1}{2}}+y_{\frac{3}{2}}+\ldots+y_{n-\frac{3}{2}}+y_{n-\frac{1}{2}}\right) \qquad \text{where } h=\frac{b-a}{n}$$

The trapezium rule:

$$T_n=\frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\} \qquad \text{where } h=\frac{b-a}{n}$$

Simpson's rule

$$S_{2n}=\frac{1}{3}h\{(y_0+y_{2n})+4(y_1+y_3+\ldots+y_{2n-1})+2(y_2+y_4+\ldots+y_{2n-2})\}$$

$$\text{where } h=\frac{b-a}{2n}$$

These are related as follows:

$$T_{2n}=\frac{1}{2}(M_n+T_n)$$

$$S_{2n}=\frac{1}{3}(2M_n+T_n)=\frac{1}{3}(4T_{2n}-T_n)$$

Interpolation

Newton's forward difference interpolation formula:

$$\mathrm{f}(x)=\mathrm{f}(x_0)+\frac{(x-x_0)}{h}\Delta\mathrm{f}(x_0)+\frac{(x-x_0)(x-x_1)}{2!h^2}\Delta^2\mathrm{f}(x_0)+\ldots$$

Lagrange's polynomial:

$$\mathrm{P}_n(x)=\sum \mathrm{L}_r(x)\mathrm{f}(x_r) \quad \text{where} \quad \mathrm{L}_r(x)=\prod_{\substack{i=0\\ i\neq r}}^{n}\frac{x-x_i}{x_r-x_i}$$

Statistics

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with $\mathrm{P}(X = x_i) = p_i$

Expectation: $\mu = \mathrm{E}(X) = \sum x_i p_i$

Variance: $\sigma^2 = \mathrm{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

	Probability	E(X)	Var(X)
Uniform distribution over 1, 2, …, n	$P(X = r) = \frac{1}{n}$	$\frac{n+1}{2}$	$\frac{1}{12}(n^2 - 1)$
Geometric distribution	$P(X = r) = q^{r-1}p$ $q = 1 - p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson distribution	$P(X = r) = \mathrm{e}^{-\lambda}\frac{\lambda^r}{r!}$		

Correlation and regression

For a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n},\ S_{yy} = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n},\ S_{xy} = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

product moment correlation coefficient: $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]}}$

least squares regression line of y on x is $y - \overline{y} = b(x - \overline{x})$ where $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}}$

least squares regression line of x on y is $x - \overline{x} = b'(y - \overline{y})$ where $b' = \dfrac{S_{xy}}{S_{yy}} = \dfrac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}}$

Spearman's coefficient of rank correlation:

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)}$$

Confidence intervals

To calculate a confidence interval for a mean or difference in mean in different circumstances, use the given distribution to calculate the critical value, k.

To estimate…	Confidence interval	Distribution
a mean	$\bar{x} \pm k\frac{\sigma}{\sqrt{n}}$	N(0, 1)
a mean	$\bar{x} \pm k\frac{s}{\sqrt{n}}$	t_{n-1}
difference in mean of paired populations	treat differences as a single distribution	

Hypothesis tests

Description	Test statistic	Distribution
Pearson's product moment correlation test	$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]}}$	
Spearman's rank correlation test	$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$	
χ^2 test	$\sum \frac{(f_o - f_e)^2}{f_e}$	χ_v^2
Normal test for a mean	$\frac{\bar{x}-\mu}{\left(\sigma/\sqrt{n}\right)}$	N(0, 1)
t-test for a mean	$\frac{\bar{x}-\mu}{\left(s/\sqrt{n}\right)}$	t_{n-1}
Wilcoxon single sample test	A statistic T is calculated from the ranked data	

Continuous distributions

X is a continuous random variable with probability density function (pdf) $\mathrm{f}(x)$

Expectation: $\mu = \mathrm{E}(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$

Variance: $\sigma^2 = \mathrm{Var}(X) = \int (x-\mu)^2 \mathrm{f}(x)\,\mathrm{d}x = \int x^2 \mathrm{f}(x)\,\mathrm{d}x - \mu^2$

Cumulative distribution function $\mathrm{F}(x) = \mathrm{P}(X \le x) = \int_{-\infty}^{x} \mathrm{f}(t)\,\mathrm{d}t$

	E(X)	Var(X)
Continuous uniform distribution over $[a,b]$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$

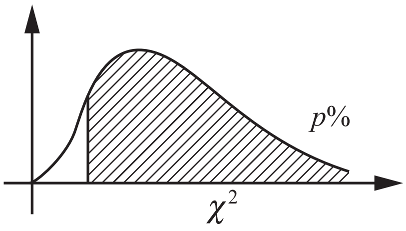
Critical values for the product moment correlation coefficient, r

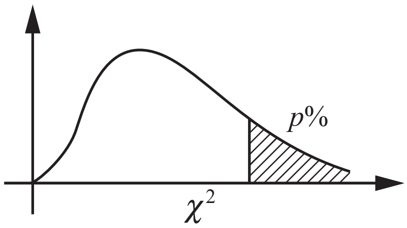
1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%
n				
1	-	-	-	-
2	-	-	-	-
3	0.9877	0.9969	0.9995	0.9999
4	0.9000	0.9500	0.9800	0.9900
5	0.8054	0.8783	0.9343	0.9587
6	0.7293	0.8114	0.8822	0.9172
7	0.6694	0.7545	0.8329	0.8745
8	0.6215	0.7067	0.7887	0.8343
9	0.5822	0.6664	0.7498	0.7977
10	0.5494	0.6319	0.7155	0.7646
11	0.5214	0.6021	0.6851	0.7348
12	0.4973	0.5760	0.6581	0.7079
13	0.4762	0.5529	0.6339	0.6835
14	0.4575	0.5324	0.6120	0.6614
15	0.4409	0.5140	0.5923	0.6411
16	0.4259	0.4973	0.5742	0.6226
17	0.4124	0.4821	0.5577	0.6055
18	0.4000	0.4683	0.5425	0.5897
19	0.3887	0.4555	0.5285	0.5751
20	0.3783	0.4438	0.5155	0.5614
21	0.3687	0.4329	0.5034	0.5487
22	0.3598	0.4227	0.4921	0.5368
23	0.3515	0.4132	0.4815	0.5256
24	0.3438	0.4044	0.4716	0.5151
25	0.3365	0.3961	0.4622	0.5052
26	0.3297	0.3882	0.4534	0.4958
27	0.3233	0.3809	0.4451	0.4869
28	0.3172	0.3739	0.4372	0.4785
29	0.3115	0.3673	0.4297	0.4705
30	0.3061	0.3610	0.4226	0.4629
31	0.3009	0.3550	0.4158	0.4556
32	0.2960	0.3494	0.4093	0.4487
33	0.2913	0.3440	0.4032	0.4421
34	0.2869	0.3388	0.3972	0.4357
35	0.2826	0.3338	0.3916	0.4296
36	0.2785	0.3291	0.3862	0.4238
37	0.2746	0.3246	0.3810	0.4182
38	0.2709	0.3202	0.3760	0.4128
39	0.2673	0.3160	0.3712	0.4076
40	0.2638	0.3120	0.3665	0.4026
41	0.2605	0.3081	0.3621	0.3978
42	0.2573	0.3044	0.3578	0.3932
43	0.2542	0.3008	0.3536	0.3887
44	0.2512	0.2973	0.3496	0.3843
45	0.2483	0.2940	0.3457	0.3801
46	0.2455	0.2907	0.3420	0.3761
47	0.2429	0.2876	0.3384	0.3721
48	0.2403	0.2845	0.3348	0.3683
49	0.2377	0.2816	0.3314	0.3646
50	0.2353	0.2787	0.3281	0.3610
51	0.2329	0.2759	0.3249	0.3575
52	0.2306	0.2732	0.3218	0.3542
53	0.2284	0.2706	0.3188	0.3509
54	0.2262	0.2681	0.3158	0.3477
55	0.2241	0.2656	0.3129	0.3445
56	0.2221	0.2632	0.3102	0.3415
57	0.2201	0.2609	0.3074	0.3385
58	0.2181	0.2586	0.3048	0.3357
59	0.2162	0.2564	0.3022	0.3328
60	0.2144	0.2542	0.2997	0.3301

Critical values for Spearman's rank correlation coefficient, r_s

1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%
n				
1	-	-	-	-
2	-	-	-	-
3	-	-	-	-
4	1.0000	-	-	-
5	0.9000	1.0000	1.0000	-
6	0.8286	0.8857	0.9429	1.0000
7	0.7143	0.7857	0.8929	0.9286
8	0.6429	0.7381	0.8333	0.8810
9	0.6000	0.7000	0.7833	0.8333
10	0.5636	0.6485	0.7455	0.7939
11	0.5364	0.6182	0.7091	0.7545
12	0.5035	0.5874	0.6783	0.7273
13	0.4835	0.5604	0.6484	0.7033
14	0.4637	0.5385	0.6264	0.6791
15	0.4464	0.5214	0.6036	0.6536
16	0.4294	0.5029	0.5824	0.6353
17	0.4142	0.4877	0.5662	0.6176
18	0.4014	0.4716	0.5501	0.5996
19	0.3912	0.4596	0.5351	0.5842
20	0.3805	0.4466	0.5218	0.5699
21	0.3701	0.4364	0.5091	0.5558
22	0.3608	0.4252	0.4975	0.5438
23	0.3528	0.4160	0.4862	0.5316
24	0.3443	0.4070	0.4757	0.5209
25	0.3369	0.3977	0.4662	0.5108
26	0.3306	0.3901	0.4571	0.5009
27	0.3242	0.3828	0.4487	0.4915
28	0.3180	0.3755	0.4401	0.4828
29	0.3118	0.3685	0.4325	0.4749
30	0.3063	0.3624	0.4251	0.4670
31	0.3012	0.3560	0.4185	0.4593
32	0.2962	0.3504	0.4117	0.4523
33	0.2914	0.3449	0.4054	0.4455
34	0.2871	0.3396	0.3995	0.4390
35	0.2829	0.3347	0.3936	0.4328
36	0.2788	0.3300	0.3882	0.4268
37	0.2748	0.3253	0.3829	0.4211
38	0.2710	0.3209	0.3778	0.4155
39	0.2674	0.3168	0.3729	0.4103
40	0.2640	0.3128	0.3681	0.4051
41	0.2606	0.3087	0.3636	0.4002
42	0.2574	0.3051	0.3594	0.3955
43	0.2543	0.3014	0.3550	0.3908
44	0.2513	0.2978	0.3511	0.3865
45	0.2484	0.2945	0.3470	0.3822
46	0.2456	0.2913	0.3433	0.3781
47	0.2429	0.2880	0.3396	0.3741
48	0.2403	0.2850	0.3361	0.3702
49	0.2378	0.2820	0.3326	0.3664
50	0.2353	0.2791	0.3293	0.3628
51	0.2329	0.2764	0.3260	0.3592
52	0.2307	0.2736	0.3228	0.3558
53	0.2284	0.2710	0.3198	0.3524
54	0.2262	0.2685	0.3168	0.3492
55	0.2242	0.2659	0.3139	0.3460
56	0.2221	0.2636	0.3111	0.3429
57	0.2201	0.2612	0.3083	0.3400
58	0.2181	0.2589	0.3057	0.3370
59	0.2162	0.2567	0.3030	0.3342
60	0.2144	0.2545	0.3005	0.3314

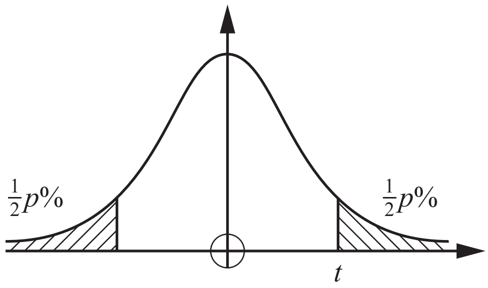
Percentage points of the χ^2 (chi-squared) distribution





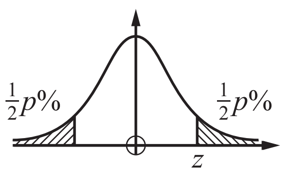
$p\%$	99	97.5	95	90	10	5	2.5	1	0.5
$v = 1$	.0001	.0010	.0039	.0158	2.706	3.841	5.024	6.635	7.879
2	.0201	.0506	0.103	0.211	4.605	5.991	7.378	9.210	10.60
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34	12.84
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28	14.86
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09	16.75
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81	18.55
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48	20.28
8	1.646	2.180	2.733	3.490	13.36	15.51	17.53	20.09	21.95
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67	23.59
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21	25.19
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72	26.76
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22	28.30
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69	29.82
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14	31.32
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58	32.80
16	5.812	6.908	7.962	9.312	23.54	26.30	28.85	32.00	34.27
17	6.408	7.564	8.672	10.09	24.77	27.59	30.19	33.41	35.72
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.81	37.16
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
35	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	60.27
40	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
100	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Percentage points of the *t* distribution



$p\%$ / v	10	5	2	1	
1	6.314	12.71	31.82	63.66	
2	2.920	4.303	6.965	9.925	
3	2.353	3.182	4.541	5.841	
4	2.132	2.776	3.747	4.604	
5	2.015	2.571	3.365	4.032	
6	1.943	2.447	3.143	3.707	
7	1.895	2.365	2.998	3.499	
8	1.860	2.306	2.896	3.355	
9	1.833	2.262	2.821	3.250	
10	1.812	2.228	2.764	3.169	
11	1.796	2.201	2.718	3.106	
12	1.782	2.179	2.681	3.055	
13	1.771	2.160	2.650	3.012	
14	1.761	2.145	2.624	2.977	
15	1.753	2.131	2.602	2.947	
20	1.725	2.086	2.528	2.845	
30	1.697	2.042	2.457	2.750	
50	1.676	2.009	2.403	2.678	
100	1.660	1.984	2.364	2.626	
∞	1.645	1.960	2.326	2.576	= percentage points of the Normal distribution N(0, 1)

Percentage points of the normal distribution



p	10	5	2	1
z	1.645	1.960	2.326	2.576

Critical values for the Wilcoxon Single Sample test

1-tail	5%	2½%	1%	½%	1-tail	5%	2½%	1%	½%
2-tail	10%	5%	2%	1%	2-tail	10%	5%	2%	1%
n					*n*				
					26	110	98	84	75
2	–	–	–	–	27	119	107	92	83
3	–	–	–	–	28	130	116	101	91
4	–	–	–	–	29	140	126	110	100
5	0	–	–	–	30	151	137	120	109
6	2	0	–	–	31	163	147	130	118
7	3	2	0	–	32	175	159	140	128
8	5	3	1	0	33	187	170	151	138
9	8	5	3	1	34	200	182	162	148
10	10	8	5	3	35	213	195	173	159
11	13	10	7	5	36	227	208	185	171
12	17	13	9	7	37	241	221	198	182
13	21	17	12	9	38	256	235	211	194
14	25	21	15	12	39	271	249	224	207
15	30	25	19	15	40	286	264	238	220
16	35	29	23	19	41	302	279	252	233
17	41	34	27	23	42	319	294	266	247
18	47	40	32	27	43	336	310	281	261
19	53	46	37	32	44	353	327	296	276
20	60	52	43	37	45	371	343	312	291
21	67	58	49	42	46	389	361	328	307
22	75	65	55	48	47	407	378	345	322
23	83	73	62	54	48	426	396	362	339
24	91	81	69	61	49	446	415	379	355
25	100	89	76	68	50	466	434	397	373

BLANK PAGE