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Examiners' report

FURTHER MATHEMATICS B (MEI)

H635

For first teaching in 2017

Y413/01 Summer 2024 series

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y413/01 series overview

June 2024 was the fifth summer examination series for the Linear H635 AS Further Maths B (MEI) optional paper Y413 (Modelling with Algorithms). Most candidates were very well prepared for the examination, making strong attempts on all questions and parts. Candidates seemed to have enough time to answer the questions and nearly all seemed to make good use of the Printed Answer Booklet (very few responses to one question appeared in the part for another, or in the additional answer space). Based on the responses seen by examiners for this series, the following general points should be considered by centres in preparation for candidates for future sittings of this paper.

Diagrams should be completed in pencil so that candidates can rub out and replace their response rather than trying to correct in pen on what is likely to be a diagram, which does not have enough space for multiple attempts. This was especially true in Question 3 (d) (scheduling) and Question 5 (c) (d) (Dijkstra's algorithm), where it was difficult at times for examiners to know what work candidates were genuinely trying to correct or what was in fact their final attempt.

This unit is primarily a methods examination and so it is vital that candidates make their method and application of any corresponding algorithm clear; spotting and writing down the solution will rarely be given marks.

The specification (on page 8) contains a section on the meanings of several instructions that will be used in examination questions. One of the key words that could appear (as it did in this paper) is the word 'determine' – the word 'determine' in this mathematical context requires that justification should be given for any results found, and not just the writing down or stating of the answer.

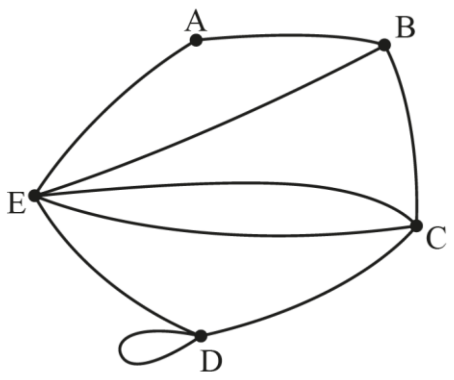
Candidates are reminded to use the number of marks available as the main guide to how detailed their answer should be and not the space given in the Printed Answer Booklet. It is always hoped that the Printed Answer Booklet provides enough space for the candidate's response, which often includes additional space so the candidate can correct any errors.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• applied the corresponding algorithm correctly and showed detailed working/reasoning (and therefore did not rely on ad-hoc methods) • read questions carefully and provided the responses that were requested • made efficient use of their calculator.	• did not give sufficient detail in questions which required an explanation or which used the command word 'determine' • used imprecise mathematical notation or incorrect language.

Question 1 (a) (i)

1 **Fig. 1** shows a graph.

Fig. 1



(a) **(i)** State the order of node B. **[1]**

Most candidates correctly stated the order of node B as 3.

Question 1 (a) (ii)

(ii) State the order of node D. **[1]**

While most candidates gave the correct response of 4, it was common to see the incorrect response of 3.

Question 1 (b)

(b) Complete the incidence matrix for the graph in **Fig. 1** in the Printed Answer Booklet. **[1]**

Although the majority of candidates correctly completed the incidence matrix, it was common for the entry from D to D to be given as either 1 or 0 (rather than the correct 2).

Misconception

It is important that all entries in such a matrix are completed with a numerical value. Therefore, a dash cannot be implied to mean a 0.

Question 1 (c)

(c) Explain why the graph in **Fig. 1** is not simple. **[1]**

Most candidates used correct mathematical language here. They either stated that the graph was not simple due to the loop at D, or that there was a repeated edge between nodes C and E. The most common incorrect response was to say that the graph was not simple because it contained a cycle.

Question 1 (d)

(d) Write down the number of arcs that need to be removed from the graph in **Fig. 1** for it to be a tree. **[1]**

Although many candidates gave the correct response of 5, it was common to see a response of 4. In this case, examiners assumed that candidates may have misread the question and were therefore stating the number of arcs that would be in a tree with 5 nodes.

Question 2 (a)

2 The list below shows the sizes of nine items.

31 13 12 17 25 18 11 8 15

(a) Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[1]**

This part was answered extremely well with most candidates giving the correct 4-bin solution. The most common error was to place the 18 in Bin 3, rather than Bin 2.

Question 2 (b)

(b) Apply the quick sort algorithm to sort the list of numbers above into **ascending** order. You should use the **first** value as the pivot for each sublist. **[3]**

This part was also answered extremely well with almost all candidates sorting the list into ascending order correctly. The most common error was to sort into descending order or to perform too many passes. Candidates are reminded that once a sub-list contains only one item, no further pass is required on that part of the list.

Question 2 (c)

(c) Show the result of applying the first fit decreasing algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[2]**

Similar to part (a), candidates were well prepared at performing first fit decreasing on a correctly sorted list. Occasionally, examiners reported that some candidates did first fit increasing (and so were given no marks). Other common mistakes were placing the 18 in Bin 2 or not placing the 8 in Bin 2.

Question 2 (d)

(d) By finding an optimal packing using a full bin strategy, explain why both first fit and first fit decreasing are examples of heuristic algorithms. **[2]**

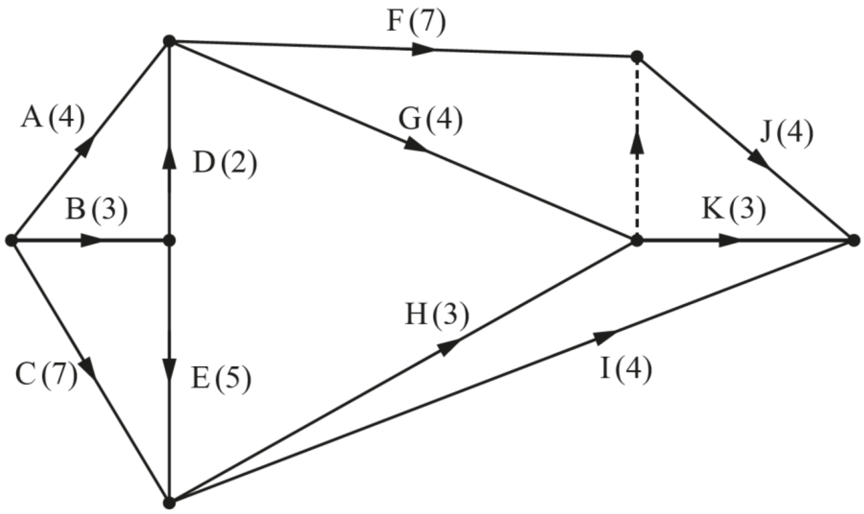
Some candidates did not read the question carefully and did not find an optimal packing using a full-bin strategy (it was possible to pack all nine items into three bins, each having a capacity of exactly 50). Of those that did, only the most successful responses went on to explain correctly why both first fit and first fit decreasing where examples of heuristic algorithms.

Misconception

While it was clear that most candidates appreciated that a heuristic algorithm does not necessarily find an optimal solution to a problem, it is true that a heuristic does find a solution to the problem (so both packing algorithms do find a solution to the original problem). Candidates are reminded that heuristic algorithms do find a solution, but the solution may not necessarily be an optimal one.

Question 3 (a)

3 The diagram shows an activity network for a project. The number in brackets show the duration of these activities in hours.



(a) Complete the table in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

This part was answered extremely well with most candidates given at least 1 mark for correctly completing five rows of the table. The most common error was due to the dummy activity (and so did not include G and/or H in the immediate predecessors for activity J).

Question 3 (b) (i)

(b) **(i)** Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of the network. **[3]**

While most candidates were given the method marks for attempting both a forward and backward pass through the network, it was uncommon for them to be given the corresponding accuracy mark as many candidates did not consider correctly the dummy activity when completing either one or both passes (most notably the backward pass).

Misconception

? The start event node should be labelled with a double zero and neither of these two boxes should be left blank.

Question 3 (b) (ii)

(ii) State the minimum project completion time. **[1]**

This part was answered correctly by most candidates.

Question 3 (b) (iii)

(iii) State the critical activities of the project. **[1]**

This part was answered extremely well with most candidates stating the correct four critical activities of B, D, F and J.

Question 3 (c)

(c) Calculate the independent float for activity I. **[1]**

The most common error here was to calculate the total float rather than the independent float of activity I.

Question 3 (d)

Each activity requires one person. When an activity is started it must be completed without interruption.

(d) Use the diagram in the Printed Answer Booklet to show how **three** people can complete the project in the minimum time found in **3(b)(ii)**. Each column in the diagram represents 1 hour. For each person, write the letter of the activity they are doing in each box, or leave the box blank if the person is resting for that 1 hour. **[2]**

While several candidates left this part blank, many did schedule the project correctly using three people. The most common errors were due to not maintaining precedence of activities (for example, starting activity K before either G or H had finished).

Question 4 (a)

4 A student sets up the following initial tableau in **Fig. 4.1** to solve a maximisation LP problem in x, y, and z.

Fig. 4.1

P	x	y	z	s_1	s_2	s_3	RHS
1	−2	3	−1	0	0	0	0
0	1	2	3	1	0	0	k
0	3	−1	1	0	1	0	45
0	2	4	−1	0	0	1	50

(a) Formulate the information from **Fig. 4.1** as a linear programming problem, stating the objective and listing all the constraints as simplified inequalities with integer coefficients. **[3]**

Most candidates were given at least 1 mark in this part for correctly stating two correct inequalities from the initial tableau. While many were given a second mark for correctly stating the objective function (together with 'maximise'), it was rare for candidates to be given the final mark for including all three non-trivial constraints.

Assessment for learning

Candidates are reminded that when formulating an LP problem, it must be stated whether the objective is being maximised or minimised. Furthermore, the trivial non-negative constraints for *x*, *y* and *z* must be included too.

Question 4 (b)

It is given that k is a constant where $15 < k < 135$.

(b) Perform **one** iteration of the simplex method, using an entry in the x column as the pivot element. Give each entry as an exact value. **[3]**

Overall, candidates were very proficient in carrying out the simplex method and many identified the correct pivot value (the 3 in the *x*-column) and used this to carry out the iteration as requested. The most common mistakes were sign errors on certain values or difficulty in dealing with fractions. Some candidates pivoted on an incorrect value and there were some candidates that used a negative value as a pivot.

Question 4 (c)

The tableau after a second iteration of the simplex method, is given in **Fig. 4.2**.

Fig. 4.2

P	x	y	z	s_1	s_2	s_3	RHS
1	0	$\frac{21}{8}$	0	$\frac{1}{8}$	$\frac{5}{8}$	0	$\frac{k+225}{8}$
0	0	$\frac{7}{8}$	1	$\frac{3}{8}$	$-\frac{1}{8}$	0	$\frac{3(k-15)}{8}$
0	1	$-\frac{5}{8}$	0	$-\frac{1}{8}$	$\frac{3}{8}$	0	$\frac{-k+135}{8}$
0	0	$\frac{49}{8}$	0	$\frac{5}{8}$	$-\frac{7}{8}$	1	$\frac{5(k+17)}{8}$

(c) Explain how the tableau in **Fig. 4.2** shows that the solution obtained after a second iteration is optimal. **[1]**

Almost all candidates correctly stated that the solution obtained after the second iteration was optimal due to there being no negative values in the objective row. The most common incorrect response was to say that all the values in the objective row were now positive (so ignoring the fact that some of the entries were in fact zero).

Question 4 (d)

(d) Given that the optimal value of P is 36, determine the corresponding value of each basic variable after the second iteration. **[3]**

Most candidates were given the first 2 marks in this part for correctly finding the value of *k* (which, if correct, was 63) and using this to find at least one of the three basic variables. Several candidates were not given the final mark as they either found the values of *x*, *y* and *z* only (so possibly not reading the question carefully) or instead found the values of all variables without making it clear which were the basic ones.

Question 5 (a)

5 The objective function for a LP formulation to find the shortest path from A to G in a network with seven vertices, A, B,…, G is given below.

Minimise 5AB + 13AC + 8AD + 6BC + 6CB + 14BF + 14FB + 12BE + 12EB + 6DE + 6ED + 6CF + 6FC + 2EF + 2FE + 22DG + 11EG + 7FG

(a) Explain why both BC and CB appear in the objective function. **[1]**

Most candidates correctly stated that both BC and CB appeared in the objective function, to allow for the possibility that the shortest path from A to G could go from B directly to C or from C directly to B (the edge BC is undirected).

Question 5 (b)

One of the constraints of the LP formulation is

AD + ED − DE − DG = 0

(b) Explain the purpose of this constraint in the LP formulation. **[1]**

It was rare for this mark to be given. Most candidates incorrectly argued that this constraint was due to the fact that 'the flow into D must be equal to the flow out of D' even though the question had nothing to do with the flow through a network. Only the most successful responses were able to articulate that this constraint was to make sure that if the shortest path passes through D, then the path can enter D via A or E and leave via E or G (which accounts for all the arcs incident to D).

Question 5 (c)

(c) Using the vertices given in the Printed Answer Booklet draw a network with **12** arcs to represent the information given in the above objective function. **[2]**

This part was answered extremely well with nearly all candidates adding all the required arcs to the vertices provided in the Printed Answer Booklet, and including the correct corresponding weights too.

Question 5 (d) (i)

(d) **(i)** Apply Dijkstra's algorithm to the network drawn in part **(c)**, to find the weight of the shortest path from A to G. **[5]**

Many candidates were given full marks in this part. Centres should note that it is vital when applying Dijkstra's algorithm that candidates show all the working values at each node. In this case, examiners had to see the correct working values of 13 and 11 at C to indicate that the algorithm was initially being applied correctly. Candidates also need to make sure that the order of labelling is correct (a few labelled two different nodes with the same label). Some candidates correctly applied the algorithm, but then did not state the shortest path from A to G.

Assessment for learning

The Printed Answer Booklet provided the standard key for the completion of the boxes at each node together with a specific instruction to candidates that they should not cross out their temporary labels. Several candidates still did so. It is vital that examiners can read all the values at each node (so that they can check that the algorithm has been applied correctly) and it is strongly recommended that centres emphasise that no temporary labels are crossed out in classroom practice.

Question 5 (d) (ii)

(ii) Write down the shortest path from A to G. **[1]**

Most candidates correctly stated the shortest path from A to G as ADEFG.

Question 5 (e)

(e) A computer takes 0.014 seconds to solve a shortest path problem on a network with seven vertices using Dijkstra's algorithm. Approximately how long, in seconds, will it take the same computer to solve a shortest path problem on a network with seven hundred vertices using Dijkstra's algorithm? **[2]**

Many candidates recognised that Dijkstra's algorithm has quadratic complexity (so they were given at least 1 mark in this part); of those that recognised this, most then went on to correctly find the required time.

Question 5 (f) (i)

(f) **(i)** Apply Prim's algorithm, starting at A, to find the minimum spanning tree for the network drawn in part **(c)**. You must, as part of your solution, state the order in which the arcs were included in the tree. **[2]**

This part was answered extremely well with most correctly stating all six arcs (in the correct order) that formed the required minimum spanning tree (MST). Occasionally, candidates only stated the nodes (so were given only 1 mark) or simply drew the MST (which, as it was unclear whether or not a correct (or any) algorithm had been applied, were given no marks). Although rare, some candidates applied Kruskal rather than the required Prim's algorithm to find the MST.

Question 5 (f) (ii)

(ii) State the weight of the minimum spanning tree. **[1]**

Almost all candidates who found the correct MST in Question 5 (f) (i) went on to correctly find the correct corresponding weight.

Question 6 (a)

6 Each year, to raise money for charity, Henry makes and sells three different sizes of T-shirt, small, medium, and large.

This year, the number of T-shirts that Henry plans to make are subject to the following constraints.

Each small T-shirt requires 25 minutes to make, each medium T-shirt requires 40 minutes to make, and each large T-shirt requires 60 minutes to make.

The total time that Henry can spend making all the T-shirts must not exceed 100 hours.

From his experience in previous years, Henry decides to make at most 50 medium T-shirts and at least 100 T-shirts in total.

Furthermore, Henry decides to make small and large T-shirts in the ratio of 2 small T-shirts for every 5 large T-shirts.

Henry plans on selling each small T-shirt for £5, each medium T-shirt for £6 and each large T-shirt for £8. Henry wants to maximise the total income from the sale of the T-shirts. Total income is the money made from selling T-shirts, ignoring the cost of materials.

Let x, y and z represent the number of small T-shirts, medium T-shirts, and large T-shirts respectively that Henry makes.

(a) By representing the feasible region for x and y graphically, determine how many of each size of T-shirt Henry should make to maximise the total income from selling the T-shirts. You must show all your working. **[10]**

As expected, the responses to this part were mixed. Most candidates were given some marks for dealing with some of the constraints correctly. Many candidates stated that:

$25x + 40y + 60z \leq 6000$ (time of production constraint)

$y \leq 50$ ('at most 50 medium' constraint)

$x + y + z \geq 100$ ('at least 100' constraint)

together with the correct objective $(5x + 6y + 8z)$. However, it was the constraint that 'for every 2 small T-shirts made, 5 large T-shirts would also be made' that candidates found the most difficult. Candidates were almost equally split between giving the correct response of $5x = 2z$ compared to the incorrect response of $5z = 2x$. Of those that did have the correct constraints, most used the relationship involving *x* and *z* to eliminate *z* and then proceeded to set up the correct 2D problem in *x* and *y* only. However, only the most successful responses (who had the correct feasible region) realised that the optimal solution to this ILP was some distance from the optimal lattice point (in this case the optimal lattice point was at (22.9, 50) while the optimal solution was at (24, 45)).

Exemplar 1

6(a)

time:

$25x + 40y + 60z \leq 6000$ $\quad 5x + 8y + 12z \leq 1200$

medium:

$y < 50$

total shirts:

$x + y + z \geq 100$

ratio:

$2x = 5z$, $5z = 2x$ $\quad x = \frac{5}{2}z$ $\quad z = \frac{2}{5}x$

maximise:

$5x + 6y + 8z$

$25x + 40y + 24x \leq 6000$ $\quad y \leq 50$

$49x + 40y \leq 6000$ $\quad 40y \leq -49x + 6000$

$y \leq \frac{-49}{40}x + 150$

$x + y + \frac{2}{5}x \geq 100$

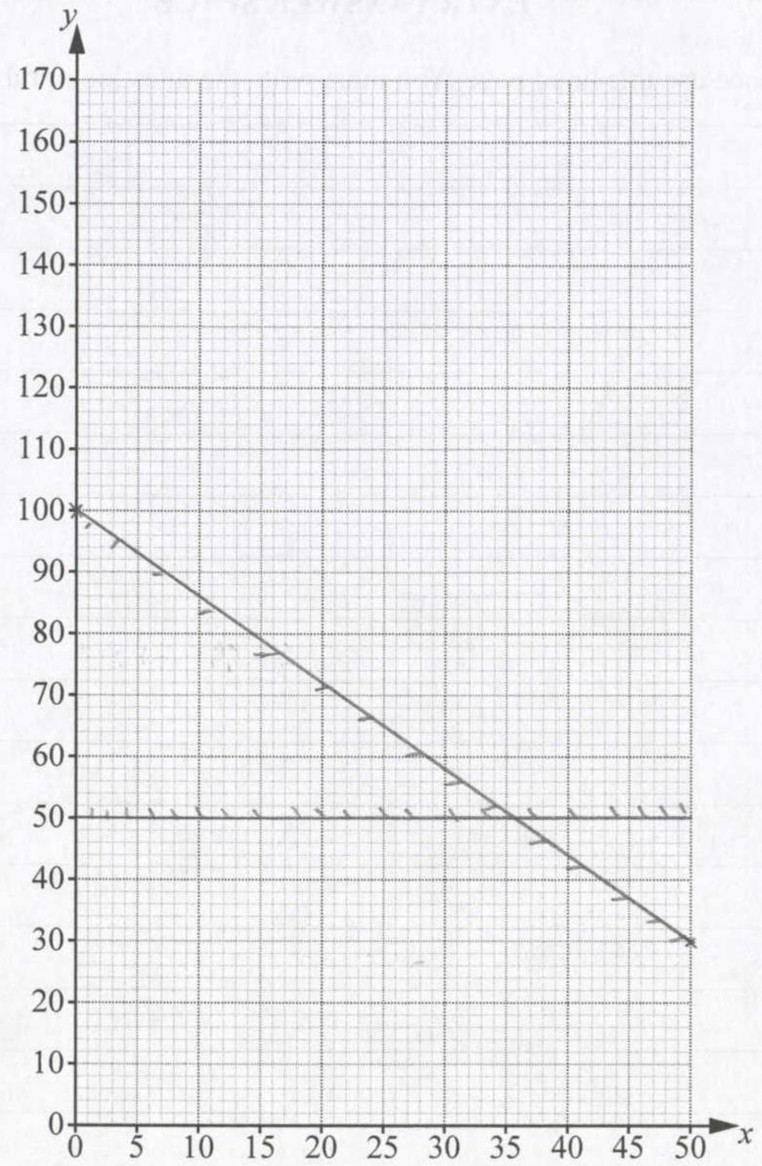
$\frac{7}{5}x + y \geq 100$

$7x + 5y \geq 500$ $\quad 5y \geq -7x + 500$

$y \geq -\frac{7}{5}x + 100$

$\frac{5}{12}x + \frac{2}{3}y + z \leq 100$

6(a) **(continued)**

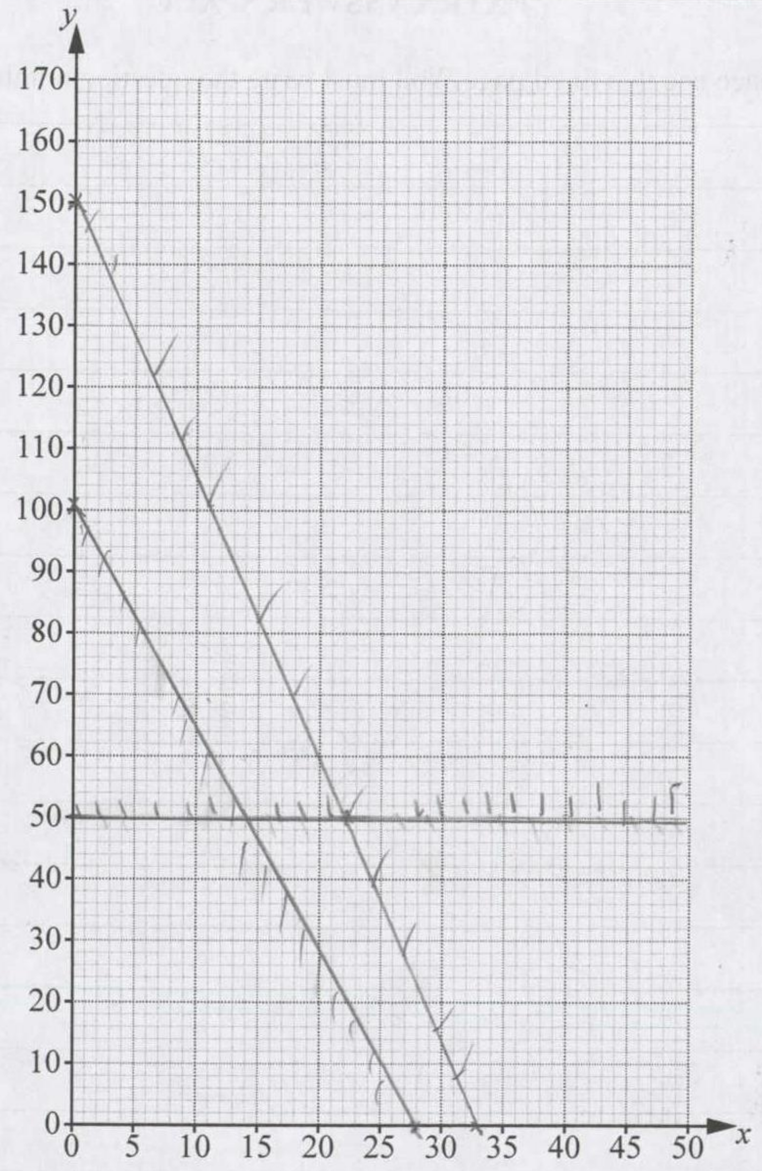


Exemplar 1 is a response given 5 of the 10 marks available for obtaining: three of the correct constraints, the correct objective function and for illustrating one of the constraints correctly. The common error of having $2x = 5z$ means that very little further progress could be made.

Exemplar 2

6(a)	x = Small y = medium z = large $25x + 40y + 60z \leq 6000$ $y \leq 50$ $x + y + z \geq 100$ ratio $x : z$ 2 : 5 $5x = 2z$ $\frac{5}{2}x = z$ $25x + 40y + 60 \times \frac{5}{2}x \leq 6000$ $y \leq 50$, $x, y, z \geq 0$ $\frac{5}{2}x \geq 0$ $x + y + \frac{5}{2}x \geq 100$ $175x + 40y \leq 6000$ $35x + 8y \leq 1200$ $y \leq 50$ $\frac{7}{2}x + y \geq 100$ maximise $5x + 6y + 8z$ $5x + 6y + 8 \times \frac{5}{2}z$ maximise $25x + 6y$

6(a) **(continued)**



(14.286, 50) = 657
(22.9, 50) = 872.5

Exemplar 2 is a response given 9 of the 10 marks available. The candidate states all constraints correctly, reduces the problem to the correct 2D one, draws the correct feasible region (FR) and attempts to find the correct optimal solution by testing two points in the FR. They are not given the final mark because they do not obtain the correct final solution to the problem.

Question 6 (b)

(b) Using the solution found in part **(a)**, find the maximum possible total income. **[1]**

As this part required the correct optimal solution from Question 6 (a), it was very rare for this mark to be given.

Question 6 (c)

(c) Give a reason why the total income found in part **(b)** may not be achieved. **[1]**

This question was answered extremely well, with most candidates giving a correct reason for why the total income found may not be achieved. Of these correct reasons, the most common one was to state that not all the T-shirts would necessarily be sold.

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