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Examiners' report

FURTHER MATHEMATICS B (MEI)

H635

For first teaching in 2017

Y411/01 Summer 2024 series

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y411/01 series overview

Most candidates were able to demonstrate sound knowledge and competence over a range of topics, notably in Questions 1 (forces), 2 (dimensional analysis), 4 (collisions) and 5 (work and energy). Questions 3 (equilibrium) and 6 (centres of mass) proved to be challenging for the majority of candidates. The marks were slightly higher than last year, with around half the candidates scoring 32 marks or more (out of 60). There were some very good scripts, with about 5% of the candidates scoring marks in the 50s.

Assessment for learning

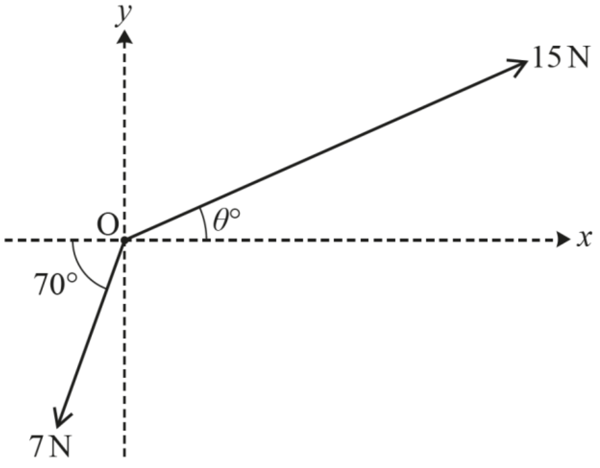
Candidates should be encouraged to set out their working clearly and to explain what they are doing. For example, 'taking moments about A', 'resolving perpendicular to the beam' or 'by the work-energy principle'.

This is particularly important when asked to 'show' or 'prove' an answer given in the question.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• produced substantially correct responses to most parts of Questions 1, 2, 4 and 5 • attempted all parts of all the questions • set out their working clearly and in a logical order • drew diagrams • explained where their equations came from • understood the concept of a 'total contact force'.	• left several parts of questions unanswered • produced work that was disorganised and hard to follow • wrote down equations without explaining what they were doing • did not understand the concept of a 'total contact force'.

Question 1 (a)

1 Two horizontal forces of magnitudes 7 N and 15 N act at a point O.
The 15 N force acts an angle of $\theta°$ above the positive x-axis.
The 7 N force acts at an angle of 70° below the negative x-axis (see diagram).



The resultant of the two forces acts only in the positive x-direction.

(a) Calculate the value of θ. **[2]**

Most candidates answered this correctly by resolving in the *y*-direction. Some confused sine and cosine and some had a sign error in their equation.

Question 1 (b)

(b) Calculate the magnitude of the resultant of the two forces. **[2]**

Most candidates also answered this correctly, usually by resolving the forces. Sign errors were more frequent in this part, while giving the resultant force as $\sqrt{15^2 + 7^2}$ was a fairly common error. Some candidates used a triangle of forces, sometimes successfully, but often with the 7 N force drawn in the wrong direction.

Question 2 (a)

2 **(a)** Find the dimensions of energy. **[1]**

Almost all candidates answered this correctly. There were no common errors, although some candidates did not respond.

Question 2 (b)

The moment of inertia, I, of a rigid body rotating about a fixed axis is measured in kg m^2.

(b) State the dimensions of I. **[1]**

Almost every candidate answered this correctly.

Question 2 (c)

The kinetic energy, E, of a rigid body rotating about a fixed axis is given by the formula

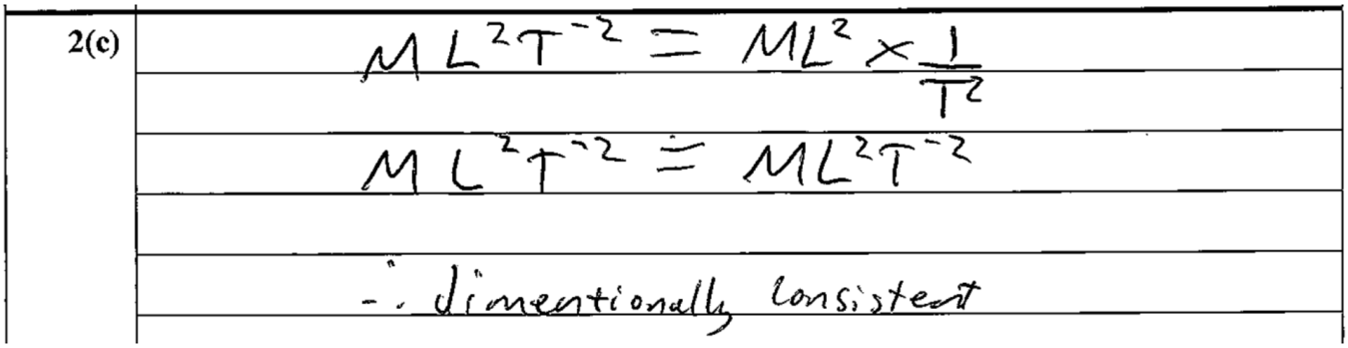
$$E = \frac{1}{2}I\omega^2,$$

where ω is the angular velocity (angle per unit time) of the rigid body.

(c) Show that the formula for E is dimensionally consistent. **[2]**

In a 'show that' question such as this, candidates are expected to set out their reasoning clearly. The most important step is that the dimensions of angular velocity are T^{-1} and ideally this should be stated explicitly. Common errors were $[\omega] = LT^{-1}$ (presumably confusing angular velocity with velocity) and $[\omega] = T^{-2}$. At the end, candidates were also required to give some indication that they had completed the task. Many candidates did produce fully convincing responses, but very many did not. The following exemplar illustrates a very common response, which did not score any marks.

Exemplar 1


2(c)

$ML^2T^{-2} = ML^2 \times \frac{1}{T^2}$

$ML^2T^{-2} = ML^2T^{-2}$

∴ dimentionally consistent

The first line, although true, is meaningless without some context and the angular velocity ω is not mentioned at all. There is a conclusion, but this will not earn a mark if the task has not been done satisfactorily. If the candidate had written the equation $E = ½I\omega^2$ above their first line, that would have explained what they were doing and they would have been given both marks.

Question 2 (d)

When a rigid body is pivoted from one of its end points and allowed to swing freely, it forms a pendulum. The period, t, of the pendulum is the time taken for it to complete one oscillation. A student conjectures the formula

$$t = k(mg)^{\alpha} r^{\beta} I^{\gamma},$$

where

- k is a dimensionless constant,
- m is the mass of the rigid body,
- g is the acceleration due to gravity,
- r is the distance between the pivot point and the rigid body's centre of mass.

(d) Use dimensional analysis to find the values of α, β and γ. **[4]**

The use of dimensional analysis to find indices in an equation appeared to be very well understood and most candidates scored full marks in this part. Some had the dimensions of *g* and *r* wrong (usually as MT^{-2} and M respectively, presumably confusing units with dimensions). Sometimes *β* and *γ* were confused when solving the equations.

Question 2 (e)

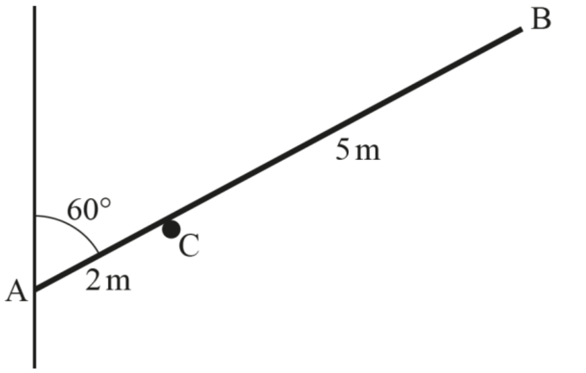
The moment of inertia of a thin uniform rigid rod of mass 1.5 kg and length 0.8 m, rotating about one of its endpoints, is $0.32\,\text{kg m}^2$. The student suspends such a rod from one of its endpoints and allows it to swing freely. The student measures the period of this pendulum and finds that it is 1.47 seconds.

(e) Using the formula conjectured by the student, determine the value of k. **[3]**

Most candidates substituted the values into the equation using their values of *α*, *β* and *γ* and obtained a value for *k*, however many used *r* = 0.8 instead of *r* = 0.4.

Question 3 (a)

3 The diagram shows a uniform beam AB, of weight 80 N and length 7 m, resting in equilibrium in a vertical plane. The end A is in contact with a rough vertical wall, and the angle between the beam and the upward vertical is 60°. The beam is supported by a smooth peg at a point C, where AC = 2 m.



(a) Complete the diagram in the **Printed Answer Booklet** to show all the forces acting on the beam. **[2]**

The weight acting at the centre of the beam was almost always shown correctly. The contact force at C was quite often shown as acting vertically upwards instead of perpendicular to the beam. At A there should be a frictional force (which could be shown acting upwards or downwards) and a normal reaction (which was sometimes omitted). Some candidates showed extra forces acting at A, such as a force in the direction of the beam AB.

Candidates who had an incorrect or incomplete diagram rarely scored many marks in the remainder of this question.

Question 3 (b) (i)

(b) **(i)** Show that the magnitude of the frictional force exerted on the beam by the wall is 25 N. **[3]**

Some candidates answered this efficiently, taking moments about A to find the reaction at C, then resolving vertically to find the frictional force. This is shown in the following exemplar.

Many candidates attempted other methods, such as resolving parallel and perpendicular to the beam and taking moments about C; these equations should include both the frictional force and the normal reaction at A, but one of these forces was frequently omitted. A few did complete this work successfully, sometimes after lengthy manipulation. Taking moments about C and resolving parallel to the beam produces an elegant solution, as it avoids the reaction at C and gives two simultaneous equations for the frictional force and the normal reaction at A.

Some candidates who had shown the frictional force acting upwards correctly achieved *F* = -25, but then stopped (or tried to adjust their working to avoid the minus sign). To gain full marks they needed to say that the magnitude is 25 N, or the frictional force is 25 N downwards.

Exemplar 2

3(b)(i)	resolve horizontal. moments about A
	$S - R\sin 30 = 0$ $\quad 80\sin 60 \times 3.5 = 2R$
	Resolve vert.
	$80 + Fr = R\cos 30$ $\quad \therefore \frac{80\sin 60 \times 3.5}{2} = R.$
	sub in
	$80 + Fr = \frac{80\sin 60 \times 3.5}{2} \times \cos 30$
	$Fr = 105 - 80$
	$Fr = 25$

This candidate offers an efficient solution, but also has explained very clearly what they are doing at each step.

Question 3 (b) (ii)

(ii) Hence determine the magnitude of the total contact force exerted on the beam by the wall. **[3]**

The normal reaction at A was quite often found correctly, by resolving horizontally after finding the contact force at C in part (b) (i), or by an alternative method. However, many attempts ended here.

Candidates who had a value for the normal reaction and understood the concept of the total contact force were usually able to calculate its magnitude.

Common misconception

?

Very many candidates did not recognise that the 'total contact force' is the resultant of the frictional force and the normal reaction. Instead, they usually took it to mean the normal reaction. These candidates could not make any further progress in part (b)(ii) and could not earn any marks in part (c) or part (e).

Question 3 (c)

(c) Determine the direction of the total contact force exerted on the beam by the wall. **[2]**

Those who had calculated the magnitude could usually find the direction as well.

Question 3 (d)

The coefficient of friction between the beam and the wall is μ.

(d) Find the range of possible values for μ. **[2]**

Candidates who had a value for the normal reaction R_A usually calculated the critical value $\frac{25}{R_A}$. Most responses however gave this single value, instead of the requested range of possible values.

Question 3 (e)

(e) Explain how your answer to part **(b)(ii)** would change if the peg were situated closer to A but the angle between the beam and the upward vertical remained at 60°. **[1]**

Not many candidates earned the mark for this part, but a few gave good responses. Some argued that the force at C, the frictional force and the normal reaction would all increase. Others considered moments about C, where the moment of the weight is balanced by the moment of the total contact force and that if C were moved towards A, the force at A would increase.

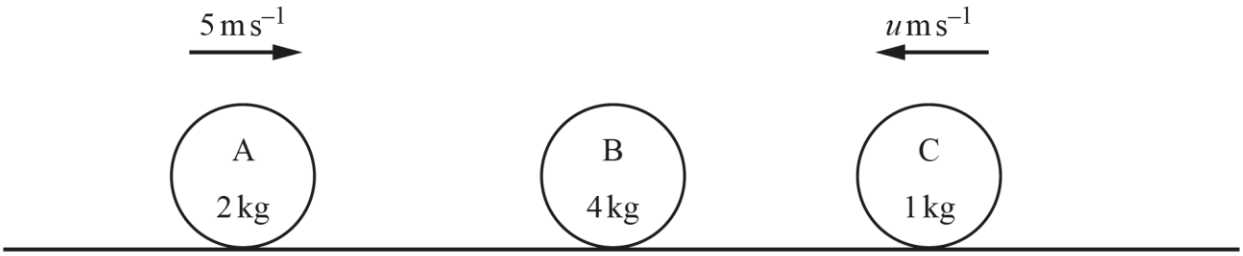
Many candidates misread this as referring to the previous part (d) and commented about μ.

Question 4 (a)

4 Three spheres A, B, and C, of equal radius are in the same straight line on a smooth horizontal surface. The masses of A, B and C are 2 kg, 4 kg and 1 kg respectively.

Initially the three spheres are at rest.

Spheres A and C are each given impulses so that A moves towards B with speed $5\,\text{m s}^{-1}$ and C moves towards B with speed $u\,\text{m s}^{-1}$ as shown in the diagram below.



The coefficient of restitution between A and B is $\frac{4}{5}$.
It is given that the first collision occurs between A and B.

(a) State how you can tell from the information given above that kinetic energy is lost when A collides with B. **[1]**

Most candidates correctly stated that this was because the coefficient of restitution is less than one (or not equal to one).

Question 4 (b)

(b) Show that the combined kinetic energy of A and B decreases by 24% during their collision. **[5]**

Most candidates set up the momentum and restitution equations correctly and very many scored full marks in this part. There were no common errors.

Question 4 (c)

Sphere B next collides with C. The coefficient of restitution between B and C is $\frac{2}{3}$.

(c) Given that a third collision occurs, determine the range of possible values for u. **[6]**

Analysing the second collision proved to be much more difficult than the first, with the unknown *u* and the spheres B and C moving in opposite directions. Very many candidates had sign errors in their equations. Most made some attempt to compare the velocities of A and B to obtain the condition for a third collision to occur, but not very many achieved the correct answer. Some compared the velocities of B and C (which have just collided) instead of, or as well as, A and B.

Some candidates found the critical value of *u* by assuming, from the outset, that the final velocity of B is equal to the velocity of A ($1\,\text{ms}^{-1}$ to the left). They could then conclude that a third collision will occur if *u* exceeds the critical value; this was a perfectly acceptable method.

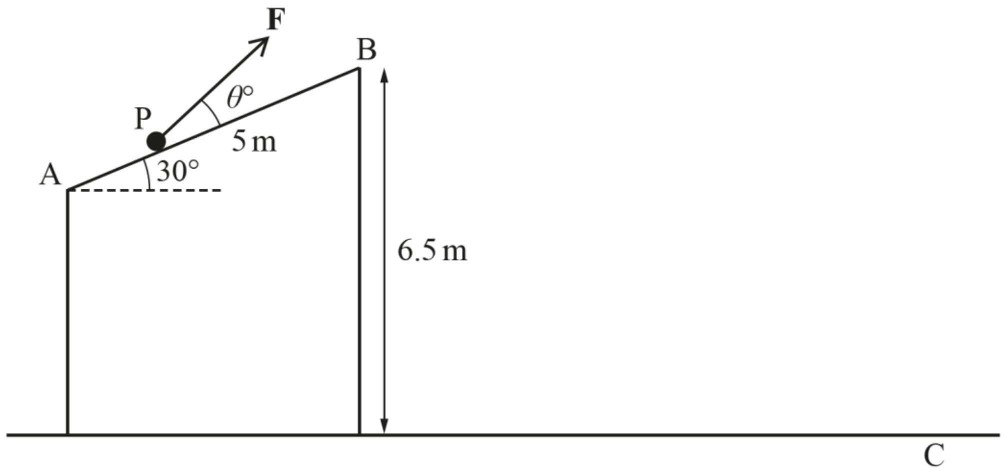
Question 4 (d)

(d) State **one** limitation of the model used in this question. **[1]**

Most candidates gave a suitable limitation, usually referring to resistances that are not accounted for.

Question 5 (a)

5 In the diagram below, points A, B and C lie in the same vertical plane. The slope AB is inclined at an angle of 30° to the horizontal and AB = 5 m. The point B is a vertical distance of 6.5 m above horizontal ground. The point C lies on the horizontal ground.



Starting at A, a particle P, of mass m kg, moves along the slope towards B, under the action of a constant force **F**. The force **F** has a magnitude of 50 N and acts at an angle of $\theta°$ to AB in the same vertical plane as A and B. When P reaches B, **F** is removed, and P moves under gravity landing at C.

It is given that

- the speed of P at A is $3\,\text{m}\,\text{s}^{-1}$,
- the speed of P at B is $6\,\text{m}\,\text{s}^{-1}$,
- the speed of P at C is $12\,\text{m}\,\text{s}^{-1}$,
- 58 J of work is done against non-gravitational resistances as P moves from A to B,
- 42 J of work is done against non-gravitational resistances as P moves from B to C.

(a) By considering the motion from B to C, show that $m = 4.33$ correct to **3** significant figures. **[3]**

Most candidates understood how to apply the work-energy principle to this section of the motion and very many were able to obtain the given answer. Sometimes the reasoning shown was not clear enough to earn the marks, and quite often there were sign errors in the working.

Question 5 (b)

(b) By considering the motion from A to B, determine the value of θ. **[4]**

Candidates needed to consider the work done by the force **F**, the work done against the resistances, the change in kinetic energy and the change in potential energy. When putting these into an equation, many omitted one of the terms (usually the 58 J of work done against the resistances, but sometimes the potential energy) and many made sign errors. Some used the component of the force 50 cos*θ* instead of the work done by the force (250 cos*θ*). Nevertheless, a good number did obtain the correct value of *θ*.

Some candidates calculated the resistive force as $\frac{58}{5}$, then used a constant acceleration formula to find the acceleration and then applied Newton's second law. This is not a valid method, as it assumes that the resistive force (and hence also the acceleration) is constant and this is not necessarily the case. Although marks could still be earned, full marks were not earned even if the correct value of *θ* was obtained.

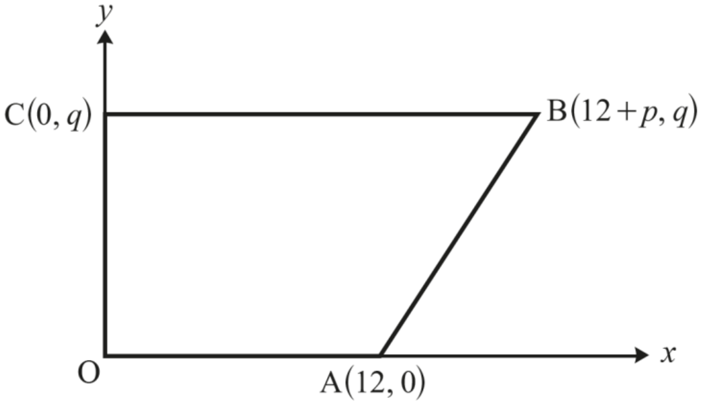
Question 5 (c)

(c) Calculate the power of **F** at the instant that P reaches B. **[2]**

The power was often found correctly by multiplying the component of **F** (50 cos*θ*) by the speed of P at B (6). A very common incorrect response was 300 W.

Question 6 (a)

6 A uniform lamina OABC is in the shape of a trapezium where O is the origin of the coordinate system in which the points A, B and C have coordinates (12, 0), $(12+p, q)$ and $(0, q)$ respectively.



(a) Determine, in terms of p and q, the coordinates of the centre of mass of OABC. **[4]**

Most candidates considered the lamina as a rectangle plus a right-angled triangle and demonstrated that they understood the method for finding the centre of mass of a combined object. However, errors in the centre of mass of the triangle (such as y-coordinate $\frac{q}{3}$ or $\frac{q}{2}$ instead of $\frac{2q}{3}$) and the complicated algebraic manipulation involving the two unknowns p and q, meant that not very many achieved correct expressions for $\bar{x}$ and $\bar{y}$.

Some candidates considered the system as a framework of four uniform rods. Some gave the response as $\bar{x} = 6 + \frac{p}{4}$, $\bar{y} = \frac{q}{2}$, which is obtained by averaging the coordinates of the four vertices.

Question 6 (b)

The point D has coordinates $(7.6, q)$. When OABC is suspended from D, the lamina hangs in equilibrium with BC horizontal.

(b) Determine the value of p. **[3]**

Most candidates deduced that $\bar{x} = 7.6$ and those with a correct expression for $\bar{x}$ in part (a) very often obtained the correct value of p. Many however had an expression for $\bar{x}$ that depended on both p and q and so could not progress beyond stating that $\bar{x} = 7.6$.

Question 6 (c)

When OABC is suspended from C, the lamina hangs in equilibrium with BC at an angle of 35° to the downward vertical.

(c) Determine the value of q, giving your answer correct to **3** significant figures. **[3]**

Candidates who had successfully answered parts (a) and (b) usually identified 35° as the angle CBG and often found the value of *q* correctly. Many candidates were unable to answer this due to previous errors and this part was frequently omitted.

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